Comments on “A New Theoretical Model for the Prediction of Rapid Fading Variations in an Indoor Environment”

Ali Abdi

Department of Electrical and Computer Engineering,

University of Minnesota, Minneapolis, Minnesota, USA

***Abstract* — In this paper,[1] a new distribution, named POCA, is introduced for modeling envelope fluctuations due to fast fading in indoor environments. The main motivation has been the small number of scatterers in those environments, which makes the central limit theorem invalid for in-phase and quadrature components. Hence they are no longer Gaussian and envelope is not Rayleigh-distributed. POCA, the new envelope distribution,[1] is obtained by assuming *t* distribution for in-phase and quadrature components. In this comment, first a short summary of three possible approaches for deriving the envelope distribution for indoor channels, where the number of scatterers is usually small, has been provided. Thereafter, based on various detailed theoretical considerations and also measurement results,[1] the deficiency of POCA for prediction of rapid envelope variations in indoor channels has been demonstrated.**

Having the number of scatterers large is the main assumption in modeling the envelope probability density function (PDF) for fast fading (or local fading, if spatial variations are concerned instead of temporal fluctuations). Whenever other conditions of central limit theorem (CLT), like independence of multipath components received from scatterers, etc. are met, the in-phase (I) and quadrature (Q) components of the received signal become Gaussian variables. In

[1] D. S. Polydorou and C. N. Capsalis, *IEEE Trans. Vehic. Technol.*, vol. 46, pp. 748-754, 1997.

general, they are dependent Gaussian variables with different non-zero means and unequal variances. The envelope PDF for this general case is presented in [1, eq. (4.6-28)]. The classical Rayleigh and Rice PDFs and the less-known Hoyt PDF are special cases of [1, eq. (4.6-28)].

However, as is pointed out,[1] for indoor propagation channel the number of scatterers is usually small. So CLT does not hold and it does not make sense to consider [1, eq. (4.6-28)] or its special cases like Rayleigh, Rice, ..., as the envelope PDF for indoor environments. In order to find a suitable indoor envelope PDF, we may consider any of the following three approaches which have mainly been used for modeling the envelope PDF in other propagation environments. Each of them has its own advantages and disadvantages, not discussed here due to space limitations:

Model-free approach: In this approach no assumption is made about the physical mechanism that generates envelope fluctuations. Usually a known and flexible PDF with two or three parameters is picked up and then its appropriateness is verified by doing statistical goodness-of-fit tests on real data. Weibull and lognormal PDFs are two examples of this kind, used for indoor channels [2]. Stacy PDF [3] is also promising for indoor environments. More suitable candidates may be found in [4] [5]. A more general method is to consider an infinite orthogonal expansion for the envelope PDF and then obtain the first few coefficients from data, as is done in [6] for statistical modeling of radar cross section using Legendre orthogonal polynomials. Since envelope is a positive-valued random process, Laguerre polynomials constitute the natural set of orthogonal basis for expanding the envelope PDF [7].

Random-vector-model approach: Here each multipath component is considered as a random vector with random length and angel. Then the superposition of multipath components at the receiver corresponds to the addition of random vectors. In this way, finding the envelope PDF reduces to calculating the PDF of the resulting vector. There are a couple of numerical and analytic methods for calculating the PDF of the resulting vector (see [7] and references therein). Expansion in terms of Laguerre polynomials is discussed in [7], along with a numerical example.

Non-Gaussian I-Q approach: According to a comprehensive literature survey reported in [8], only few authors have considered joint non-Gaussian PDFs for *I* and *Q* components, and then have derived the envelope PDF for different applications [1, eq. (4.7-15)] (see also [9]) [10]-[12]. Although the envelope PDF is not discussed in [13] and [14], the joint PDF of *I* and *Q* components there is expanded in terms two-dimensional Hermite polynomials. The envelope PDF can be easily obtained afterwards, which in fact is a generalization of the result reported in [1, eq. (4.7-15)].

The approach taken[1] for modeling the indoor envelope PDF falls into the third category, i.e. non-Gaussian *I-Q* approach. Basically they[1] have considered *I* and *Q* components as two independent and identically distributed random variables with *t* PDF, i.e. (7)[1]. Then they have derived the PDF of the envelope, named POCA, in (12)[1]. In what follows, several comments regarding POCA PDF are provided. Unless otherwise mentioned, the same notation and terms[1] will be used throughout the comment:

1) According to [15, p. 133],[1] when $y_1,\ldots,y_n$ are independent and identically distributed variables with mean μ_0 and standard deviation *s*, then the sample mean $\bar{y} = \sum_{j=1}^{n} y_j \big/ n$ is a variable with mean μ_0 and standard deviation $s/\sqrt{n}$. Therefore, for *large n* and by CLT, the variable $z = (\bar{y} - \mu_0)\big/(s/\sqrt{n})$ in (1)[1] is *approximately* Gaussian with mean 0 and standard deviation 1. Note that when *n* is small, there is no simple closed-form PDF for *z*, contrary to the claim[1] that *z* has the *t* distribution. Actually the *t* distribution appears in another way. Let $y_1,\ldots,y_n$ introduced above be also Gaussian. Then $\bar{y}$ is a Gaussian variable with the same mean and standard deviation as above, i.e., μ_0 and $s/\sqrt{n}$, respectively. Let $\hat{s}^2$ be the unbiased maximum likelihood estimate of s^2, i.e. $\hat{s}^2 = \sum_{j=1}^{n} (y_j - \bar{y})^2 \big/ (n-1)$. Then, for any *arbitrary n*, the variable $\zeta = (\bar{y} - \mu_0)\big/(\hat{s}/\sqrt{n})$ is *exactly* *t*-distributed [15, pp. 133-134] (note that ζ and $\hat{s}$ are defined here). If the drop the assumption of Gaussianity for $y_1,\ldots,y_n$, the variable ζ will no longer be *t*-distributed. Based on the above discussion, it is not clear how the authors[1] have proposed *t* PDF for *I* and *Q* components, as far as no Gaussian variable is considered.[1] Assumption of Gaussianity is also vital for generalized forms of *t* distribution, like non-central

and doubly non-central *t* distributions [16].

2) As is correctly addressed,[1] removing the effect of path loss and slow fading is necessary to study the effect of fast fading on the received envelope [17, chap. 6]. Since envelope is of concern, normalization for removing those effects should be done on the envelope ([18, p. 170] [19] [20, sec. 5.15 and sec. 8.3]), not on the *I* and *Q* components, as is shown in (1)[1] for the variable z which is a representative for *I* and *Q* components z_c and z_s.

3) Although in appendix B[1] the relationship between Gaussian variables and *t* distribution is precisely stated, still it seems wrong to assign *t*-distribution to z_c and z_s, again because no Gaussian variable is considered throughout the paper[1] and also normalization is improperly applied to *I* and *Q* components instead of the envelope.[1]

4) Based on the assumptions made,[1] z_c and z_s are uncorrelated (see (6c)[1]). However, z_c and z_s are not necessarily independent in general (Only for jointly *Gaussian* variables, zero correlation means independence). In fact, as a special case of [7], the joint characteristic function of z_c and z_s, i.e. $\Psi_{z_c,z_s}(\omega_1,\omega_2) = E[\exp(i\omega_1 z_c + i\omega_2 z_s)]$ with E as expectation and $i^2 = -1$, has the form:

$$\Psi_{z_c,z_s}(\omega_1,\omega_2) = \prod_{j=1}^{n} E_{c_j}[J_0(c_j\sqrt{\omega_1^2+\omega_2^2})],$$

where $J_0(.)$ is the Bessel function of order zero. The characteristic functions of z_c and z_s are given by $\Psi_{z_c}(\omega_1) = \Psi_{z_c,z_s}(\omega_1,0)$ and $\Psi_{z_s}(\omega_2) = \Psi_{z_c,z_s}(0,\omega_2)$, respectively:

$$\Psi_{z_c}(\omega_1) = \prod_{j=1}^{n} E_{c_j}[J_0(c_j|\omega_1|)], \quad \Psi_{z_s}(\omega_2) = \prod_{j=1}^{n} E_{c_j}[J_0(c_j|\omega_2|)].$$

In general, $\Psi_{z_c,z_s}(\omega_1,\omega_2) \neq \Psi_{z_c}(\omega_1)\Psi_{z_s}(\omega_2)$ (A simple obvious form of this inequality can be observed when $n=1$ and c_1 is a deterministic constant). Therefore, we conclude that z_c and z_s are not necessarily independent (They are approximately independent when n is large and $\{c_j\}$ are deterministic constants, because for large n we have $J_0(\xi)^n \approx \exp(-n\xi^2/4)$ [21, p. 421], which in turn yields $\Psi_{z_c,z_s}(\omega_1,\omega_2) \approx \Psi_{z_c}(\omega_1)\Psi_{z_s}(\omega_2)$. There may be other sufficient conditions for the independence of z_c and z_s). So, (7a)[1] and the results derived based on that, including POCA PDF for the envelope, are not correct in general, even if we assume that z_c and z_s are *t*-

distributed variables. Nevertheless, if we insist on modeling z_c and z_s with *t* distribution, we should use a bivariate *t* PDF [22] as the joint PDF of z_c and z_s.

5) Non-Gaussian *I*-*Q* approach employed by authors[1] is a good way for modeling envelope fluctuations due to fast fading. In other words, it is advantageous to consider a non-Gaussian PDF (that includes Gaussian PDF as a special case) for *I* and *Q* components, because the resulting non-Rayleigh envelope PDF will have Rayleigh PDF as a special case. The main point in selecting a non-Gaussian PDF for *I* and *Q* components is how well the resulting envelope PDF fits to data. It is a key point that cannot be ignored. Even if we disregard all of the previous four comments made earlier and just consider *t* distribution as a possibly good candidate for *I* and *Q* components, without any attempt to justify it theoretically as is improperly done by authors,[1] the poor fit of the so called POCA PDF to real data shows the inappropriateness of *t* distribution for envelope modeling. A quick look at Figs. 7-14[1] confirm this fact. For $n_{\text{POCA}} = 1$ (Figs. 7 and 11[1]), POCA and Rayleigh are significantly different and both of them provide poor fit over the entire range of fading depths, specially over the deep fade region. So here POCA is inferior to Rayleigh because with one more free parameter and more complicated mathematical form, provides no enhancement. For $n_{\text{POCA}} = 2, 5, 50$ (Figs. 8-10 and 12-14[1]), POCA and Rayleigh are basically the same. In these cases POCA is still inferior to Rayleigh because again with one more free parameter and much more complexity, it shows the same behavior as Rayleigh. In other words, POCA is either different from Rayleigh but without any improvement (Figs. 7 and 11[1]), or very similar to Rayleigh (Figs. 8-10 and 12-14[1]), which inherently is an improper PDF for indoor channels.

6) Acceptance of POCA distribution as the null hypothesis based on the values of X^2 statistic,[1] which is contrary to the poor fits observed in Figs. 7-14,[1] is somehow misleading. This dilemma comes from the way the authors[1] have used Pearson’s chi-square goodness-of-fit test. According to [23, pp. 363-364], asymptotic distribution of X^2 under null hypothesis is chi-square with $k-1-m$ degrees of freedom, where *k* is the number of subintervals[1] and *m* is the number of parameters that must be estimated using a *maximum likelihood estimator* (for POCA,

$m = 2$). So if we find the maximum likelihood estimates of σ and *n*, the parameters of POCA, then under null hypothesis, X^2 in (16)[1] follows a chi-square distribution with $k-1-2=k-3$ degrees of freedom, provided that the number of data is large. Under this circumstances, one is allowed to use tables of chi-square distribution, in order to test the goodness-of-fit of POCA for a given confidence level, say 0.05. Obviously any deviation from the conditions stated above introduces some errors which may be serious or not, depending on the case [23, p. 364]. The authors[1] have used an ad hoc and not-clearly-described version of the correct Pearson’s chi-square goodness-of-fit test. They have estimated σ using the method of moments by equating $E(n,\sigma)$ in (14),[1] the mean of POCA, with the sample mean. However, it is not evident how this can be done when the value of *n* is unknown. Since no clear statement is made about the way they have estimated *n*, motivated by Figs. 3-4[1] one may assume that for each data set (office, corridor, ...) they have considered fifty integer values for *n*, i.e. $n = 1, 2, \ldots 50$. Then they have estimated σ using the method of moments for each of those fifty values of *n*, thereafter have calculated X^2 statistic for each *n* (see Figs. 3-4[1]). Among those fifty values, the optimum value of *n* (depicted in Table I[1] with shadowing) for each data set is the one that yields minimum value for X^2. So far, the authors[1] have presented an ad hoc method for estimating σ and *n* of POCA. If just for a moment we suppose that POCA is a good PDF for indoor environments, the ad hoc method for estimating POCA parameters may be useful in practice. In fact, because of the complicated form of POCA PDF, the maximum likelihood estimators for its parameters must be quite involved. The crucial point is that once the X^2 statistic is utilized for parameter estimation, it cannot be used to perform Pearson’s chi-square goodness-of-fit test based on the tables of chi-square distribution, as is done[1] for 0.05 confidence level and $k-2$ degrees of freedom (different from $k-3$ mentioned earlier in this comment). In summary, since the ad hoc goodness-of-fit test of authors[1] is different from the correct Pearson’s chi-square test, acceptance of POCA as the null hypothesis within 0.05 confidence level and based on the tables of chi-square distribution[1] is questionable. This fact can be confirmed by visual inspection of Figs. 7-14.[1] The shortage of POCA distribution for prediction of fast fading is also stated by authors[1] explicitly.

7) It is worthwhile to note that the deep fade region is of fundamental importance because it seriously affects the performance of a communication system over fading channels [24]. So, a suitable envelope PDF must show satisfactory fit over the deep fade region. From this point of view and according to Figs. 7-14,[1] POCA offers no improvement against Rayleigh.

8) Only in Figs. 5 and 6[1] (specially in Fig. 5[1]) we can see rather reasonable fit of POCA to data, comparing with Rayleigh. But it is not surprising because it has one more free parameter that can be adjusted to yield better fit. The cost of having better fit with the complicated POCA PDF is the difficulties that we encounter in analytic and numerical manipulation of POCA for prediction of average bit error rate (BER) for various modulation schemes, evaluating the effect of different diversity methods, incorporating the shadowing phenomenon, ... For example, even for the simple BER expression of binary DPSK, it seems very hard, if not impossible, to express average BER in a closed form, assuming POCA fast fading for the envelope. Therefore, if for a moment we assume that POCA has strong theoretical and experimental supports, we should look for a simple PDF with few parameters that can reasonably approximate POCA PDF.

9) Those entries of Table I[1] which are listed under the title “Office” don’t comply with the corresponding curves in Fig. 3.[1]

10) Although not so important, in some cases common letters are used for different quantities,[1] for example, α for both envelope in (8b) [1] and confidence level, v as the parameter of t-distribution in (7a) [1] and also as the number of degrees of freedom for chi-square distribution, k for the number of weak scattered waves, summation index in (13)-(15),[1] and number of subintervals in (16).[1] Of course most of them can be correctly identified from the context.

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For the English abstract, see: http://it.ucsd.edu/IT/Society/dissertations/abdi.html

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