DISCRETE COSINE TRANSFORM ALGORITHMS FOR FPGA DEVICES

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Outline

- Motivation
- Discrete Cosine Transform
- One-dimensional Algorithms
- Multidimensional 2-D algorithm

Motivation

- on the efficient hardware implementations of 1-D DCT & 2-D DCT
- will be to research computational savings, accuracy improvements and chip area savings that result from the application of mutildimetional transforms to DCT

Discrete Cosine Transform

- Discrete Fourier Transform

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-i 2\pi \frac{k}{N} n}.$$

- Discrete Cosine Transform

$$X_k = \sum_{n=0}^{N-1} x_n \cos\left[\frac{\pi}{N}\left(n + \frac{1}{2}\right)k\right] \qquad k = 0, \ldots, N-1.$$

One-dimensional Algorithms

$$\begin{bmatrix} y0 \\ y1 \\ y2 \\ y3 \\ y4 \\ y5 \\ y6 \\ y7 \end{bmatrix} = \begin{bmatrix} a & a & a & a & a & a & a & a \\ b & d & e & g & -g & -e & -d & -b \\ c & f & -f & -c & -c & -f & f & c \\ d & -g & -b & -e & e & b & g & -d \\ a & -a & -a & a & a & -a & -a & a \\ e & -b & g & d & -d & -g & b & -e \\ f & -c & c & -f & -f & c & -c & f \\ g & -e & d & -b & b & -d & e & -g \end{bmatrix} \begin{bmatrix} x0 \\ x1 \\ x2 \\ x3 \\ x4 \\ x5 \\ x6 \\ x7 \end{bmatrix},$$

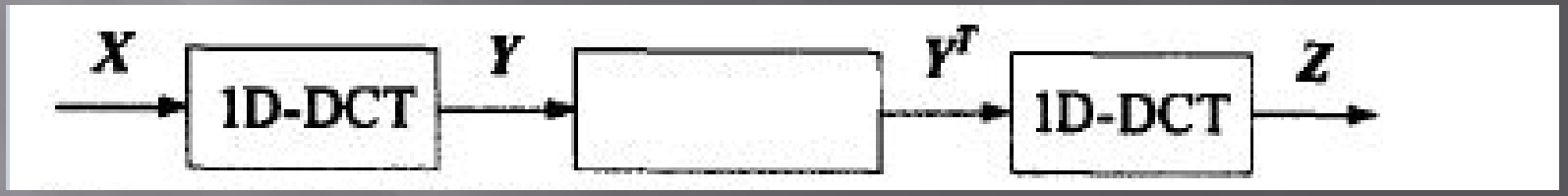
$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \cos\frac{4\pi}{16} \\ \cos\frac{\pi}{16} \\ \cos\frac{2\pi}{16} \\ \cos\frac{3\pi}{16} \\ \cos\frac{5\pi}{16} \\ \cos\frac{6\pi}{16} \\ \cos\frac{7\pi}{16} \end{bmatrix}$$

$$\begin{bmatrix} y0 \\ y2 \\ y4 \\ y6 \end{bmatrix} = \begin{bmatrix} a & a & a & a \\ c & f & -f & -c \\ a & -a & -a & a \\ f & -c & c & -f \end{bmatrix} \begin{bmatrix} x0 + x7 \\ x1 + x6 \\ x2 + x5 \\ x3 + x4 \end{bmatrix}, \quad \begin{bmatrix} y1 \\ y3 \\ y5 \\ y7 \end{bmatrix} = \begin{bmatrix} b & d & e & g \\ d & -g & -b & -e \\ e & -b & g & d \\ g & -e & d & -b \end{bmatrix} \begin{bmatrix} x0 - x7 \\ x1 - x6 \\ x2 - x5 \\ x3 - x4 \end{bmatrix}$$

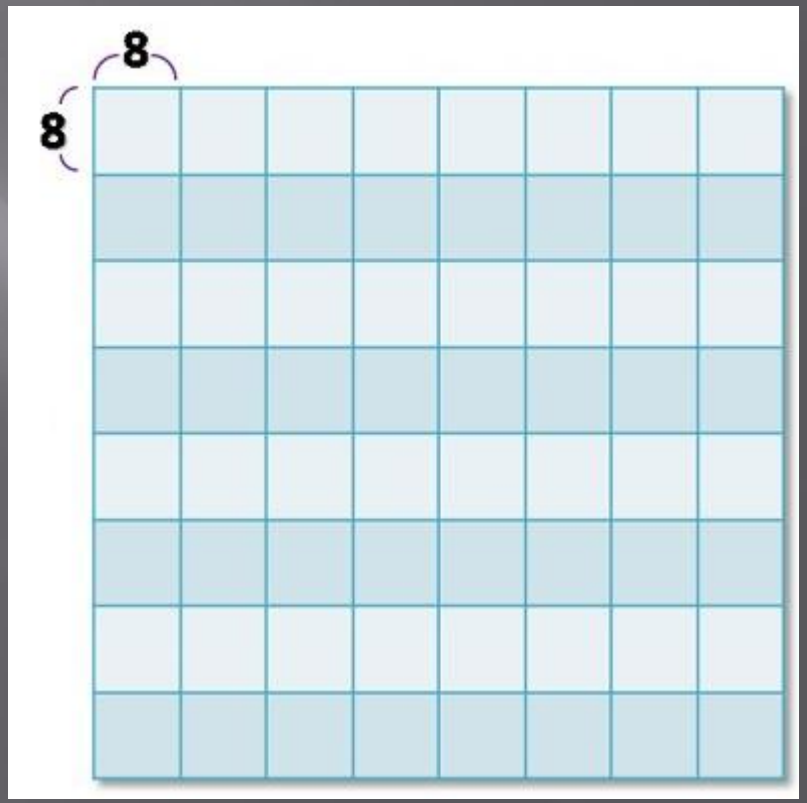
Multidimensional 2-D algorithm

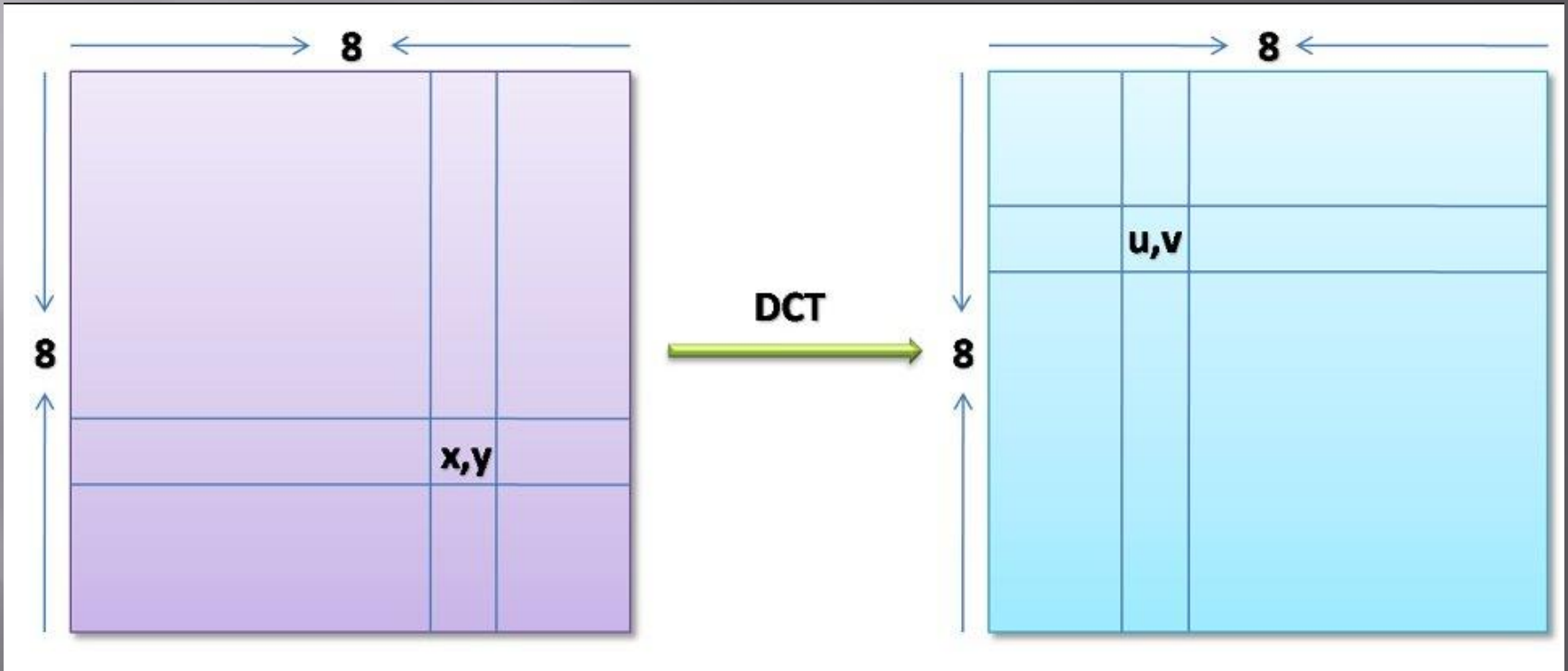
- 2-D DCT

$$X_{k_1,k_2} = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} x_{n_1,n_2} cos \left[\frac{2\pi(2n_1+1)k_1}{4N_1} \right] cos \left[\frac{2\pi(2n_2+1)k_2}{4N_2} \right] \quad (5.5)$$

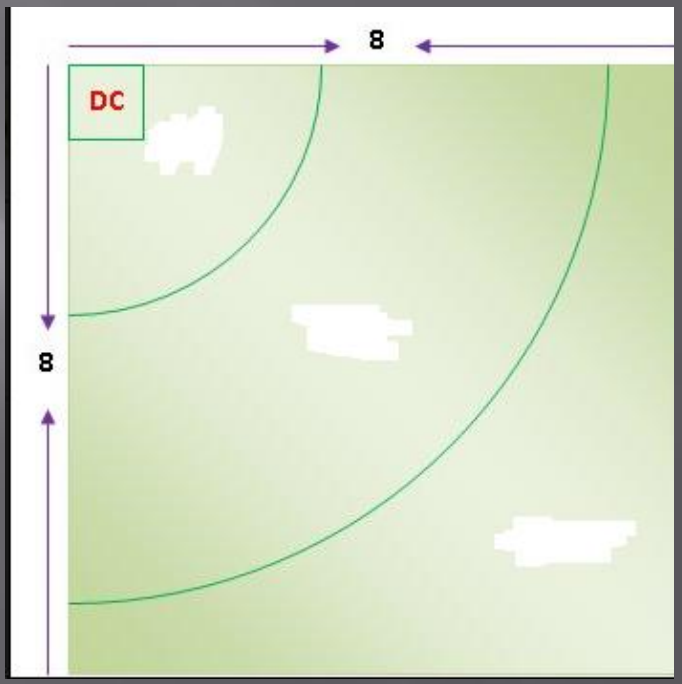





8
8


8
8
x,y
DCT
8
8
u,v

DC	AC	AC	AC	AC	AC	AC	AC
AC	AC	AC	AC	AC	AC	AC	AC
AC	AC	AC	AC	AC	AC	AC	AC
AC	AC	AC	AC	AC	AC	AC	AC
AC	AC	AC	AC	AC	AC	AC	AC
AC	AC	AC	AC	AC	AC	AC	AC
AC	AC	AC	AC	AC	AC	AC	AC
AC	AC	AC	AC	AC	AC	AC	AC



4-point 1-D DCT on FPGA

0.5000	0.5000	0.5000	0.5000
0.6533	0.2706	-0.2706	-0.6533
0.5000	-0.5000	-0.5000	0.5000
0.2706	-0.6533	0.6533	-0.2706

```
>> dct([1 0 0 0])

ans =

    0.5000    0.6533    0.5000    0.2706

>> dct([1 1 0 0])

ans =

    1.0000    0.9239         0   -0.3827

>> dct([1 0 1 0])

ans =

    1.0000    0.3827         0    0.9239

>> dct([1 1 1 0])

ans =

    1.5000    0.6533   -0.5000    0.2706
```

- Still some errors in the experiment, since the values assigned on the FPGA and accuracy of multiplier

- Thank You