On the Second-Order Statistics of a New Simple Model for Land Mobile Satellite Channels

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***Abstract* -A new simple model has recently been proposed for land mobile satellite channels. In this paper we study the second-order statistics of the new model. First we develop a characteristic function-based approach for calculating the level crossing rate of the new model in diversity systems, where the branches can have different fading statistics. Then we compare the level crossing rate and the average fade duration of the new model with measured data. Similar to the first-order statistics, our results show that the second-order statistics of the new model can be expressed in compact closed forms, convenient for both mathematical analysis and numerical calculations.**

I. Introduction

In mobile satellite channels, fading degrades the performance of communications systems significantly and it is therefore vital to characterize the signal envelope fluctuations in terms of the fading statistics. The level crossing rate (LCR), how often the envelope crosses a certain threshold, and the average fade duration (AFD), how long the envelope stays below a given threshold, are two important second-order statistics of fading channels, which, for example, carry useful information about the burst error statistics [1]. Therefore, for different system engineering issues such as choosing the frame length for coded packetized systems, designing interleaved or non-interleaved concatenated coding methods [1], optimizing the interleaver size, choosing the buffer depth for adaptive modulation schemes [2], throughput estimation of communication protocols [3], etc., we need an exact method for calculating the LCR and AFD of the fading model of interest. In particular, the method should also work for diversity systems, which have proven to be useful in combating the deleterious effects of fading.

Among the available fading models for satellite channels, the shadowed Rice model is of particular interest. In this model, the amplitude of the line-of-sight (LOS) component is reasonably assumed to be random due to the LOS path obstructions by buildings, trees, etc. In Loo's model [4], the LOS amplitude is characterized by the lognormal distribution, while in a recently proposed model by the authors [5], the amplitude of the LOS follows the Nakagami distribution. In [5], the first-order statistics of the new model are analyzed. Interestingly, the first-order statistics have shown very good fit to the measured satellite channel data. Moreover, they have simple compact closed forms, very convenient for mathematical analysis and systems performance prediction. In this paper, we focus on the second-order statistics of the newly proposed model, with application to the calculation of the LCR and AFD of diversity systems.

This paper is organized as follows. In Section II we develop a characteristic function (CF)-based approach [6] for calculating the LCR of a maximal ratio combiner (MRC) in a shadowed Rice fading channel with arbitrary power spectrum. The branches of the MRC are assumed to be independent, though with different fading statistics. Such a general scenario has practical importance and is not just of theoretical interest. For example, in the application of spread spectrum techniques such as direct sequence-code division multiple access (DS-CDMA) to satellite systems, to increase capacity, fading resistance, and security [7], the first arriving path of the wideband satellite channel can be modeled by a shadowed Rice distribution, while the remaining paths may be modeled by Rayleigh distributions with different powers [8] [9]. The traditional probability density function (PDF)-based method, employed in [4] to derive an approximate LCR expression for a non-diversity (single branch) receiver in a lognormally-shadowed Rice channel, cannot be easily applied to diversity systems. Section III is devoted to the comparison of the LCR and AFD of the new model with measured data. Concluding remarks are given in Section IV.

II. Level Crossing Rate of the Model in Diversity Systems

The LCR of a signal is the average number of times per second that the signal crosses a specific level with positive slope. The signal AFD is defined as the average time that the signal remains below a certain level. Assume that the diversity combiner, an MRC, has L branches. In the presence of additive white Gaussian noise, which is independent of fading, the instantaneous signal-to-noise ratio (SNR) per bit of the ℓ th branch is given by $\gamma_\ell(t) = \gamma_b R_\ell^2(t)$. In this formula, $R_\ell(t)$ is the signal envelope of the ℓ th branch, and $\gamma_b = E_b/N_0$, where E_b is the energy per bit, and N_0 is the one-sided power spectral density of the noise. The total instantaneous SNR per bit at the output of MRC is [10]

$$\gamma(t) = \sum_{\ell=1}^{L} \gamma_\ell(t) \, . \qquad (1)$$

It is shown in [6] that the LCR of $\gamma(t)$ at the given threshold γ_{th}, $N_\gamma(\gamma_{th})$, can be written as

$$N_\gamma(\gamma_{th}) = \frac{-1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\omega_2} \frac{d}{d\omega_2} \prod_{\ell=1}^{L} \Phi_{\gamma_\ell \dot{\gamma}_\ell}(\omega_1, \omega_2) e^{-j\omega_1 \gamma_{th}} d\omega_1 d\omega_2 \, , \qquad (2)$$

with $j^2 = -1$, assuming $\{\gamma_\ell(t)\}_{\ell=1}^{L}$ are stationary independent random processes. In the above expression, $\Phi_{\gamma_\ell \dot{\gamma}_\ell}(\omega_1, \omega_2)$ is the joint CF of $\gamma_\ell(t)$ and $\dot{\gamma}_\ell(t)$ at the same time instant *t*, defined by $E[\exp(j\omega_1 \gamma_\ell + j\omega_2 \dot{\gamma}_\ell)]$, where $\dot{\gamma}_\ell(t)$ is the time derivative of $\gamma_\ell(t)$.

In [6] and for an arbitrary power spectrum, an expression is derived for $\Phi_{\gamma_\ell \dot{\gamma}_\ell}(\omega_1, \omega_2)$, assuming Rice fading model for the ℓ th branch. Needless to say, the $\Phi_{\gamma_\ell \dot{\gamma}_\ell}(\omega_1, \omega_2)$ in [6] holds for the Rayleigh fading model as well.

Let us represent the complex envelope of the new shadowed Rice model [5] in the ℓ th branch as $\Re_\ell(t) = A_\ell(t)\exp[j\alpha_\ell(t)] + Z_\ell(t)\exp[j\zeta_\ell(t)]$, where $A_\ell(t)$ and $Z_\ell(t)$ are the amplitudes of the diffuse and LOS components, following Rayleigh and Nakagami distributions, respectively. The phase $\alpha_\ell(t)$ is a uniform random process, while $\zeta_\ell(t)$ is a deterministic signal [11]. The diffuse and the LOS components are also independent. The first-order statistics of $R_\ell(t) = |\Re_\ell(t)|$ are discussed in [5]. It is shown in [12] that for the new shadowed Rice fading model, the joint CF of $\gamma_\ell(t)$ and $\dot{\gamma}_\ell(t)$ can be written in the following closed form

$$\Phi_{\gamma_\ell \dot{\gamma}_\ell}(\omega_1, \omega_2) = m_\ell^{m_\ell + \frac{1}{2}} \vartheta^{m_\ell - \frac{1}{2}} (\chi_\ell \, \Omega_\ell \, b_{\ell,0} \, \omega_2^2 + m_\ell \, \vartheta)^{-\frac{1}{2}}$$
$$\left[\left\{ 2b_{\ell,2} + \frac{\chi_\ell \, \Omega_\ell}{2(\chi_\ell \, \Omega_\ell \, b_{\ell,0} \, \omega_2^2 + m_\ell \, \vartheta)} \right\} \Omega_\ell \, \omega_2^2 - j\,\omega_1 \, \Omega_\ell + m_\ell \, \vartheta \right]^{-m_\ell}, \qquad (3)$$

in which $\vartheta = 1 + 4(b_{\ell,0} b_{\ell,2} - b_{\ell,1}^2)\omega_2^2 - j2b_{\ell,0}\omega_1$. The parameter $b_{\ell,n}$, $n = 0,1,2$, is the *n*th spectral moment of the diffuse component of the ℓ th branch, defined by $b_{\ell,n} = j^{-n} C_{\Re_\ell}^{(n)}(0)$, where $C_{\Re_\ell}(\tau)$ is the autocovariance of the complex envelope $\Re_\ell(t)$. Regarding the LOS component of the ℓ th branch, m_ℓ and Ω_ℓ are the shape and scale parameters of the Nakagami distribution, i.e., $m_\ell = (E[Z_\ell^2])^2 / Var[Z_\ell^2]$ and $\Omega_\ell = E[Z_\ell^2]$ [10], where $Var[.]$ is the variance. The parameter χ_ℓ in (3) is the average power of the time derivative of $Z_\ell^2(t)$, defined by $\chi_\ell = |D''_{Z_\ell^2}(0)|$ [13], where $D_{Z_\ell^2}(\tau)$ is the normalized autocovariance of $Z_\ell^2(t)$ and prime represents differentiation with respect to τ.

As expected, $\Phi_{\gamma_\ell \dot{\gamma}_\ell}(j\eta, 0)$ matches the moment generating function $E[\exp(-\eta R_\ell^2)]$ given in [5], while as $\chi_\ell \to 0$ and $m_\ell \to \infty$ (time-invariant non-random LOS with amplitude $\sqrt{\Omega_\ell}$), $\Phi_{\gamma_\ell \dot{\gamma}_\ell}(\omega_1, \omega_2)$ in (3) converges to the corresponding result for the Rice fading model, given in [6]. By substituting (3) into (2), the LCR of an MRC in a shadowed Rice satellite channel with different branch statistics can be calculated, numerically, using a standard mathematical software such as Mathematica.

III. Comparison with Measured Data

A. LCR for a Single Antenna Receiver

In order to compare the LCR of the new model of [5] with measured data, collected by a single antenna receiver [4], now we present an LCR expression for a single branch receiver, $L = 1$, under the assumption of $\chi = 0$. This simplifying assumption comes from the empirical observations that the rate of change of the LOS component (several Hz) is significantly less than that of the diffuse component (several hundred Hz) [14]. For the case in which $Z(t) = Z$, a random variable and not a random process over the observation time slot, we have derived [12] the following PDF-based result by averaging Eqs. (4.13) and (4.14) in [15] with respect to the Nakagami distributed LOS amplitude

$$N_\gamma(\gamma_{th}) = \frac{1}{\sqrt{2\pi}\Gamma(m)} \left(\frac{2b_0 m}{2b_0 m + \Omega} \right)^m \sqrt{\frac{b_0 b_2 - b_1^2}{b_0}} \frac{\sqrt{\gamma_{th}}}{b_0} e^{\frac{-\gamma_{th}}{2b_0}}$$
$$\sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)_n (-1)^n}{n!} \left[\xi_n(\gamma_{th}) + \xi_{n+1}(\gamma_{th}) \right], \qquad (4)$$

where

$$\xi_n(\gamma_{th}) = \frac{\Gamma(n+m)}{2^n n!} \left[\frac{b_1^2}{b_0(b_0 b_2 - b_1^2)} \right]^n \left(\frac{2b_0 \Omega}{2b_0 m + \Omega} \right)^n$$
$${}_1F_1\left(n+m, n+1, \frac{\Omega \, \gamma_{th}}{2b_0(2b_0 m + \Omega)} \right). \qquad (5)$$

In the above formulas, $\Gamma(.)$ is the gamma function, $(x)_n = x(x+1)\cdots(x+n-1), (x)_0 = 1$, and ${}_1F_1(.,.,.)$ is the confluent hypergeometric function [16]. Obviously, for $m \to \infty$ (non-random LOS with amplitude $\sqrt{\Omega}$), (4) simplifies to Eqs. (4.13) and (4.14) in [15]. On the other hand, in the case of $b_1 = 0$, symmetric power spectrum resulting from isotropic scattering [10], we have $\xi_n(\gamma_{th}) = 0, \, n = 1,2,...$, which simplifies (4) drastically to

$$N_\gamma(\gamma_{th}) = \frac{1}{\sqrt{2\pi}} \left(\frac{2b_0 m}{2b_0 m + \Omega} \right)^m \frac{\sqrt{b_2 \, \gamma_{th}}}{b_0} e^{\frac{-\gamma_{th}}{2b_0}}$$
$${}_1F_1\left(m, 1, \frac{\Omega \, \gamma_{th}}{2b_0(2b_0 m + \Omega)} \right). \qquad (6)$$

Using the fact that ${}_1F_1(x,1,y/x) \to I_0(2\sqrt{y})$ as $x \to \infty$ [16], where $I_k(.)$ is the modified *k*th order Bessel function of the first kind, we observe that (6) reduces to Eq. (4.8) in [15] as $m \to \infty$ (i.e., constant LOS). Under the same condition, i.e., a time-invariant LOS, a double-fold integral is given in Eq. (29) of [11], assuming a lognormally distributed LOS amplitude.

B. Discussion

Now we compare the LCR of the new model with published data in [4], assuming the LOS is a time-invariant random variable over the period of measurement ($\chi = 0$). Firs we assume isotropic scattering, which gives rise to the Clarke's correlation model $C_\Re(\tau) = b_0 J_0(2\pi f_{\max}\tau)$ [17], with $J_0(.)$ as the zero-order Bessel function of the first kind and $f_{\max}$ as the maximum Doppler-shift frequency. In this case we have $b_1 = 0$ and $b_2 = 2\pi^2 f_{\max}^2 b_0$. Using the estimated values for b_0, m, and Ω in [5], Eq. (6), normalized to $f_{\max}$, is plotted in Fig. 1 and Fig. 2 against the measured points for light and heavy shadowing conditions, respectively. To improve the fits, now we consider non-isotropic scattering, with the correlation model $C_\Re(\tau) = b_0 \, I_0([\kappa^2 - 4\pi^2 f_{\max}^2 \tau^2 + j4\pi\kappa\cos(\mu) f_{\max}\tau]^{1/2}) / I_0(\kappa)$, where $\mu \in [-\pi, \pi)$ is the mean direction of the angle of arrival (AOA) and $\kappa \geq 0$ is the width control parameter of the AOA [18]. This correlation function is a natural generalization of the Clarke's model ($\kappa = 0$). For $\kappa > 0$ we have $b_1 = b_0 \, 2\pi f_{\max} \cos(\mu) I_1(\kappa) / I_0(\kappa)$ and $b_2 = b_0 \, 2\pi^2 f_{\max}^2 [I_0(\kappa) + \cos(2\mu) I_2(\kappa)] / I_0(\kappa)$ [19]. By minimizing the squared error between (4) and the data, the parameters μ and κ are estimated and listed in Table I, together with resulting minimized squared errors (MSEs). Using these estimated values, Eq. (4), normalized to $f_{\max}$, is plotted in Fig. 1 and Fig. 2, together with the measured data, for light and heavy shadowing conditions, respectively. It is clear from Fig. 1 and Fig. 2

Table I. ESTIMATED PARAMETERS IN THE LEVEL CROSSING RATE EXPRESSIONS OF LOO'S MODEL AND THE PROPOSED MODEL

	Loo's model		The proposed model		
	ρ	MSE	μ	κ	MSE
Light shadowing	0.792	0.0685	1.55	9.96	0.0412
Heavy shadowing	0.773	0.0315	1.55	24.2	0.01882

that the introduction of the parameters μ and κ through the model $C_{\Re}(\tau) = b_0\, I_0([\kappa^2 - 4\pi^2 f_{\max}^2 \tau^2 + j4\pi\kappa\cos(\mu) f_{\max}\tau]^{1/2}) / I_0(\kappa)$ has significantly improved the fits.

The Loo's LCR expression, Eq. (25) in [4], is also plotted in Fig. 1 and Fig. 2, where the free parameter ρ is chosen to minimize the squared error between his LCR model and the data. The estimated parameters along with the MSEs are also given in Table I. According to Table I, the MSE of our model is always less than that of Loo's model. However, visual inspection of the two figures reveals that the difference between the two models is much more pronounced under the light shadowing condition.

To complete the comparison, in Fig. 3 and Fig. 4 we have plotted the AFD of the new model and the Loo's model against the measured data, with the optimized parameters taken from Table I. According to these two figures, our model provides a better fit to empirical data.

IV. CONCLUSION

Recently, a new simple model has been proposed for land mobile satellite channels. The first-order statistics of the new model have been studied previously. In this paper, the key second-order statistic of the model, i.e., the level crossing rate, is analyzed. The flexibility of the new model has allowed us to calculate the level crossing rate for the general case of a multichannel receiver with different channel statistics. Comparison of the level crossing rate and the average fade duration of the new model with published measured data has demonstrated the flexibility of the model in describing a variety of different satellite channel conditions.

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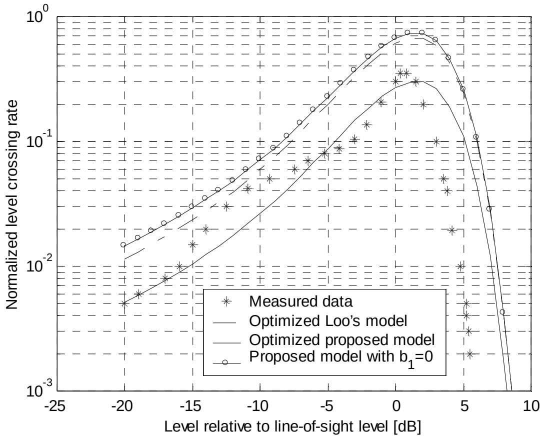

Fig. 1. Level crossing rate of the signal envelope in a land mobile satellite channel with infrequent light shadowing: Measured data [4], Loo's model [4], and the proposed model with two different types of correlation.

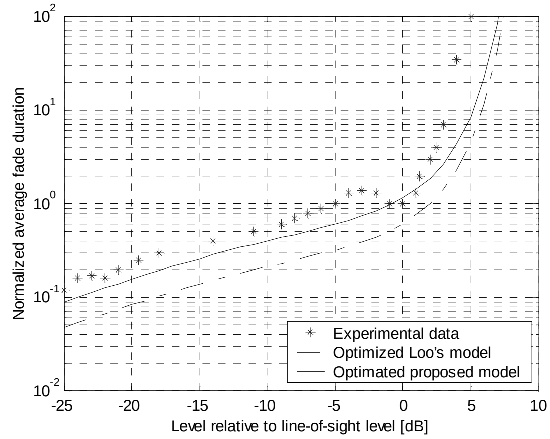

Fig. 3. Average fade duration of the signal envelope in a land mobile satellite channel with infrequent light shadowing: Measured data [4], Loo's model [4], and the proposed model with the non-isotropic scattering-based correlation.

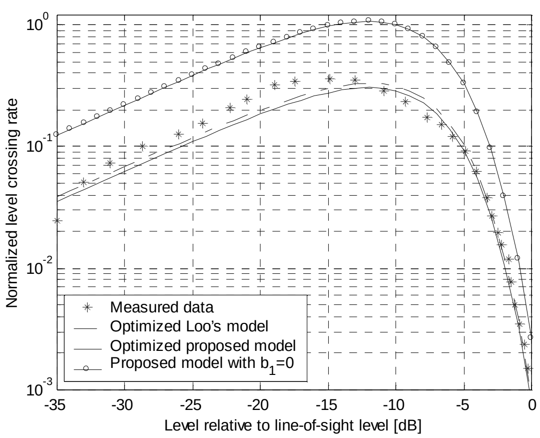

Fig. 2. Level crossing rate of the signal envelope in a land mobile satellite channel with frequent heavy shadowing: Measured data [4], Loo's model [4], and the proposed model with two different types of correlation.

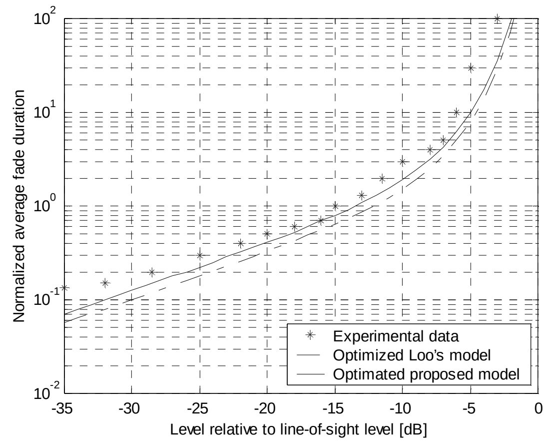

Fig. 4. Average fade duration of the signal envelope in a land mobile satellite channel with frequent heavy shadowing: Measured data [4], Loo's model [4], and the proposed model with the non-isotropic scattering-based correlation.