A Simple Alternative to the Lognormal Model of Shadow Fading in Terrestrial and Satellite Channels

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***Abstract* - The distribution of the local average power at the land mobile is widely accepted to be lognormal in both terrestrial and satellite channels. However, the mathematical form of the lognormal distribution is not convenient for analytic calculations. In this paper, we show the utility of the gamma distribution for shadow fading, in both terrestrial and satellite channels, using empirical data. Furthermore, we show that the application of the gamma model of shadow fading, in place of the lognormal model, results in closed-form and mathematically-tractable solutions for key system performance measures such as the average symbol error rate of different modulations with a variety of diversity combining techniques.**

I. Introduction

The local average power of the signal at the mobile station (MS), the instantaneous power averaged over a distance of a few wavelengths, fluctuates randomly from place to place in a mobile radio channel. This phenomenon is generally referred to as shadow fading. The probability density function (PDF) of the local average power is widely accepted to be lognormal for both terrestrial channels [1] and satellite channels [2]. However, the mathematical form of the lognormal PDF is not convenient for the analytic calculations that arise in connection with shadow fading in wireless channels. In fact, with lognormal model of shadow fading, we cannot obtain closed-form and easy-to-use expressions for many performance measures of interest such as the average symbol error rate of different modulation schemes with a variety of diversity combining techniques [3], outage probability in the presence of multiple cochannel interferers [1], and so on. The complicated closed form average bit error rate expression for the simple binary phase shift keying (BPSK) modulation with a single antenna receiver [4] is a good example of the intractability of analytic calculations with the lognormal PDF. Even the numerical integration of the expressions involving the lognormal PDF using the Hermite method of integration is not a straightforward procedure and one should carefully control the error using another technique such as the Simpson integration rule [5].

While for multipath fading numerous PDFs such as Rayleigh, Rice, Nakagami, Weibull, etc., have been examined, it appears that there are few comparative theoretical and/or experimental studies on alternatives to the lognormal PDF for shadow fading [6].

In this paper, we first demonstrate the utility of the gamma PDF for shadow fading in terrestrial and satellite channels using empirical data. For terrestrial channels, we employ the data recently collected in urban and suburban areas [7], as well as the empirical parameters given in sections 2.4 and 2.5 of [1], whereas for satellite channels we use the extensive empirical information published in [8]. We then show how the application of the gamma PDF, in conjunction with the Rayleigh model of multipath fading, results in closed-form expressions for key system performance measures in both terrestrial and satellite channels.

II. Data Analysis

Let R denote the envelope of the narrowband received signal. Let S_0 represent the local average power of the envelope, i.e., $S_0 = E[R^2 | S_0]$. For the gamma model of shadow fading we have

$$f_{S_0}(s_0) = \left(\frac{\nu}{\Omega}\right)^{\nu} \frac{s_0^{\nu-1}}{\Gamma(\nu)} \exp\left(\frac{-\nu s_0}{\Omega}\right), \qquad s_0 \geq 0 , \tag{1}$$

where $\Gamma(.)$ is the gamma function, and $\nu = \left(E[S_0]\right)^2 / Var[S_0]$ and $\Omega = E[S_0]$ are the shape and scale parameters, respectively, with $Var[.]$ as the variance. The PDF of the lognormal model is

$$f_{S_0}(s_0) = \frac{1}{\sqrt{2\pi\sigma^2}\, s_0} \exp\left[\frac{-(\ln s_0 - \mu)^2}{2\sigma^2}\right], \quad s_0 \geq 0 , \tag{2}$$

where $\mu = E[\ln S_0]$, $\sigma^2 = Var[\ln S_0]$, and $\ln(.)$ is the logarithm to the base e. In addition to the references given in [6], some other physical/statistical explanations for the gamma distribution of S_0 can be inferred from [9], [10], and [11]. The interesting property of the gamma distribution discussed in [12] may open up another door to the justification of this model for shadow fading.

A. Terrestrial Channels

A typical envelope record, $R(t)$, out of twelve terrestrial channel measurements [7] is shown in Fig. 1[1]. The square root of the running mean of the envelope squared, $\sqrt{S_0(t)}$, is also shown on the same graph. The length of the sliding temporal window was 1 second, equivalent to a spatial window of length 20λ, with λ as the wavelength of a 910 MHz carrier. As is discussed in [16], the length of the sliding window has little effect of the histogram of $S_0(t)$ (18λ and 26λ are compared in [16] at 836 MHz).

In order to compare the empirical distribution of $S_0(t)$ with the gamma and lognormal distributions, we resort to statistical goodness-of-fit tests. Since the performance of any goodness-of-fit test can be seriously affected by dependent samples [17][2], we first decimated each record of $S_0(t)$ such that the correlation between any two adjacent samples was reduced to 0.5. The number of

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[1] More details about the collected data, the locations, and other statistical characteristics of the data can be found in [13], [14], and [15].

[2] Among the huge number of papers on the statistical analysis of the fading channel data collected at different propagation environments, only few papers such as [18] and [19], have considered this effect.

(approximately) uncorrelated samples varies between 8 and 22 over all the twelve records.

The Kolmogorov-Smirnov (KS) and Pearson's chi-square (PCS) tests are two common tests that have been widely used in studying the goodness of fit of a variety of fading distributions to channel measurements. In most applications, however, these two tests have been used incorrectly, without noticing the underlying assumptions behind the tests, which in turn may lead to erroneous conclusions. Two Examples of the incorrect use of the KS and PCS tests can be found in [20] and [21], respectively. Some common pitfalls regarding the KS and PCS tests are discussed in [22] and [23] [24], respectively. These pitfalls are generally overlooked in many elementary and intermediate statistical textbooks.

According to the nature of $S_0(t)$, KS test is a better choice as it is directly applicable to ungrouped continuous data, while the inevitable grouping of the data in the PCS test, which can be done in many different ways, discards some information [25]. However, since the power of the KS test (the probability of accepting the alternative hypothesis when the alternative hypothesis is true) is small [26], we use the more powerful Cramer-von Mises (CV) test [26]. The CV statistic is nothing but the integrated squared error between the empirical and theoretical distribution functions.

Using the uncorrelated sets of $S_0(t)$ samples, we estimated the parameters of the gamma and lognormal distributions via the method of maximum likelihood. We used the percentage points given in Table 4.21 of [26] for the CV statistics in testing the gamma distribution. For testing the lognormal distribution, we used the percentage points given in Table 4.7 of [26] for the modified CV statistics. The results of the test for all the twelve sets of data, at the significance level of 0.05, are listed in Table I, in terms of the rejection or acceptance of the null hypothesis. According to this table, there seems to be no preference in choosing between the gamma PDF and the lognormal PDF for our data sets.

As a point of caution, notice that prior to the goodness-of-fit test, the unknown parameters of the gamma and lognormal distributions are estimated from the data. Hence, the values of the CV statistic, or any other test statistic, cannot be directly compared to decide which model fits better (see section 4.19 in p. 180 of [26]). In fact, for a given significance level, say 0.05, the associated *p*-values are suitable indicators of a better fit and subsequently, a better model. Apparently, this point has been generally ignored in most of the propagation studies. For example, in [27] and [28], only the value of the CV statistic is used to select the best distribution.

To compare the gamma and lognormal distributions for other sets of terrestrial measurements, we have used the empirical path loss models and estimated shadow standard deviations, to obtain some practical values for μ and σ, respectively. Specifically, we have used the two-slope street microcell path loss model, Eq. (2.248) [1], with its parameters taken from the bottom of p. 113 in [1], and also Lee's area-to-area macrocell path loss model, Eq. (2.233) [1], with its parameters computed under the nominal conditions. The numerical values of σ for the microcell and macrocell models are taken from p. 113 and p. 99 of [1], respectively. The corresponding parameters of the gamma distribution, Ω and ν, are calculated using the relations in [29]

$$\mu = \ln(\Omega/\nu) + \Psi(\nu), \quad \sigma^2 = \Psi'(\nu), \tag{3}$$

where $\Psi(.)$ and $\Psi'(.)$ are the psi function and its derivative, respectively [30]. In Fig. 2 we have plotted the distribution functions of gamma and lognormal for four different sets of parameters. We observe that the gamma and the lognormal distributions are close enough over a wide range of S_0 values and for different propagation environments.

Interestingly, for almost all of the μ and σ values listed in Table I of [31], which correspond to the measured local average power at different locations in an extensive series of ultra-wide bandwidth propagation experiments, the plots of the gamma and lognormal distributions are almost indistinguishable.

B. Satellite Channel

For this type of channel, we have used the parameters of the lognormal distribution, estimated from a comprehensive set of data collected at many different places and elevation angles [8]. In Fig. 3 we have plotted the distribution functions of the gamma and lognormal distributions, with μ and σ taken from Table II of [8], whereas Ω and ν are computed using (3). As these figures show, the gamma and the lognormal distributions are reasonably close over the practical ranges of μ and σ. For other sets of the parameters μ and σ given in [32] and [33], which fall within the ranges of the parameters of [8], we expect close agreement between the gamma and the lognormal distributions as well.

III. System Performance Analysis

In this section we show how the application of the gamma distribution for shadow fading allows straightforward and convenient system performance analysis, with closed-form results and mathematically tractable solutions. The same system analysis procedures with the lognormal distribution most often end up with integrals that have to be solved numerically. In this paper we focus on the calculation of average symbol error rate in diversity receivers over multipath fading/shadow fading wireless channels.

Let $R(t)$ denote the received signal envelope in the fading channel. As is discussed in [3], the moment generating function (MGF) of R^2, $\mathrm{M}_{R^2}(\xi) = E[\exp(-\xi R^2)]$, $\xi \geq 0$, and the characteristic function (CF) of *R*, $\Phi_R(\omega) = E[\exp(j\omega R)]$, $j^2 = -1$, play key roles in the performance analysis of multichannel reception schemes over generalized fading channels. To show this, we note that the power at the output of an *L*-branch maximal ratio combiner (MRC) can be written as $S_{out,MRC} = \sum_{i=1}^{L} R_i^2$, while for the envelope at the output of an *L*-branch equal gain combiner (EGC) we have $R_{out,EGC} = \sum_{i=1}^{L} R_i$ [3]. If the *L* branches are independent, then the MGF of $S_{out,MRC}$ and the CF of $R_{out,EGC}$, which we need for calculating the average symbol error rate of MRC and EGC, respectively [3], can be easily obtained by multiplying *L* MGFs and CFs, respectively. The function $\mathrm{M}_{R^2}(\xi)$, also named as the image function of *R*, is of significant importance in the performance analysis of mobile radio networks as well [34].

If we assume that multipath fading follows the Rayleigh distribution, then we obtain the composite Rayleigh-lognormal distribution (also known as Suzuki distribution), which is a common model for the signal envelope in both terrestrial [35] and satellite channels [8]. However, the functions $\mathrm{M}_{R^2}(\xi)$ and $\Phi_R(\omega)$ for the Rayleigh-lognormal model can be expressed only in terms of definite integrals, which can not be expressed and/or simplified in terms of known mathematical functions. To show this, note that conditioned on S_0, the Rayleigh PDF of multipath fading is given by $f_R(r|S_0) = (2r/S_0)\exp(-r^2/S_0)$. It is easy to verify that

$E[\exp(-\xi R^2)|S_0] = (1+\xi S_0)^{-1}$, while from p. 285 of [3], it can be shown that $E[\exp(j\omega R)|S_0] = D_{-2}(-j\omega\sqrt{S_0}/\sqrt{2})\exp(-S_0\omega^2/8)$, where $D_{-2}(.)$ is the parabolic cylinder function of order –2 [36]. According to the lognormal distribution of S_0 in (2), the functions $\mathrm{M}_{R^2}(\xi)$ and $\Phi_R(\omega)$ for the Rayleigh-lognormal model are

$$\mathrm{M}_{R^2}(\xi) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_0^\infty \frac{\exp\left[-(\ln s_0 - \mu)^2/(2\sigma^2)\right]}{s_0(1+\xi s_0)} \, ds_0 , \quad (4)$$

$$\Phi_R(\omega) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_0^\infty s_0^{-1} \exp\left[-(\ln s_0 - \mu)^2/(2\sigma^2)\right] \exp(-s_0\omega^2/8) \, D_{-2}(-j\omega\sqrt{s_0}/\sqrt{2}) \, ds_0 . \quad (5)$$

Because of the special mathematical form of the lognormal PDF, the integrals in (4) and (5) cannot be further simplified.

On the other hand, if we replace the lognormal model of shadow fading with the gamma model in (1), then the PDF of *R*, $f_R(r)$, and also $\mathrm{M}_{R^2}(\xi)$ and $\Phi_R(\omega)$ can be readily expressed in terms of tabulated functions. For $f_R(r)$ we obtain the *K* distribution [29]

$$f_R(r) = E_{S_0}\left[f_R(r|S_0)\right] = \frac{4}{\Gamma(\nu)}\left(\frac{\nu}{\Omega}\right)^{\frac{\nu+1}{2}} r^\nu K_{\nu-1}\left(\sqrt{\frac{4\nu}{\Omega}}\, r\right), \quad r \geq 0 , \quad (6)$$

where $K_{\nu-1}(.)$ is the modified Bessel function of the second kind and order $\nu - 1$ [36]. Some binary error probability calculations for single branch receivers in *K*-distributed fading channels are presented in [37] and [38]. Based on the result derived in [39], one may obtain a generic error probability expression for binary signals in channels with gamma shadow fading. For a survey on the applications of the family *K* distributions in biology, optics, and radar, we refer the reader to [40].

Under the assumption of gamma shadow fading, the MGF of R^2 can be obtained by averaging $E[\exp(-\xi R^2)|S_0] = (1+\xi S_0)^{-1}$ with respect to the gamma PDF of S_0 in (1)

$$\mathrm{M}_{R^2}(\xi) = \left(\frac{\nu}{\Omega\xi}\right)^\nu \exp\left(\frac{\nu}{\Omega\xi}\right) \Gamma\left(1-\nu, \frac{\nu}{\Omega\xi}\right), \quad (7)$$

where $\Gamma(.,.)$ is the incomplete gamma function, defined as $\Gamma(a,z) = \int_z^\infty e^{-t} t^{a-1} dt$ [36]. Moreover, by calculating the average of $\exp(j\omega R)$ with respect to the *K* PDF in (6), using Eq. (6.621-3) in p. 733 of [36], the CF of *R* can be written as

$$\Phi_R(\omega) = \frac{\sqrt{\pi}(4\nu)^\nu \Gamma(2\nu)}{\Gamma(\nu)\Gamma(\nu+3/2)} \left(2\sqrt{\nu} - j\sqrt{\Omega}\omega\right)^{-2\nu} F\left(2\nu, \nu - \frac{1}{2}, \nu + \frac{3}{2}, \frac{2\sqrt{\nu} + j\sqrt{\Omega}\omega}{-2\sqrt{\nu} + j\sqrt{\Omega}\omega}\right), \quad (8)$$

where $F(.,.,.,.)$ is the Gauss hypergeometric function [36]. Notice the clean and compact forms of equations (7) and (8) for the gamma model of shadow fading, in contrast with the complex integral forms of the expressions in (4) and (5) for the lognormal distribution. The mathematical properties of $\Gamma(.,.)$ and $F(.,.,.,.)$ such as recursive relations, asymptotic expansions, etc., are extensively discussed in any standard book on special functions such as [30]. In addition, any standard mathematical software such as Mathematica has built-in procedures for efficient numerical evaluation of these functions. These facts make the gamma distribution an attractive model for shadow fading, from a performance analysis point of view.

If the *L* branches are independent, then the MGF of $S_{out,MRC}$ and the CF of $R_{out,EGC}$ can be easily obtained by multiplying *L* terms according to (7) and (8), respectively. However, there are some cases of interest in which we can obtain much simpler closed-form solutions for the MGF or the CF of the output of the combiner. For example, consider an *L*-branch diversity combiner where the multipath Rayleigh components of the branches are independent, but all the branches experience a common local average power S_0 [5]. Let $S_{out,SC} = \max(R_1^2, R_2^2, ..., R_L^2)$ denote the power at the output of a selection combiner (SC) [5]. For the gamma distributed S_0 in (1), the MGF of $S_{out,MRC}$ and $S_{out,SC}$ in such a setting can be shown to be

$$\mathrm{M}_{S_{out,MRC}}(\xi) = \left(\frac{\nu}{\Omega\xi}\right)^\nu U\left(\nu, \nu - L + 1, \frac{\nu}{\Omega\xi}\right), \quad (9)$$

$$\mathrm{M}_{S_{out,SC}}(\xi) = 1 + \nu\left(\frac{\nu}{\Omega\xi}\right)^\nu \sum_{i=1}^{L} \frac{(-1)^i i^\nu L!}{(L-i)!\, i!} \Gamma\left(-\nu, \frac{i\nu}{\Omega\xi}\right) \exp\left(\frac{i\nu}{\Omega\xi}\right), \quad (10)$$

where $U(.,.,.)$ is the Tricomi function, defined in Eq. (48:3:5) of [41] as $U(a,c,z) = \Gamma(a)^{-1}\int_0^\infty t^{a-1} e^{-zt}(1+t)^{c-a-1} dt$ (also available in Mathematica). Using the relations $\Gamma(a,z) = \exp(-z)U(1-a, 1-a, z)$ and $\Gamma(a+1,z) = a\Gamma(a,z) + z^a \exp(-z)$ [36], it is easy to verify that for $L = 1$, both (9) and (10) simplify to (7), as expected. Needless to say, the lognormal model of shadow fading for S_0 results in integral expressions, which in contrast to (9) and (10), cannot be written in terms of known mathematical functions [5].

In Fig. 4 we have plotted the average bit error rates of binary differential phase shift keying (BDPSK) and BPSK modulations with MRC and SC, $L = 3$, assuming *K* and Suzuki fading models for two satellite channels with $\sigma = 2$ dB and $\sigma = 6$ dB. The bit error rate of BDPSK and BPSK are calculated according to $\mathrm{M}_{S_{out}}(\gamma)/2$ and $\pi^{-1}\int_0^{\pi/2} \mathrm{M}_{S_{out}}(\gamma/\cos^2\eta)\, d\eta$ [5], where γ is the signal-to-noise- ratio (SNR) per branch of the unfaded link [5]. Note that $\mathrm{M}_{S_{out}}(.)$ for the *K* model is given by (9) and (10) in simple closed forms, for both combiners. For the Suzuki model, we used the numerical integration routine of Mathematica to compute $\mathrm{M}_{S_{out}}(.)$. As Fig. 4 demonstrates, all the bit error rate curves all very close over a wide range of SNRs. Notice that for large SNR and σ, the *K* model predicts a slightly higher bit error rate.

IV. Conclusion

In this paper and based on experimental data, we have demonstrated that the gamma distribution models the shadow fading in both terrestrial and satellite channels, as well as the lognormal distribution. Furthermore, we have shown that the application of the gamma model, in place of the lognormal model, introduces a lot of analytical/computational facilities in calculating the average symbol error rate of diversity receivers over wireless channels. Although not discussed in this paper due to space limitations, application of the gamma model of shadow fading to the analysis of cochannel interference in cellular systems, in place of the lognormal model [42], seems to provide significant mathematical tractability and computational convenience as well. For example, a closed-form expression is derived in [43] for the PDF of the signal-to-interference power ratio under the assumption of gamma shadow fading and no multipath fading. Close agreement between the theoretical gamma-based expression and the lognormal-based simulation verifies the applicability of the gamma model. As another example, the cutoff rate of coded systems in fading channels, which is a key factor in system analysis and design, can be expressed in terms of closed-from formulas using the gamma model of shadow fading, in place of the lognormal-based integrals [44]. It seems to us that there are many other cases where the gamma model of shadow

fading can drastically simplify the calculation of other performance measures of interest, and provide closed-form solutions which are numerically close to the results obtained via the lognormal model.

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References

[1] G. L. Stuber, *Principles of Mobile Communication*, 2nd ed., Boston, MA: Kluwer, 2001.

[2] F. Vatalaro and F. Mazzenga, "Statistical channel modeling and performance evaluation in satellite personal communications," *Int. J. Satell. Commun.*, vol. 16, pp. 249-255, 1998.

[3] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels: A Unified Approach to Performance Analysis*. New York: Wiley, 2000.

[4] D. Cygan, "Analytical evaluation of average bit error rate for the land mobile satellite channel," *Int. J. Sat. Commun.*, vol. 7, pp. 99-102, 1989.

[5] C. Tellambura, A. J. Mueller, and V. K. Bhargava, "Analysis of *M*-ary phase-shift keying with diversity reception for land-mobile satellite channels," *IEEE Trans. Vehic. Technol.*, vol. 46, pp. 910-922, 1997.

[6] A. Abdi and M. Kaveh, "On the utility of the gamma PDF in modeling shadow fading (slow fading)," in *Proc. IEEE Vehic. Technol. Conf.*, Houston, TX, 1999, pp. 2308-2312.

[7] H. Allen Barger, "Dynamic characteristics of a narrowband land mobile communication channel," *IEEE Trans. Vehic. Technol.*, vol. 47, pp. 216-224, 1998.

[8] E. Lutz, D. Cygan, M. Dippold, F. Dolainsky, and W. Papke, "The land mobile satellite communication channel – Recording, statistics, and channel model," *IEEE Trans. Vehic. Technol.*, vol. 40, pp. 375-386, 1991.

[9] D. Middleton, "New physical-statistical methods and models for clutter and reverberation: the KA-distribution and related probability structures," *IEEE J. Oceanic Eng.*, vol. 24, pp. 261-284, 1999.

[10] K. D. Ward, C. J. Baker, and S. Watts, "Maritime surveillance radar-Part 1: Radar scattering from the ocean surface," *IEE Proc. F, Radar, Signal Processing*, vol. 137, pp. 51-62, 1990.

[11] R. Barakat, "Second-order statistics of integrated intensities and of detected photoelectrons," *J. Modern Opt.*, vol. 34, pp. 91-102, 1987.

[12] C. Roberts and S. Geisser, "A necessary and sufficient condition for the square of a random variable to be gamma," *Biometrika*, vol. 53, pp. 275-278, 1966.

[13] A. Abdi, H. Allen Barger, and M. Kaveh, "A parametric model for the distribution of the angle of arrival and the associated correlation function and power spectrum at the mobile station," 2nd revision submitted to *IEEE Trans. Vehic. Technol.*, and accepted for publication in *Proc. Asilomar Conf. Signals, Systems, Computers*, Pacific Grove, CA, 2001.

[14] A. Abdi, H. A. Barger, and M. Kaveh, "Signal modeling in wireless fading channels using spherically invariant processes," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, Istanbul, Turkey, 2000, pp. 2997-3000.

[15] A. Abdi, K. Wills, H. A. Barger, M. S. Alouini, and M. Kaveh, "Comparison of the level crossing rate and average fade duration of Rayleigh, Rice, and Nakagami fading models with mobile channel data," in *Proc. IEEE Vehic. Technol. Conf.*, Boston, MA, 2000, pp. 1850-1857.

[16] W. C. Y. Lee and Y. S. Yeh, "On the estimation of the second-order statistics of lognormal fading in mobile radio environment," *IEEE Trans. Commun.*, vol. 22, pp. 869-873, 1974.

[17] L. J. Gleser and D. S. Moore, "The effect of dependence on chi-squared and empiric distribution tests of fit," *Ann. Statist.*, vol. 11, pp. 1100-1108, 1983.

[18] F. Babich and G. Lombardi, "Statistical analysis and characterization of the indoor propagation channel," *IEEE Trans. Commun.*, vol. 48, pp. 455-464, 2000.

[19] P. C. Fannin and A. Molina, "Analysis of mobile radio channel sounding measurements in inner city Dublin at 1.808 Ghz," *IEE Proc. Commun.*, vol. 143, pp. 311-316, 1996.

[20] B. Belloul, S. R. Saunders, M. A. N. Parks, and B. G. Evans, "Measurements and modelling of wideband propagation at L- and S-bands applicable to the LMS channel," *IEE Proc. Microw. Antennas Propag.*, vol. 147, pp. 116-121, 2000.

[21] A. Abdi, "Comments on "A new theoretical model for the prediction of rapid fading variations in an indoor environment"," *IEEE Trans. Vehic. Technol.*, vol. 50, pp. 331-334, 2001.

[22] H. L. Crutcher, "A note on the possible misuse of the Kolmogorov-Smirnov test," *J. Appl. Meteor.*, vol. 14, pp. 1600-1603, 1975.

[23] P. Albrecht, "On the correct use of the chi-square goodness-of-fit test," *Scand. Actuarial J.*, vol. 1, pp. 149-160, 1980.

[24] P. Albrecht, "A remark on Albrecht's "On the correct use of the chi-square goodness-of-fit test"," *Scand. Actuarial J.*, vol. 3, pp. 168-170, 1982.

[25] D. S. Moore, "Tests of chi-squared type," in *Goodness-of-Fit Techniques*. R. B. D'Agostino and M. A. Stephens, Eds., New York: Marcel Dekker, 1986, pp. 63-95.

[26] M. A. Stephens, "Tests based on EDF statistics," in *Goodness-of-Fit Techniques*. R. B. D'Agostino and M. A. Stephens, Eds., New York: Marcel Dekker, 1986, pp. 97-193

[27] M. Lecours, J. Y. Chouinard, G. Y. Delisle, and J. Roy, "Statistical modeling of the received signal envelope in a mobile radio channel," *IEEE Trans. Vehic. Technol.*, vol. 37, pp. 204-212, 1988.

[28] R. Ganesh and K. Pahlavan, "Statistical modeling and computer simulation of indoor radio channel," *IEE Proc. I, Commun., Speech, Vision*, vol. 138, pp. 153-161, 1991.

[29] A. Abdi and M. Kaveh, "*K* distribution: an appropriate substitute for Rayleigh-lognormal distribution in fading-shadowing wireless channels," *Electron. Lett.*, vol. 34, pp. 851-852, 1998.

[30] W. Magnus, F. Oberhettinger, and R. P. Soni, *Formulas and Theorems for the Special Functions of Mathematical Physics*, 3rd ed., New York: Springer, 1966.

[31] M. Z. Win and R. A. Scholtz, "On the robustness of ultra-wide bandwidth signals in dense multipath environments," *IEEE Commun. Lett.*, vol. 2, pp. 51-53, 1998.

[32] M. A. N. Parks, B. G. Evans, and G. Butt, "High elevation angle propagation results, applied to a statistical model and an enhanced empirical model," *Electron. Lett.*, vol. 29, pp. 1723-1725, 1993.

[33] A. Jahn, H. Bischl, and G. Heiss, "Channel characterization for spread spectrum satellite communications," in *Proc. IEEE Int. Symp. Spread Spectrum Techniques Applications*, Mainz, Germany, 1996, pp. 1221-1226.

[34] J. P. Linnartz, *Narrowband Land-Mobile Radio Networks*. Boston, MA: Artech House, 1993.

[35] J. D. Parsons, *The Mobile Radio Propagation Channel*. New York: Wiley, 1992.

[36] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 5th ed., A. Jeffrey, Ed., San Diego, CA: Academic, 1994.

[37] A. Abdi and M. Kaveh, "Comparison of DPSK and MSK bit error rates for *K* and Rayleigh-lognormal fading distributions," *IEEE Commun. Lett.*, vol. 4, pp. 122-124, 2000.

[38] M. Nesenbergs, "Binary error probability due to an adaptable fading model," *IEEE Trans. Commun. Syst.*, vol. 12, pp. 64-73, 1964.

[39] M. S. Alouini and M. K. Simon, "Generic form for average error probability of binary signals over fading channels," *Electron. Lett.*, vol. 34, pp. 949-950, 1998.

[40] M. C. Teich and P. Diament, "Multiply stochastic representations for *K* distributions and their Poisson transforms," *J. Opt. Soc. Am. A*, vol. 6, pp. 80-91, 1989.

[41] J. Spanier and K. B. Oldham, *An Atlas of Functions*. Washington: Hemisphere, 1987.

[42] L. C. Wang, G. L. Stuber, and C. T. Lea, "Effects of Rician fading and branch correlation on a local-mean-based macrodiversity cellular system," *IEEE Trans. Vehic. Technol.*, vol. 48, pp. 429-436, 1999.

[43] A. Sathyendran, K. W. Sowerby, and M. Shafi, "Capacity estimation for 3rd generation CDMA cellular systems," in *Proc. IEEE Vehic. Technol. Conf.*, Houston, TX, 1999, pp. 356-360.

[44] Y. T. Su and J. Y. Chen, "Cutoff rates of multichannel MFSK and DPSK signals in mobile satellite communications," *J. Select. Areas Commun.*, vol. 13, pp. 213-221, 1995.

Table I. COMPARISON OF GAMMA AND LOGNORMAL FOR SHADOW FADING DATA IN TERRESTRIAL CHANNELS USING THE CRAMER-VON MISES GOODNESS-OF-FIT TEST

Record No.	Gamma PDF	Lognormal PDF
0011	accept	accept
0012	reject	reject
0013	accept	accept
0014	accept	accept
0015	accept	accept
0016	accept	accept
0017	accept	accept
0018	accept	accept
0019	reject	reject
0020	accept	accept
0021	reject	accept
0022	accept	reject

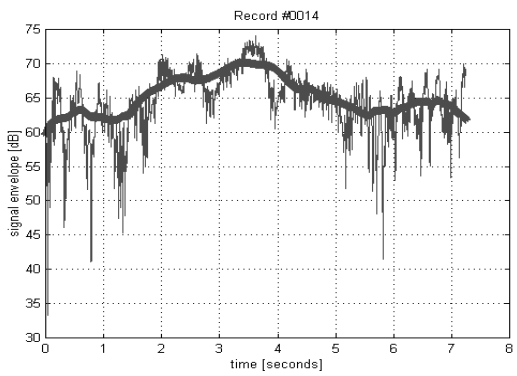

Fig. 1. The envelope, $R(t)$, of record #0014, collected in a terrestrial channel in a suburban area. The bold slowly-varying signal is the square root of the running mean of the envelope squared, $\sqrt{S_0(t)}$.

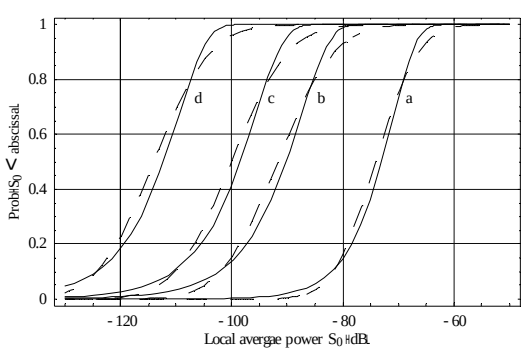

Fig. 2. Comparison of gamma and lognormal for terrestrial channels.
(a) Street microcell, $\mu = -44$ dBm , $\sigma = 6$ dB, $\Omega = 7.6\times10^{-8}$, $\nu = 0.9$.
(b) North American suburban macrocell, $\mu = -61.7$ dBm , $\sigma = 8$ dB, $\Omega = 1.8\times10^{-9}$, $\nu = 0.62$.
(c) North American urban macrocell (Philadelphia), $\mu = -70$ dBm , $\sigma = 8$ dB, $\Omega = 2.7\times10^{-10}$, $\nu = 0.62$.
(d) Japanese urban macrocell (Tokyo), $\mu = -84$ dBm , $\sigma = 8$ dB, $\Omega = 1.1\times10^{-11}$, $\nu = 0.62$.

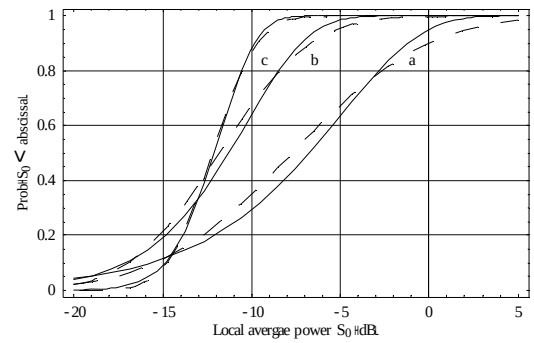

Fig. 3. Comparison of gamma and lognormal for satellite channels.
(a) Data collected in a highway with the satellite elevation angle equal to 24° , $\mu = -7.7$ dB, $\sigma = 6$ dB, $\Omega = 0.32$, $\nu = 0.9$.
(b) Data collected in a city with the satellite elevation angle equal to 18° , $\mu = -11.8$ dB, $\sigma = 4$ dB, $\Omega = 0.093$, $\nu = 1.6$.
(c) Data collected in a city with the satellite elevation angle equal to 34° , $\mu = -12.2$ dB, $\sigma = 2$ dB, $\Omega = 0.066$, $\nu = 5.2$.

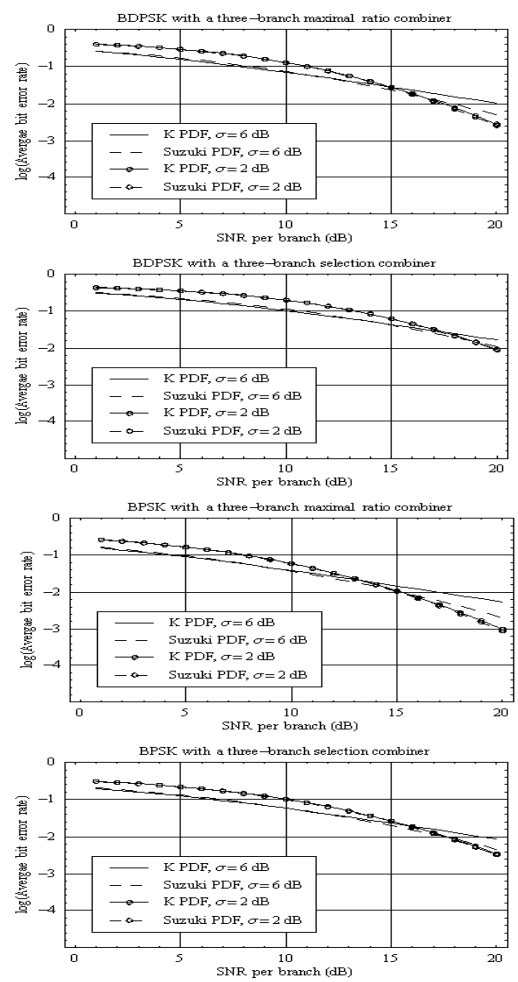

Fig. 4. Comparison of the average bit error rates of BDPSK and BPSK modulations with MRC and SC, in *K* and Suzuki fading satellite channels:
1) Data collected in a city with the satellite elevation angle equal to 34° , $\mu = -12.2$ dB, $\sigma = 2$ dB, $\Omega = 0.066$, $\nu = 5.2$,
2) Data collected in a highway with the satellite elevation angle equal to 24° , $\mu = -7.7$ dB, $\sigma = 6$ dB, $\Omega = 0.32$, $\nu = 0.9$.