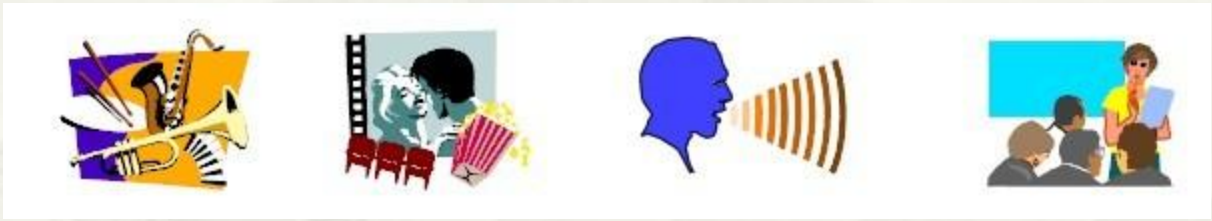
ECE 641
LABORATORY FOR DIGITAL SIGNAL PROCESSING WITH FIELD PROGRAMMABLE GATE ARRAYS

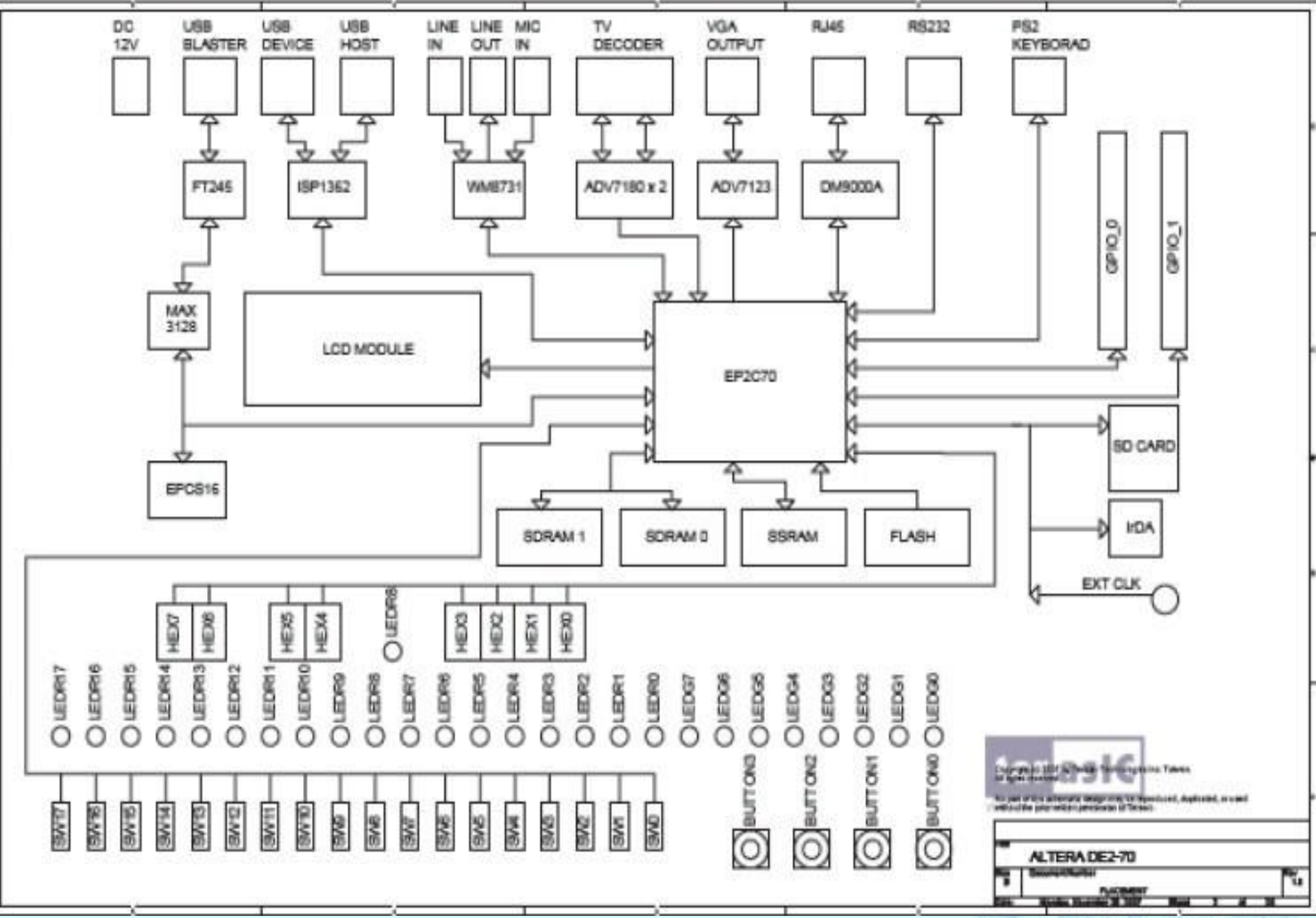


DCT and Transform Coding

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NewJersy Institude of Technology, NJ
4.28.2011

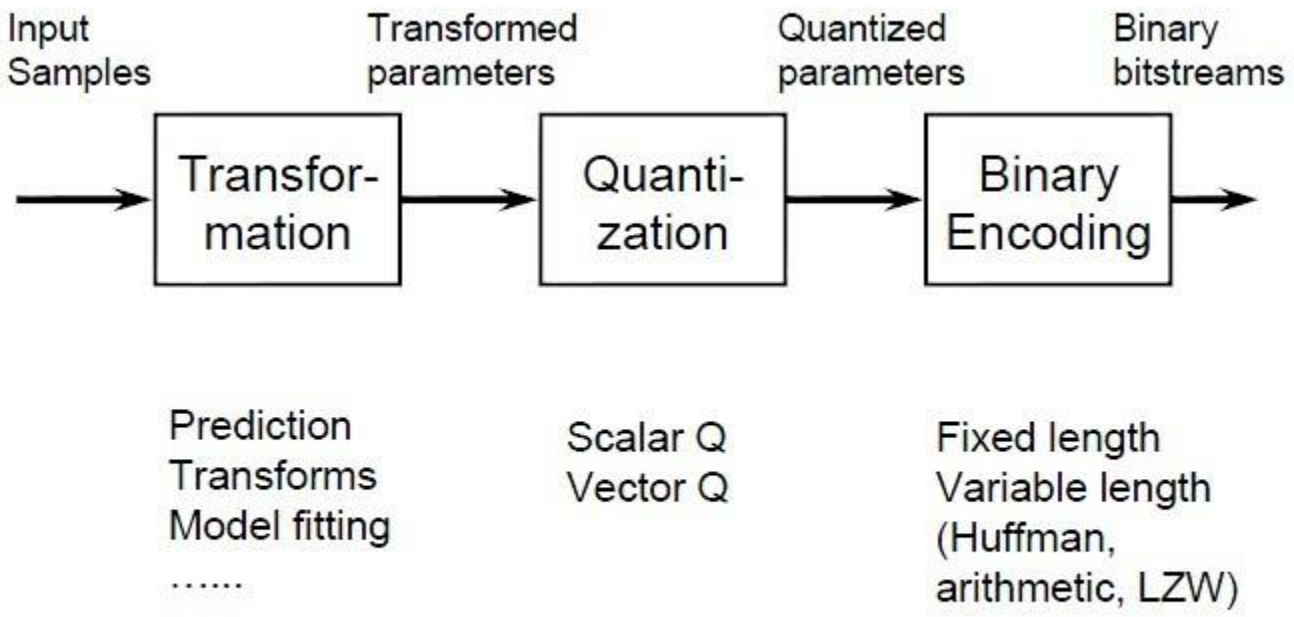
outlines

- • Transform coding
- – typical procedure.
- • DCT
- – Definition.
- – How it works in image transform.(Simulation on FPGA)
- – Advantage .



DE2_70_schematics

A typical compression system

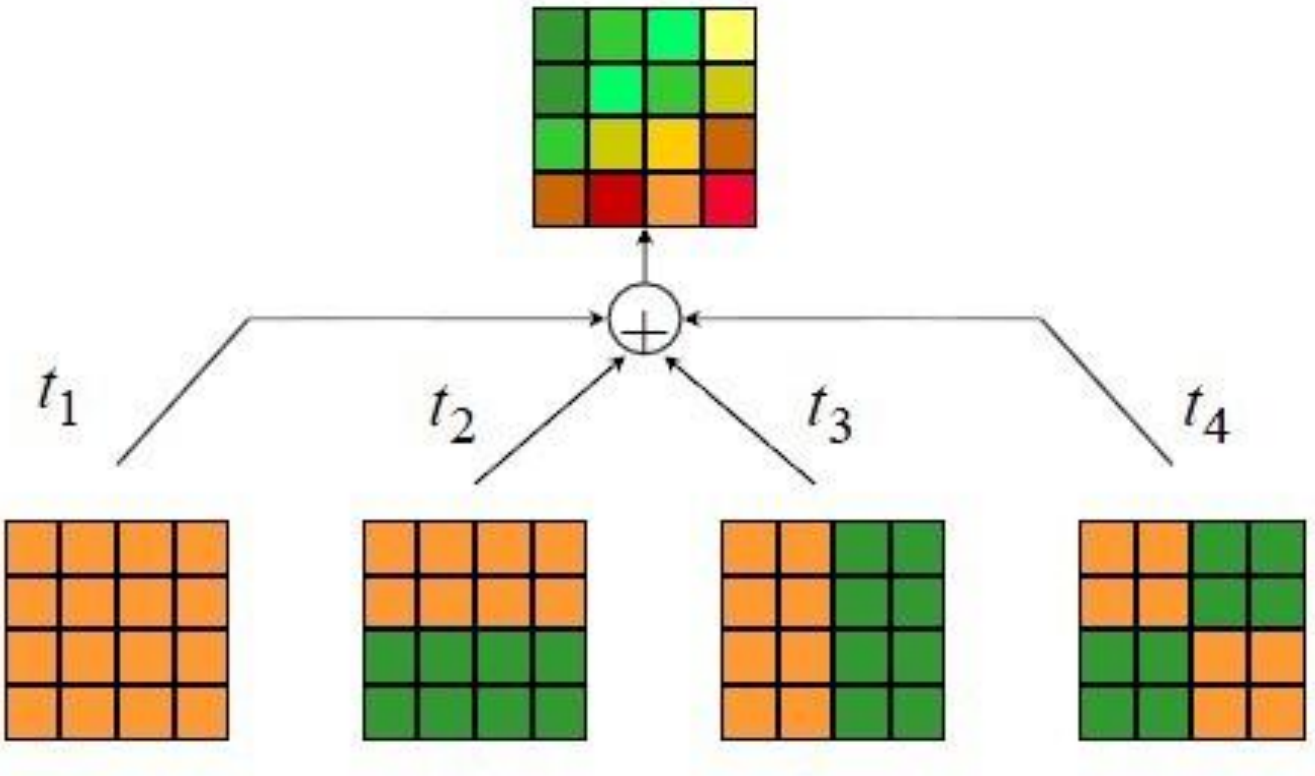


Motivation for transformation

- • Motivation for transformation:
- – To yield a more efficient representation of the original samples.
- – The transformed parameters should require fewer bits to code.

Transform coding

• Represent an image as the linear combination of some basis images and specify the linear coefficients.



1D DCT transform

- Can be considered "real" version of DFT
  - Basis vectors contain only co-sinusoidal patterns

DFT

$$u_{k,n} = \frac{1}{\sqrt{N}} \exp\left(j \frac{2\pi k}{N} n \right) = \frac{1}{\sqrt{N}} \left(\cos\left(\frac{2\pi k}{N} n \right) + j \sin\left(\frac{2\pi k}{N} n \right) \right)$$

$$\mathbf{u}_k = \frac{1}{\sqrt{N}} \begin{bmatrix} \cos\left(\frac{2\pi k}{N} 0 \right) \\ \cos\left(\frac{2\pi k}{N} 1 \right) \\ \cdots \\ \cos\left(\frac{2\pi k}{N} (N-1) \right) \end{bmatrix} + j \frac{1}{\sqrt{N}} \begin{bmatrix} \sin\left(\frac{2\pi k}{N} 0 \right) \\ \sin\left(\frac{2\pi k}{N} 1 \right) \\ \cdots \\ \sin\left(\frac{2\pi k}{N} (N-1) \right) \end{bmatrix}$$

DCT

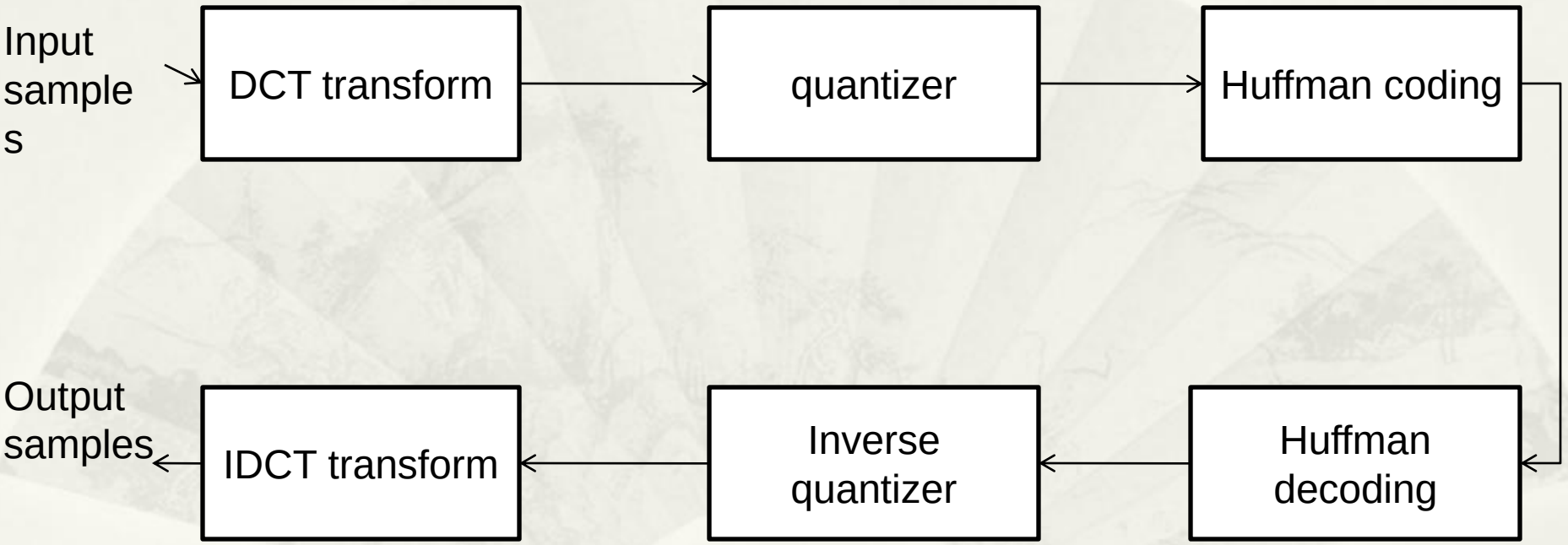
$$u_{k,n} = \alpha(k) \cos\left(\frac{\pi k}{2N} (2n+1) \right)$$

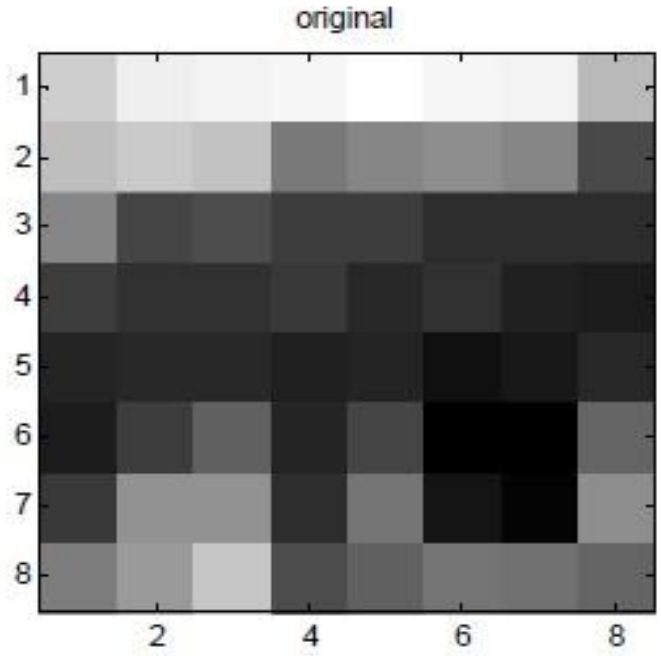
$$\alpha(0) = \sqrt{\frac{1}{N}}, \alpha(k) = \sqrt{\frac{2}{N}}, k = 1,2,\ldots,N-1$$

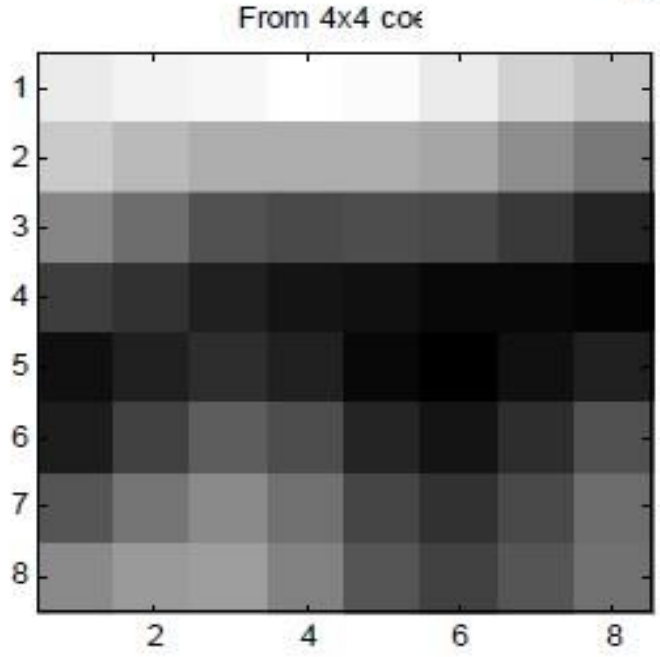
$$\mathbf{u}_k = \alpha(k) \begin{bmatrix} \cos\left(\frac{\pi k}{2N} 1 \right) \\ \cos\left(\frac{\pi k}{2N} 3 \right) \\ \cdots \\ \cos\left(\frac{\pi k}{2N} (2N+1) \right) \end{bmatrix}$$

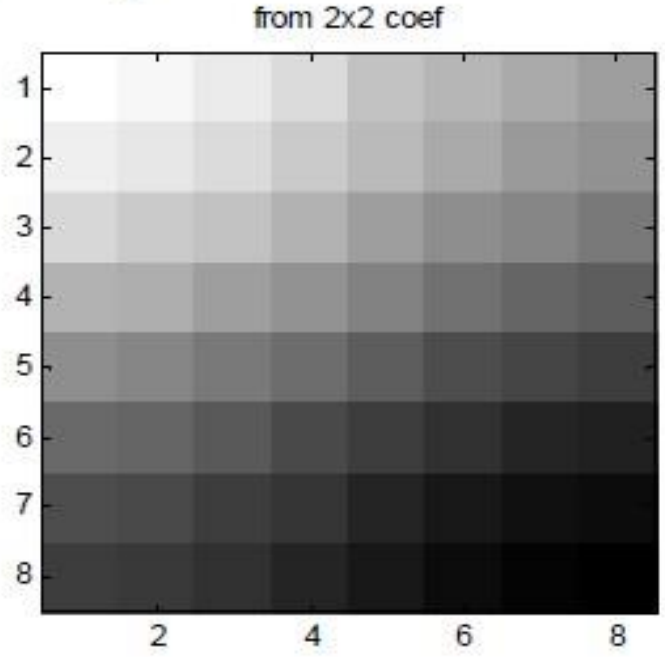
DCT Transform

*




original
1
2
3
4
5
6
7
8
2
4
6
8


From 4x4 coe
1
2
3
4
5
6
7
8
2
4
6
8


from 2x2 coef
1
2
3
4
5
6
7
8
2
4
6
8

Original



With 16/64 Coefficients

With 8/64 Coefficients

With 4/64 Coefficients

Example

$$S = \begin{bmatrix} 120 & 137 & 146 & 147 & 143 & 138 & 131 & 123 \\ 111 & 125 & 140 & 135 & 156 & 138 & 135 & 119 \\ 99 & 114 & 146 & 147 & 150 & 155 & 144 & 135 \\ 94 & 105 & 129 & 153 & 146 & 138 & 149 & 143 \\ 97 & 97 & 119 & 140 & 155 & 144 & 137 & 141 \\ 88 & 102 & 125 & 141 & 134 & 134 & 147 & 146 \\ 85 & 103 & 122 & 126 & 143 & 140 & 141 & 147 \\ 76 & 97 & 117 & 128 & 126 & 140 & 143 & 135 \end{bmatrix}$$

DCT

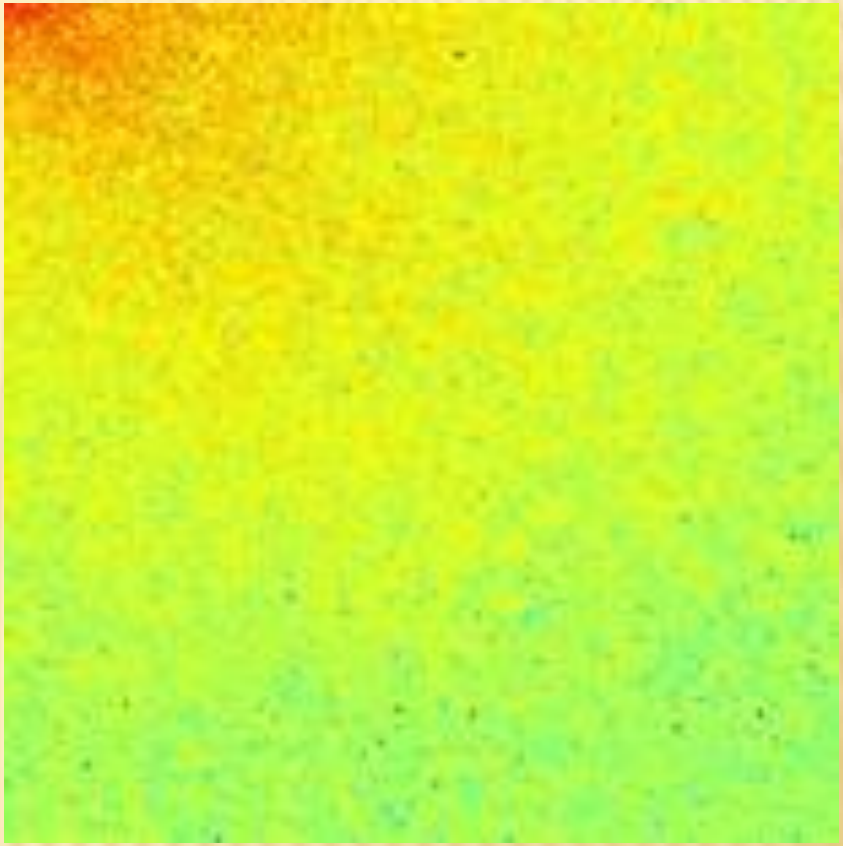
$$C = \begin{bmatrix} 478 & -101 & -78 & -20 & -6 & 3 & -1 & 7 \\ 37 & 51 & -10 & 7 & -3 & -3 & 0 & 5 \\ -9 & 22 & 8 & 1 & -16 & -7 & -1 & -1 \\ 1 & 11 & 8 & -1 & 9 & -3 & -3 & 0 \\ -5 & 1 & 0 & 1 & 6 & 5 & -3 & -7 \\ 10 & -3 & 4 & -10 & -1 & 2 & 0 & -4 \\ 4 & -1 & 0 & -9 & -8 & 10 & 3 & -9 \\ 5 & -1 & -4 & 9 & 1 & -9 & 10 & 1 \end{bmatrix}$$

$$W = \begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix}$$

$$C_{ij} = \begin{bmatrix} 480 & -99 & -80 & -16 & 0 & 0 & 0 & 0 \\ 36 & 48 & -14 & 0 & 0 & 0 & 0 & 0 \\ -14 & 26 & 16 & 0 & 0 & 0 & 0 & 0 \\ 0 & 17 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Quantification matrix

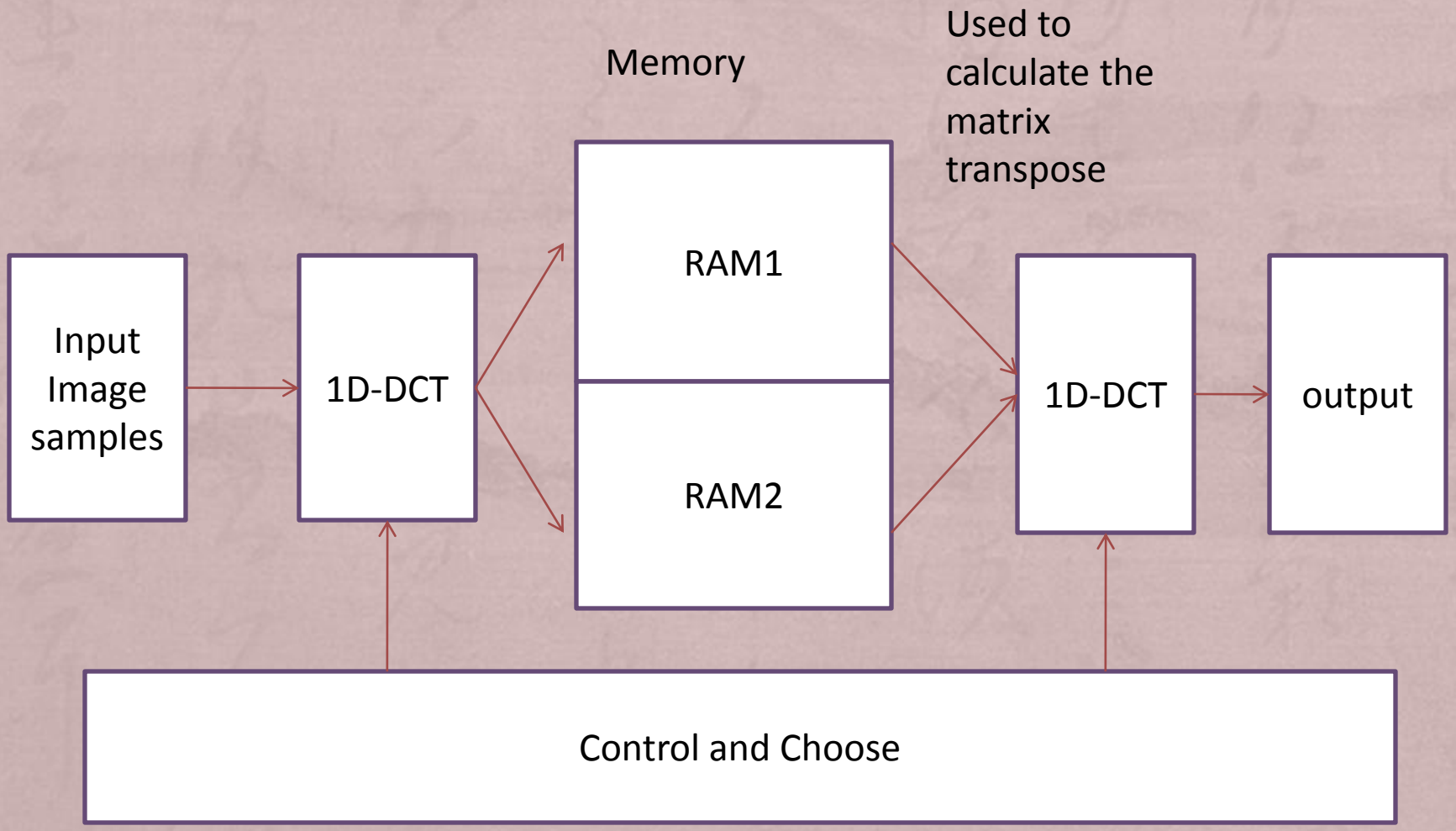
Energy distribution of the image after DCT



Original image



2D-DCT structure



1D dct image sample

The vector used for experiment

A=(1 8 12 3)

Result vector

C=(12 -2 -8 2)

DCT Matrix

dct_matrix <4x4 double>

	1	2	3	4
1	0.5000	0.5000	0.5000	0.5000
2	0.6533	0.2706	-0.2706	-0.6533
3	0.5000	-0.5000	-0.5000	0.5000
4	0.2706	-0.6533	0.6533	-0.2706

Result from matlab

```
>> dct([ 1 8 12 3])

ans =

   12.0000   -2.3890   -8.0000    2.0719
```

Flow Status	Successful - Wed May 04 02:48:51 2011
Quartus II Version	9.1 Build 222 10/21/2009 SJ Web Edition
Revision Name	DCT
Top-level Entity Name	DCT
Family	Cyclone II
Device	EP2C70F896C6
Timing Models	Final
Met timing requirements	Yes
Total logic elements	196 / 68,416 (< 1 %)
Total combinational functions	196 / 68,416 (< 1 %)
Dedicated logic registers	0 / 68,416 (0 %)
Total registers	0
Total pins	81 / 622 (13 %)
Total virtual pins	0
Total memory bits	0 / 1,152,000 (0 %)
Embedded Multiplier 9-bit elements	0 / 300 (0 %)
Total PLLs	0 / 4 (0 %)

RESULT ANALYSIS

- DCT cut off the correlations between the coefficients and focus the energy, reduce the taken space of the image.
- DCT cause little image distortion in compression.
- The errors in the experiment are caused by the accuracy of multiplier and the approximate values.

THANK YOU!