Givens Rotation Matrix Generator

Boyan Zhang

ECE 641

Spring 2011

- Motivation
- Eigen Value Decomposition & Applications
- Popular Decomposition Scheme

  QR Iteration

  Exact Jacobi
- Exact Jacobi
- Conclusion

EVD Methods

- **Exact Jacobi**
- **Systolic Array**
- Approximate Jacobi
- Algebraic Method (only for 3x3 matrix)

Eigen Value Decomposition (EVD)

- Special case of Singular Value Decomposition (SVD) where the Matrix is Square-Symmetric
- Consider a Matrix $A \epsilon R^{mxn}$

  SVD: $A = UDV^T$

  EVD: $A = UDU^T$

  where,

  $D \epsilon R^{mxn}$ is diagonal matrix,

  $U \epsilon R^{mxn}$ & $V \epsilon R^{mxn}$ are orthogonal

Exact Jacobi

- Aims at annihilating the off diagonal elements using a series of orthogonal transformations

- $A^{(k+1)} = J^{T}_{pq} A^{(k)} J_{pq}$,

  where $A^{(0)}=A$

  J_{pq} is called the Jacobi Rotation Defined by the parameter (c s, -s c)

$$\boldsymbol{R}_{12}(\alpha) = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}, \quad c = \cos\alpha, \quad s = \sin\alpha$$

Limitations of Exact Jacobi Implementation

- Jacobi iterations are serial
- Inability to derive parallelism as iterations have large inter-loop **Data Dependency**
- Inability to pipeline
- Every iteration involves transfer of 4N-4 matrix elements to the processor
- Even though it is “MATRIX” operation, parallelism cannot be derived

Cordic

- Two Modules

ArcTan Module

Used to compute the $\tan^{-1}$ / angle for constructing the Jacobi Rotation Matrix

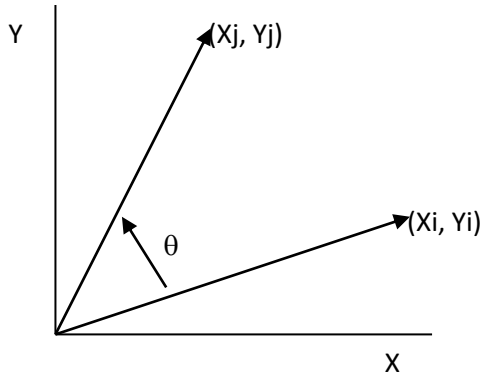
Sine/Cosine Module

$$\begin{bmatrix} \cos & \sin \\ -\sin & \cos \end{bmatrix}$$

2x2 matrix is constructed using the angle from the ArcTan module

Pipelined Cordic

- Basic Scheme



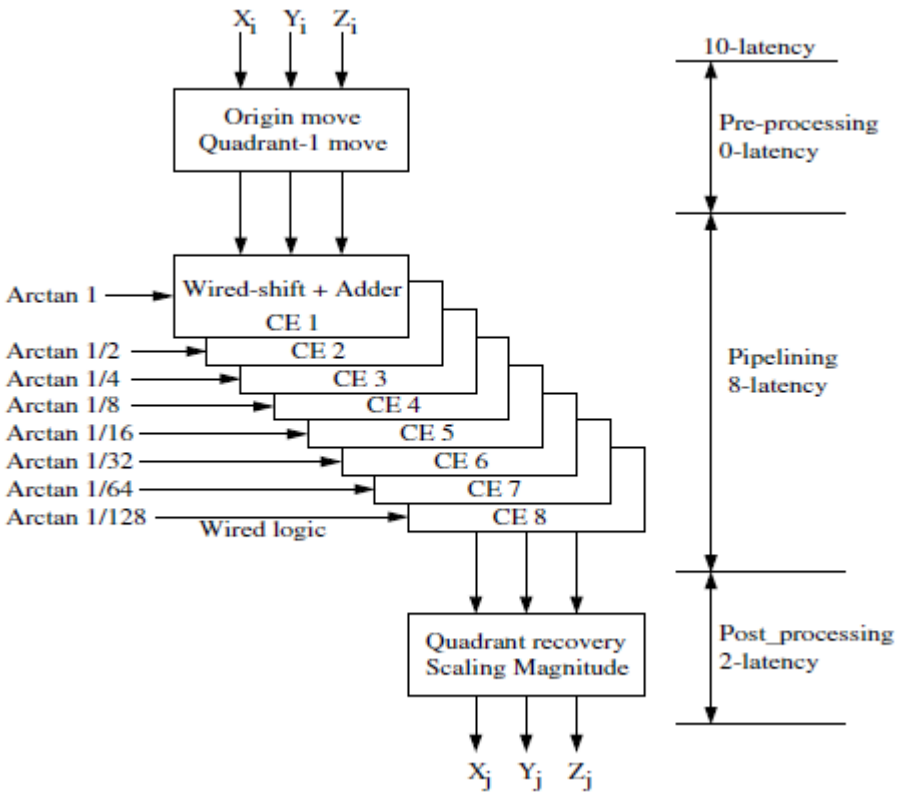
- Sine and Cosine can be calculated using the first CORDIC scheme which calculates:

$$X_i = \frac{1}{P} = \frac{1}{1.6467} \approx 0.60725$$

$$Y_i = 0$$

$$Z_i = \theta$$

$$K = \frac{1}{P} = \prod_{n=0}^{\infty} \cos\left(\arctan\left(\frac{1}{2^n}\right)\right) \approx 0.607253$$


X_i Y_i Z_i
Origin move
Quadrant-1 move
Arctan 1
Wired-shift + Adder
CE 1
Arctan 1/2
CE 2
Arctan 1/4
CE 3
Arctan 1/8
CE 4
Arctan 1/16
CE 5
Arctan 1/32
CE 6
Arctan 1/64
CE 7
Arctan 1/128
CE 8
Wired logic
Quadrant recovery
Scaling Magnitude
X_j Y_j Z_j
10-latency
Pre-processing
0-latency
Pipelining
8-latency
Post_processing
2-latency

Result

- Actual Result
- Input 30 degree
- 2^15 = 1.0
- 28451*1.0/2^15 = 0.868
- 16257*1.0/2^15 = 0.496
- Expected Result
- 0.5*2^15 = 16384
- 0.866*2^15 = 28378