Comments on "On Verifying the First-Order Markovian Assumption for a Rayleigh Fading Channel Model"

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Abstract — **In this paper,[1] the trivariate probability density function (PDF) for the envelope of a bandpass Gaussian process is derived. However, the multivariate PDF for the envelope of a bandpass Gaussian process having an arbitrary power spectrum has been previously reported. The Markovian behavior of the envelope process for the U-shaped power spectrum $1/\sqrt{1-f^2}$ is studied in[1] using an information theoretic approach. It is worth mentioning that several authors have formerly discussed that Markovian behavior for the rational power spectrum $1/(1+f^2)$, from a probabilistic point of view.**

In a small part of [1], several methods for obtaining the probability density function (PDF) of $R''(t)$, the second derivative of the envelope of a bandpass Gaussian process, are discussed. One method is based upon using the trivariate PDF of $R(t)$, i.e. the joint PDF of $R(t_1)$, $R(t_2)$, and $R(t_3)$ for three different instants of time t_1, t_2, and t_3, according to the following representation for $R''(t)$:

$$R''(t_2) = \lim_{h\to 0} \frac{R(t_1) - 2R(t_2) + R(t_3)}{h^2}, \quad h = t_2 - t_1 = t_3 - t_2 .$$

So several references on $p_{\underline{R}}(\underline{r})$, the multivariate PDF of $R(t)$, can be found in [1]. Assuming an arbitrary power spectrum for the bandpass Gaussian process, an exact and complicated closed-form formula for $p_{\underline{R}}(\underline{r})$ is presented in [2, (3.3)]; while in [3, (29)] and [4, (8) and (15)], power series expansion of $p_{\underline{R}}(\underline{r})$ is provided. In this way formula (18)[1] can be considered as a special

[1] H. S. Wang and P. C. Chang, *IEEE Trans. Vehic. Technol.*, vol. 45, pp. 353-357, 1996.

case of the above formulas. The advantage of formula (18)[1] is that it gives some ideas about the functional form of the trivariate envelope PDF in Rayleigh fading channel. It is rather hard to obtain such an insight from those previously derived general multivariate PDFs for the envelope.

Markovian assumption for a random process simplifies the mathematical analysis of that process considerably. This is why the authors[1] have verified the approximate first-order Markovian behavior of $R(t)$ for the U-shaped power spectrum $1/\sqrt{1-f^2}$, which is typical in mobile radio communication channels with isotropic scattering [5]. The rigorous theoretical support for the conclusion made in[1], based on the concept of average conditional mutual information, is this fact from information theory: A necessary and sufficient condition for vanishing the average conditional mutual information $I(R(t_3);R(t_1)|R(t_2))$, where $t_1 < t_2 < t_3$, is that the random variables $R(t_1)$, $R(t_2)$, and $R(t_3)$ form a Markov chain [6, p. 36]. In fact, the authors[1] have shown via numerical evaluation and computer simulation that for the U-shaped power spectrum $1/\sqrt{1-f^2}$, $I(R(t_3);R(t_1)|R(t_2))$ is negligible. Hence $R(t)$ can be reasonably approximated by a first-order Markov process.

It may be instructive to note that for a power spectrum of the form $1/(1+f^2)$, it is proved through various interesting probabilistic approaches that $R(t)$ is a first-order Markov process [7]-[10]. There is another reference [11], not cited in [1], that refers to rather similar issues concerning a more general rational power spectrum. Those results in [7]-[11] may shed more light on the possible Markov property of the envelope in wireless fading channels.

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