Bayesian coherent and incoherent matched-field localization and detection in the ocean

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Matched-field processing is applied to source localization and detection of sound sources in the ocean. The source spectrum is included in the set of unknown parameters and is estimated in the localization/detection process. Bayesian broadband (multi-tonal) incoherent and coherent processors are developed integrating the source spectrum estimation using a Gibbs sampler and are first evaluated in source localization via point estimates and probability density functions obtained from synthetic signals. The coherent performance is superior to the incoherent one both in terms of source location estimates and density spread. The two processors are also applied to real data from the Hudson Canyon experiment. Subsequently, using ROC curves, the two processors are evaluated and compared in the task of joint detection and localization. The coherent detector/localization processor is superior to the incoherent one, especially as the number of frequencies increases. Joint detection and localization performance is evaluated with Localization-ROC curves.

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I. INTRODUCTION

Matched-field processing (MFP)\textsuperscript{1–13} has been used extensively for source localization in the ocean. It is based on full-field calculations on a set of spatially separated receivers (replicas) for multiple candidate values of the unknown source location parameters and their comparison through a measure of correlation to real pressure fields received at the same phones. The simplest MFP scheme is the Bartlett or linear processor that evaluates squared moduli of inner products between normalized replica fields and acoustic data. The processor computes an ambiguity surface with the obtained values vs. range and depth. The Bartlett MFP estimates are equivalent to Maximum Likelihood (ML) estimates in a Gaussian noise environment\textsuperscript{14}.

In source localization and detection problems, sound is typically measured at a number of frequencies. One of the important aspects is how to combine this broadband information for optimum extraction of information. In realistic cases the source spectrum is unknown. Using the approach of\textsuperscript{14}, a broadband (multi-tonal in our case) Bartlett MFP approach multiplies narrowband ambiguity surfaces. This process is termed incoherent MFP. Coherent processing has been proposed\textsuperscript{6,7,15–21} taking into account coherence among the acoustic fields at different frequencies. Coherent MFP often provides better estimates than incoherent processing; however, this largely depends on whether source information (amplitude and phase) is available. In our work, we treat source spectrum as an unknown; we estimate it in addition to source range and depth and noise variance and we compare broadband coherent and incoherent processors using this approach (in\textsuperscript{22} it was shown that narrowband processing
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with source spectrum probability density function (PDF) estimation is superior to estimation
where an ML estimate for the source spectrum is used\textsuperscript{14}). The relative performance of the
two processors is a focal point of this work. Estimating PDFs of the field spectrum (that
includes phase) at particular frequencies, a task we undertake here, is a novel element that
can facilitate deconvolution for source identification with uncertainty quantification.

In essence the two processors are Bayesian estimators that provide estimates of source
location, source spectrum, and noise variance by maximizing posterior PDFs (the first im-
plementation of Bayesian MFP was presented in\textsuperscript{4} with a broadband extension derived in\textsuperscript{23};
more Bayesian localization work was shown in\textsuperscript{24,25}). The implementation in our work is done
using Gibbs Sampling. Sequences of samples drawn from conditional PDFs provide the joint
PDF of all unknowns and marginal PDFs for all parameters separately\textsuperscript{22,26}.

Although MFP has been widely applied to the problem of source localization, it has
hardly been employed in the problem of signal detection, with some results presented in\textsuperscript{27};
coherent MFP has not been evaluated in detection at all. It is an important task, however, to
establish whether a sound source is present while also localizing it and matched-field methods
can prove useful in this. In addition to source localization, an additional contribution of this
work is detection with the two new processors and then joint detection and localization. We
compare the two processors using ROC and LROC curves and we show that the new coherent
processor performs better than the incoherent processor by a substantial amount for the case
of four frequencies (the improvement in performance is present but not so pronounced for
the case of two frequencies).
The paper is organized as follows: Section II presents a brief summary of narrowband matched field processing and how it is implemented in\textsuperscript{22}. Section III develops Bayesian broadband incoherent and coherent processors using Gibbs Sampling, extending the method of Section II. Section IV presents localization results with synthetic data and Section V presents results from real data. Section VI discusses coherent and incoherent detection and Section VII presents joint detection and localization results for the coherent and incoherent processors. Conclusions are presented in Section VIII.

II. SINGLE FREQUENCY MATCHED-FIELD PROCESSING

Let a sound source transmit a signal at frequency $f$ that is received at $L$ vertically separated hydrophones. The $L$-dimensional complex received signal $X$ can be written as

$$X = \mu G(r, z_s, z_r) + W,$$

where $G = [G_1 \ G_2 \ldots G_L]^T$ is the solution of the Helmholtz equation, called the replica vector, $\mu$ is the source spectrum, and $W = [W_1 \ W_2 \ldots W_L]^T$ is zero-mean spatially complex white Gaussian noise with covariance matrix $\Sigma$ for both real and imaginary parts, where $\Sigma = \sigma^2 I_{L \times L}$. The remaining parameters are source range $r$, source depth $z_s$, and the vector of hydrophone depths $z_r = [z_{r1} \ z_{r2} \ldots z_{rL}]^T$. We assume here that the propagation medium and the phone depths are known exactly. We consider a single vector observation $X$.

The assumption is that we have spatially independent white Gaussian noise; this is because we are considering only sensor noise. This is an approximation as there are other noise sources such as the sea surface, shipping, and biological sounds. Combination of noise
factors and the spacing between sensors could cause a colored noise environment. In such
case, the data could be pre-whitened through multiplication by a matrix containing noise
covariance structure if some prior information is available\(^{27}\) or a non-diagonal matrix \(\Sigma\) could
be employed in the modeling of the additive noise.

For source localization, the linear-Bartlett MFP approach\(^{3}\) relies on the calculation of
ambiguity surface \(P(r, z_s)\) for range \(r\) and source depth \(z_s\) values on a grid, where:

\[
P(r, z_s) = \frac{|G^*X|^2}{||G||^2}; \quad (2)
\]

\(^\star\) stands for conjugate transpose. For simplicity, we omit the arguments \(r\) and \(z_s\) of \(G\). Maximizing \(P\) provides the estimates for range and depth. This is also derived in\(^{14}\).

Spectrum \(\mu\) and variance \(\sigma^2\) do not appear in Equation 2. The equation is obtained by using
ML estimates (MLEs) computed through the following Gaussian density:

\[
p(X \mid r, z_s, \mu, \sigma^2) = \frac{1}{(2\pi)^L\sigma^2} e^{x p\left(-\frac{1}{2\sigma^2}||X - \mu G||^2\right)}, \quad (3)
\]

where \(p(X \mid r, z_s, \mu, \sigma^2)\) is the likelihood of the unknown parameters.

It was shown in\(^{22}\) that better localization results are obtained from the posterior proba-
bility PDF calculated using a Bayesian process:

\[
p(r, z, \mu, \sigma^2 \mid X) = p(X \mid r, z_s, \mu, \sigma^2)p(r)p(z)p(\mu)p(\sigma^2)/p(X), \quad (4)
\]

where \(p(X)\) is a constant with respect to the unknown parameters. We integrate over \(\mu\) and \(\sigma^2\), rather than using MLEs, before estimating \(r\) and \(z_s\); \(p(r), p(z), p(\mu),\) and \(p(\sigma^2)\) are prior
distributions on range, source depth, source spectrum, and variance, respectively. Range is
sought in interval \([r_1, r_2]\) and source depth is sought in interval \([z_{s1}, z_{s2}]\). Here:

\[
p(r) = \frac{1}{r_2 - r_1}, \quad (5)
\]
\[ p(z_s) = \frac{1}{z_{s2} - z_{s1}}, \] (6)

\[ p(\mu) = M \infty < \text{Re}(\mu), \text{Im}(\mu) < \infty, \] (7)

\[ p(\sigma^2) = \frac{1}{\sigma^2}. \] (8)

That is, priors for \( r, z_s, \mu, \) and \( \log \sigma \) \((\sigma > 0)\) are considered uniform.

Combining the likelihood and priors using Bayes’ theorem, we get:

\[ p(r, z_s, \mu, \sigma^2 | X) = K \frac{1}{\sigma^{2L+2}} \exp(-\frac{1}{\sigma^2}||X - \mu G(r, z)||^2), \] (9)

where \( K \) is a constant. To obtain the posterior PDF for \( r \) and \( z_s \), we integrate

\[ p(r, z_s, \mu, \sigma^2 | X) \] over \( \mu \) and \( \sigma^2 \):

\[ p(r, z_s | X) = \int_\mu \int_{\sigma^2} p(r, z_s, \mu, \sigma^2 | X) d\mu d\sigma^2. \] (10)

Maximizing the density of Equation 10 provides the Maximum a Posteriori (MAP) estimates for range and depth.

To compute \( p(r, z_s, \mu, \sigma^2 | X) \) in\(^\text{22}\) we used a Gibbs Sampler.

To implement the sampler, we identified the conditional densities of each parameter on all others. For the source spectrum \( \mu \) we fix parameters \( r, z_s, \) and \( \sigma^2 \) in Equation 9 and we obtain

\[ p(\mu | r, z_s, \sigma^2, X) = C_\mu \exp((-||G||^2/(2\sigma^2))(\mu - G^*X/||G||^2)(\mu - G^*X/||G||^2)), \] (11)
where $C_\mu$ is a constant. This is recognized as a Gaussian distribution with a mean equal to $G^*X/||G||^2$ and variance $2\sigma^2/||G||^2$.

Fixing $r$, $z_s$, and $\mu$ we get an inverse $\chi^2$ distribution for $\sigma^2$:

$$p(\sigma^2 \mid r, z_s, \mu, X) = \frac{1}{\sigma^{2L+2}} \exp\left(-\frac{1}{2\sigma^2}||X - \mu G||^2\right).$$  \hspace{1cm} (12)$$

If the noise is colored, covariance matrix $\Sigma$ is not diagonal as previously mentioned and off-diagonal elements will be estimated as well with the sampler.

We cannot obtain analytically the density $p(r, z_s \mid \mu, \sigma^2, X)$, thus, we calculate it on a grid. The density that is evaluated on the grid is:

$$p(r, z_s \mid \mu, \sigma^2, X) = K \exp\left(-\frac{1}{2\sigma^2}||X - \mu G||^2\right),$$  \hspace{1cm} (13)$$

where $K$ is a constant.

The process is iterative. We start with initial values for $\mu$ and $\sigma^2$ and then sample from $p(r, z_s)$ to obtain values for $r$ and $z_s$. We continue by using these new values of $r$ and $z_s$ and the initial value for $\sigma^2$ and drawing a sample for $\mu$ from the density of Equation 11. Using this value of $\mu$ we then draw a sample for $\sigma^2$ from the density of Equation 12. After repeating the process for many iterations and omitting results from the initial “burn-in” iterations, the obtained sample of values converges to the joint density of Equation 9. Samples for individual parameters provide estimates for the marginal densities. Modes of those provide the MAP estimates of the unknown parameters, including source range and depth.
III. BROADBAND MATCHED-FIELD PROCESSING

A. Incoherent processing

Sound data are typically available at a number of frequencies and there has been much discussion as to how MFP should be implemented for broadband (multi-tonal) data, when the source spectrum is unknown. Assume that we have transmission in two frequencies:

$$X_i = \mu_i G_i(r,z) + W_i,$$  \hspace{1cm} (14)

where \(i = 1, 2\). Noise \(W_i\) is distributed similarly to \(W\). The noise variance is assumed to be the same for both frequencies.

In\(^\text{14}\) the ambiguity surface for broadband sound is derived as:

$$P_B(r,z) = \prod_{i=1}^2 \frac{|G_i^*X_i|^2}{||G_i||^2}. \hspace{1cm} (15)$$

\(P_B\) stands for broadband ambiguity surface. Derivation of this \(P_B\) is based on using ML estimates for source spectra and variance as before.

We can extend the method described in Section II to the two-frequency case. Then:

$$p(r,z, \mu_1, \mu_2, \sigma^2 | X_1, X_2) = K_1 \frac{1}{\sigma^{4L+4}} \exp\left(-\frac{1}{2\sigma^2} \left(||X_1 - \mu_1 G_1(r,z)||^2 + ||X_2 - \mu_2 G_2(r,z)||^2\right)\right). \hspace{1cm} (16)$$

In our work we implement MFP using the density of Equation 16 and estimating \(\mu_1, \mu_2,\) and \(\sigma^2\) along with \(r\) and \(z_s\). This estimation process is done using the Gibbs Sampler described above after the derivation of the conditional densities. The conditional densities are the same as before with the conditionals for \(\mu_i\) being Gaussian with means equal to \(G_i^*X_i/||G_i||^2\) and variance \(2\sigma^2/||G_i||^2\).
The conditional density for $\sigma^2$ is

$$p(\sigma^2 \mid r, z_s, \mu_1, \mu_2 \mid X_1, X_2) = \frac{1}{\sigma^{4L+4}} \exp\left(-\frac{1}{2\sigma^2}(|X_1 - \mu_1 G_1|^2 + |X_2 - \mu_2 G_2|^2)\right).$$

(17)

The process is straightforwardly extended to $N$ frequencies where the exponent of variance changes appropriately and we have $N$ terms in the summation within the exponential.

We term this approach incoherent MFP, because it does not take into account coherence among frequencies.

**B. Coherent processing**

In\textsuperscript{16} coherent MFP was implemented. In addition to spatial coherence across phones, it considered field coherence across frequencies. Towards that goal supervectors were generated for both data and replicas.

We consider source localization for $N$ frequencies with received data $X_1, \ldots, X_N$. Then the data supervector $Y$ is the vertically stacked collection

$$Y = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix}. \quad (18)$$
The replica super vector is
\[
Z = \begin{bmatrix}
G_1 \\
G_2 \\
\vdots \\
G_N
\end{bmatrix}.
\] (19)

Conventional MFP using these vectors relies on calculating the following ambiguity surface:
\[
P_{B,C}(r, z_s) = \frac{|Z^\ast Y|^2}{||Z||^2}.
\] (20)

When the source spectrum is known, the processor is superior to the one of Equation 15\(^7\).

In a realistic case and passive sonar processing, the spectrum is typically unknown and the lack of knowledge of relative phases of the frequency domain data \(X_i\) presents a difficulty. Namely, if the data vectors (and corresponding replica vectors) are not adjusted for phase, data and replicas will not match. To circumvent this problem, in\(^16\), all data and replica vectors within the supervectors are modified so that the phase at the first phone for each frequency is zero and the norm of each subvector is one. The subtraction of the phase at the first phones removes the unknown phase of the source and the normalization removes the impact of the unknown amplitudes. We call the new scaled supervectors \(Z'\) and \(Y'\) and the new ambiguity surface becomes:
\[
P'_{B,C}(r, z_s) = \frac{|Z'^\ast Y'|^2}{||Z'||^2}.
\] (21)

This processor was shown in\(^17\) to be superior to the incoherent processor of Equation 15. However, it is susceptible to poor Signal-to-Noise Ratio (SNR) at the first phone, which is
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used for the phase removal. To bypass this problem, it was proposed in\textsuperscript{19} to estimate the source spectrum phases instead of subtracting the phase at the first phone from receptions at all other phones with good results.

In a similar way we implement here a coherent processor using full PDFs as in Sections II and III. We again assume for simplicity just two frequencies, \( N = 2 \), and we form supervectors \( Y \) and \( U \), where:

\[
Y = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, \quad (22)
\]

\[
U = \begin{bmatrix} \mu_1 G_1 \\ \mu_2 G_2 \end{bmatrix}. \quad (23)
\]

The dimension of \( Y \) and \( U \) is \( 2L \). Then:

\[
Y = U(r, z_s) + V, \quad (24)
\]

where \( V \) is distributed as \( W \) but the covariance matrix now has a dimension of \( 2L \times 2L \).

The joint density of all unknowns is:

\[
p(r, z_s, \mu_1, \mu_2, \sigma^2 \mid Y) = K_C \frac{1}{\sigma^{2L+2}} \exp\left(-\frac{1}{2\sigma^2} \|Y - U(r, z)\|^2\right). \quad (25)
\]

The conditional density for \( \mu_i \), \( i = 1, 2 \), is

\[
p(\mu_i \mid r, z_s, \sigma^2, Y) = C_{\mu_i} \exp\left(-\|G_i\|^2/(2\sigma^2)\right) \left(\mu_i - G_i^* X_i/\|G_i\|^2\right)^* \left(\mu_i - G_i^* X_i/\|G_i\|^2\right), \quad (26)
\]

where \( G_i \) and \( X_i \) are the original replica and data vectors for the \( i \)th frequency.

The conditional density for \( \sigma^2 \) is

\[
p(\sigma^2 \mid r, z_s, \mu_1, \mu_2, Y) = \frac{1}{\sigma^{4L+2}} \exp\left(-\frac{1}{2\sigma^2} \|Y - U\|^2\right). \quad (27)
\]
The joint density that we estimate with the Gibbs Sampler is

\[ p(r, z, \mu_1, \mu_2, \sigma^2 \mid Y) = K_C \frac{1}{\sigma^{4L+2}} e^{\exp(-\frac{1}{2\sigma^2} ||Y - U(r, z)||^2)}, \]

(28)

where \( K_C \) is a constant.

The method can be extended to \( N \) frequencies in a straightforward manner.

As stated previously, the issue of source phase is of great importance in the coherent processor implementation. Phases are considered as unknowns in the Gibbs Sampling process through the consideration of the complex spectra as unknown parameters. At the end of the sampling process, we obtain a joint PDF of all unknowns including range, depth, spectra, and variance (Equation 28). Samples for just the spectra provide marginal PDFs for those, and, as a consequence, marginal PDFs of the phases. Samples for just range and depth form a two-dimensional PDF after integrating over the spectra and variance. Thus, estimated PDFs for range and depth (the new “ambiguity” surfaces) are the results of integration over all possible phase values.

IV. LOCALIZATION RESULTS WITH SYNTHETIC DATA

We consider the shallow water environment of\(^{16,28}\) shown in Figure 1. This is the environment of the Hudson Canyon Experiment, which was conducted off the coast of NJ in an area with relatively flat bathymetry. For the simulations, the true source location was at 2 km in range and 36 m in depth. There were 24 receiving phones with 2.5 m spacing. We generated 2000 realizations of signal plus noise for two frequencies: 175 and 375 Hz. We ran the Gibbs Sampler for 2000 iterations for all realizations; to confirm convergence we monitored the
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modes of the posterior PDFs. We removed the results of the first 500 iterations, considering them burn-in samples. We then estimated source location coordinates for each realization by maximizing the marginal PDFs for range and depth. The probability of correct localization (PCL) was computed by counting how many times the processors estimated the correct location within 200 m in range and 4 m in source depth, that is, within 10% of the true location. The results for eight SNRs are shown in Table I and Figure 2. As a benchmark we also compute results for conventional MFP, where ambiguity surfaces are computed for each frequency and are multiplied to provide a broadband surface (Equation 15). The superiority of the coherent processor is evident. The new incoherent processor and the conventional processor have very similar performances. The developed incoherent processor offers a small advantage for SNRs up to 14 dB. Interestingly the conventional processor outperforms the new incoherent processor for 15 dB. Beyond that SNR, all probabilities are very close to 1.

In Figure 3 we show PDFs of source range and depth for an SNR of 14 dB for one realization. Both approaches estimate the source location correctly. However, the spread of the PDF for the coherent processor is smaller than the one for the incoherent processor. That shows that there is a reduced variance in the coherent estimation.

Figure 4 illustrates the phase PDFs for a frequency of 175 Hz for two cases with a difference phase in the source spectrum. The correct phase for the first case is 0.75 radians; for the second one it is 0. The PDF shown in (a) has significant probability density around 0.75 and is peaked at 0.7 (a small bias exists between the MAP estimate and the true value). For the second case [(b)], the PDF mode is at 0, the true value.
FIG. 1. (Color online) The Hudson Canyon environment; $c(z)$ and $c_p(z)$ are the sound speed profiles in the water column and sediment, respectively, and are considered known from measurements and ground truth information.

Source localization was also performed with conventional incoherent MFP as mentioned above. The probability of correct localization for 10 dB was 0.32 in contrast to the 0.40 rate for the new incoherent processor and 0.48 for the new coherent processor. Figure 5(a) shows an ambiguity surface for the conventional processor, where the location estimate is highly ambiguous with the surface exhibiting multiple sidelobes. Figure 5(b) illustrates the PDF for range and depth for the same realization obtained via the Gibbs Sampler for incoherent estimation. There is a unique mode at the correct location.
TABLE I. Probability of correct localization vs. SNR.

<table>
<thead>
<tr>
<th>SNR</th>
<th>Coherent PCL</th>
<th>Incoherent PCL</th>
<th>Conv. PCL</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 dB</td>
<td>0.99</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>15 dB</td>
<td>0.97</td>
<td>0.84</td>
<td>0.93</td>
</tr>
<tr>
<td>14 dB</td>
<td>0.92</td>
<td>0.73</td>
<td>0.72</td>
</tr>
<tr>
<td>13 dB</td>
<td>0.87</td>
<td>0.72</td>
<td>0.70</td>
</tr>
<tr>
<td>12 dB</td>
<td>0.71</td>
<td>0.59</td>
<td>0.55</td>
</tr>
<tr>
<td>11 dB</td>
<td>0.55</td>
<td>0.46</td>
<td>0.44</td>
</tr>
<tr>
<td>10 dB</td>
<td>0.48</td>
<td>0.40</td>
<td>0.32</td>
</tr>
<tr>
<td>9 dB</td>
<td>0.29</td>
<td>0.24</td>
<td>0.23</td>
</tr>
</tbody>
</table>

V. HUDSON CANYON LOCALIZATION RESULTS

Source localization was also performed with real data collected during the Hudson Canyon Experiment. Data were collected at two sets of frequencies, 50, 175, 375, and 425 Hz and 75, 275, 525, and 600 Hz. Ten snapshots per source location were recorded at twenty locations. Because of high SNR and accurate knowledge of the propagation medium, all processors (with the exception of the incoherent MVDR) successfully estimated the source 18 times out of 20. However, it is interesting to observe the difference between the new coherent processor and the incoherent processor when using a single data snapshot, in which case the SNR is significantly lower. For the data that we consider here, the correct source range
FIG. 2. (Color online) Probability of correct localization vs. SNR for the coherent, incoherent, and conventional estimators.

was 3.48 km and the depth was 36 m and the data were collected at 50, 175, 375, and 425 Hz. Figure 6 shows the surfaces for the coherent and incoherent processors. The coherent processor calculates a PDF with a mode and 3.42 km in range and 36 m in depth, very close to the true values. The incoherent mode is at a source range and depth of 2.75 km and 55 m, respectively. The incoherent PDF has a secondary mode at the correct location but the density there is smaller than the one around the main mode. The results indicate the superiority of the coherent processor.

VI. DETECTION AND LOCALIZATION

In Section III, we discussed how we can localize a broadband source including source spectrum and noise variance in the estimation process. In addition to localization, we are
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FIG. 3. (Color online) PDFs for one realization for an SNR of 14 dB: (a) coherent PDF and (b) incoherent PDF.

also interested in detecting the sound-emitting source. We consider two hypotheses, \( H_1 \) when a signal is present and \( H_0 \) when there is no signal. We will evaluate the above processors by calculating probabilities of detection and probabilities of false alarm and plotting ROC curves. Combining detection and localization, we will continue our comparison using Localization-ROC (LROC) curves.

When a signal is present, we have

\[
X = \mu G(r, z_s) + W. \tag{29}
\]
When there is no signal:

\[ \mathbf{X} = \mathbf{W}. \]  

(30)

The most commonly used detector is the likelihood ratio detector. For narrowband data, we formulate the likelihood ratio using the likelihood function of Equation 3:

\[ \lambda = \frac{p(\mathbf{X} \mid H_1)}{p(\mathbf{X} \mid H_0)} = \frac{\frac{1}{(2\pi)^{d/2}} \exp\left(-\frac{1}{2\sigma^2} ||\mathbf{X} - \hat{\mu} \mathbf{G}(\hat{r}, \hat{z}_s)||^2\right)}{\frac{1}{(2\pi)^{d/2}} \exp\left(-\frac{1}{2\sigma^2} ||\mathbf{X}||^2\right)}, \]  

(31)

where \( \hat{\mu} \) and \( \hat{\sigma}^2 \) are ML estimates of source spectrum and variance, respectively. For multiple frequencies, in the presence of signals we have:

\[ \mathbf{X}_i = \mu_i \mathbf{G}_i(r, z_s) + \mathbf{W}_i \]  

(32)
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FIG. 5. (Color online) (a) Ambiguity surface for the conventional processor and (b) PDF for the new incoherent processor.

and for noise only

\[ X_i = W_i; \]  

(33)

here, \( i = 1, 2 \).

For the incoherent processor:

\[ \lambda' = \frac{p(X_1, X_2 | H_1)}{p(X | H_0)} = \frac{\frac{1}{(2\pi)^L\sigma^2}exp(-\frac{1}{2\sigma^2} \sum_{i=1}^{2} ||X_i - \hat{\mu}_i G_i(\hat{r}, \hat{z}_s)||^2)}{\frac{1}{(2\pi)^L\sigma^2}exp(-\frac{1}{2\sigma^2} \sum_{i=1}^{2} ||X_i||^2)}. \]  

(34)

Equivalently we can select sufficient statistic \( \lambda'_I \) where

\[ \lambda'_I = -\sum_{i=1}^{2} ||X_i - \hat{\mu}_i G_i(\hat{r}, \hat{z}_s)||^2 + \sum_{i=1}^{2} ||X_i||^2. \]  

(35)
We compare $\lambda'_I$ to a threshold $\beta$: when $\lambda'_I$ is larger than $\beta$ we conclude that hypothesis $H_1$ is true: a signal is present. Otherwise it is concluded that $H_0$ is true, that is, we receive only noise.

For the coherent processor for hypothesis $H_1$ we have

$$Y = U + V,$$  \hspace{1cm} (36)

and for $H_0$:

$$Y = V.$$  \hspace{1cm} (37)
For coherent likelihood-ratio detection, the statistic becomes:

$$\lambda'_C = -||\mathbf{Y} - \mathbf{U}(\hat{r}, \hat{z}_s)||^2 + ||\mathbf{Y}||^2.$$  \hspace{1cm} (38)

VII. JOINT DETECTION AND LOCALIZATION RESULTS

We consider again the shallow water environment of Figure 1. We generated 2000 realizations of signal plus noise and 2000 realizations of just noise for different SNRs. As in the localization task, we ran the Gibbs Sampler for 2000 iterations for all realizations. With a Monte Carlo process we calculated probabilities of detection and false alarm for detection and probabilities of correct localization and false alarm for localization.

We performed detection using the statistics of Equations 35 and 38 and formed ROC curves. For an SNR of 17 dB the ROC curves for both processors coincided with the upper left corner, showing perfect detection. We then decreased the SNR and computed the ROC curves of Figure 7(a) for an SNR of 14 dB. Coherent results are shown with a solid line; the dashed line illustrates incoherent detection. Curves for both processors show a high probability of detection for a small probability of false alarm. Figure 7(b) zooms on the upper left corner of the ROC curve showing that the coherent processor attains a higher probability of detection for the same false alarm albeit not by a large amount. For a probability of false alarm of 0.003 the probability of detection for the incoherent processor is 0.93; for the coherent processor for the same probability of false alarm the probability of detection is 0.96.
FIG. 7. (Color online) (a) ROC curves for an SNR of 14 dB; (b) zoom on the upper left corner of (a).
Figure 8 illustrates the PDFs of the statistics $\lambda'_{C}$ and $\lambda'_{I}$ for the two processors. The overlap for the two PDFs for hypothesis $H_0$ is slightly larger for the incoherent method than for the coherent approach, showing the advantage of coherent processing.

**FIG. 8.** (Color online) PDFs for the detection statistic for hypotheses $H_1$ and $H_0$: (a) coherent results and (b) incoherent results.

Figure 9 shows the ROC curves for 13 dB ((a) actual ROC curve and (b) zoom). Looking at the curve of Figure 9(b), we observe that for a probability of false alarm of 0.003 (the probability of false alarm at the bottom left corner), the coherent probability of detection is 0.93, while the corresponding incoherent probability is lower at 0.84.
FIG. 9. (Color online) (a) ROC curves for an SNR of 13 dB; (b) zoom on the upper left corner of (a).
Figure 10 shows LROC curves for an SNR of 14 dB. The coherent probability of correct localization reaches 92% while the incoherent probability of localization reached only 73%. Of particular interest are the results for detection and localization for four frequencies. We carried out simulations for frequencies of 75, 275, 525, and 600 Hz, one of the sets in which sound was transmitted in the Hudson Canyon experiment. Figure 11 shows (a) the ROC curves and (b) the LROC curves for the two processors for an SNR of 9 dB. Now the superiority of the coherent detector over the incoherent detector is much more pronounced. For a probability of false alarm of 0.02 the probability of detection reaches 0.98. For the
same probability of false alarm, the probability of detection of the incoherent processor is
only 0.81. The superiority is also evident in the LROC curves. The probability of correct
localization reaches 0.54 for the coherent processor and 0.43 for the incoherent processor.

VIII. CONCLUSIONS

In this work, we develop Bayesian coherent and incoherent matched-field processors for
broadband localization and detection that incorporate source spectrum estimation in the
process. The estimation is performed using a Gibbs Sampler that computes PDFs of the
unknown parameters that we then maximize. Bayesian coherent localization is clearly supe-
rior to the corresponding incoherent estimation, which is superior to conventional Bartlett
processing for most SNRs. We then perform detection with the two proposed processors.
With ROC curves and PDF calculation for the detection statistics we show that coher-
et processing is superior to incoherent processing in detection; the advantage of using the
coherent processor in detection is limited for two frequencies but it is significant for four fre-
quencies. Performing joint detection and localization demonstrates a significant advantage
of the coherent processor. Results are also presented with real data showing a coherent PDF
with a mode at the correct location. Incoherent processing provided erroneous estimates.

The coherent processor is superior to conventional processing by a considerable amount.
It is, however, more computationally demanding although not prohibitively so. The 2000
Gibbs sampling iterations performed here were computed very efficiently because of the sim-
ple form of the conditional densities. The incoherent processor also required 2000 iterations
and offered little improvement over conventional processing (it offered an advantage for sev-
FIG. 11. (Color online) (a) ROC and (b) LROC curves for an SNR of 9 dB for four frequencies.
eral SNRs that we considered but was inferior at 15 dB). Thus, the additional computational load it requires is not justified.

Summarizing, the new coherent method outperforms other processors in terms of localization accuracy and decreased uncertainty as illustrated via a comparison of PDFs, ambiguity surfaces, and their modes. The approach is also superior to the incoherent processor in terms of detection (investigated to date only in a limited fashion in terms of MFP), especially in the four-frequency case. Additionally, both coherent and incoherent methods proposed here provide estimates of the source spectra and their uncertainty as shown in Figure 4, which means that they also provide deconvolution results and corresponding uncertainty that can be used in source identification.

Being Bayesian in nature, the new work provides complete densities in addition to point estimates, enabling the understanding of uncertainty in the estimation process. In the future the proposed Gibbs sampling coherent approach can be considered for geoacoustic inversion.

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REFERENCES

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23 J. A. Shorey and L. W. Nolte, “Wideband optimal a posteriori probability source localiza-


26 S. E. Dosso and M. J. Wilmut, “Effects of incoherent and coherent source spectral infor-


The Hudson Canyon environment; $c(z)$ and $c_p(z)$ are the sound speed profiles in the water column and sediment, respectively, and are considered known from measurements and ground truth information.