

# The Binomial QMF-Wavelet Transform for Multiresolution Signal Decomposition

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**Abstract**—This paper describes a class of orthogonal binomial filters that provide a set of basis functions for a bank of perfect reconstruction (PR) finite impulse response quadrature mirror filters (FIR QMF). These binomial QMF's are shown to be the same filters as those derived from a discrete orthonormal wavelet transform approach by Daubechies. These filters are the unique maximally flat magnitude square PR QMF's. It is shown that the binomial QMF outperforms the discrete cosine transform objectively for AR(1) sources and test images considered.

## I. INTRODUCTION

Perfect reconstruction quadrature mirror filters (PR QMF's) have been proposed as structures suitable for hierarchical subband coding [1]–[4], and also for multiresolution signal decomposition as might be used in image pyramid coding [5]. More recently, multiresolution signal decomposition methods are being examined from the standpoint of the discrete wavelet transform for continuous-time signals [6]–[8]. In this paper, we describe a class of orthogonal binomial filters that provide basis functions for a perfect reconstruction bank of finite impulse response QMF's. The orthonormal wavelet filters derived by Daubechies [7] from a discrete wavelet transform approach are shown to be the same as the solutions inherent in the binomial-based filters.

The energy compaction performance of the binomial QMF decomposition is computed and shown to be better than the DCT for the Markov source models, as well as real-world images considered. The proposed binomial structure is efficient, simple to implement on VLSI, and suitable for multiresolution signal decomposition and coding applications.

## II. THE BINOMIAL-HERMITE FAMILY

The binomial-Hermite sequences [9] are a family of finite duration discrete polynomials weighted by a Gauss-

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ian-like binomial envelope. These sequences are orthogonal on  $[0, N]$  with respect to a weighting function [10]. The binomial sequence  $\binom{N}{k}$  is the generating function of this family; the other members are obtained by successive differencing of this kernel.

In this section, we summarize a few features of the binomial-Hermite family. The generating function of this family is

$$x_0(k) = \begin{cases} \binom{N}{k} = \frac{N!}{(N-k)!k!}, & 0 \leq k \leq N \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

Successive differencing

$$x_r(k) = \nabla^r \binom{N-r}{k}, \quad r = 0, 1, \dots, N \quad (2)$$

leads to

$$\begin{aligned} x_r(k) &= \binom{N}{k} \sum_{\nu=0}^r (-2)^\nu \binom{r}{\nu} \frac{k^{(\nu)}}{N^{(\nu)}} \\ &= \binom{N}{k} H_r(k), \quad k = 0, 1, \dots, N \end{aligned} \quad (3)$$

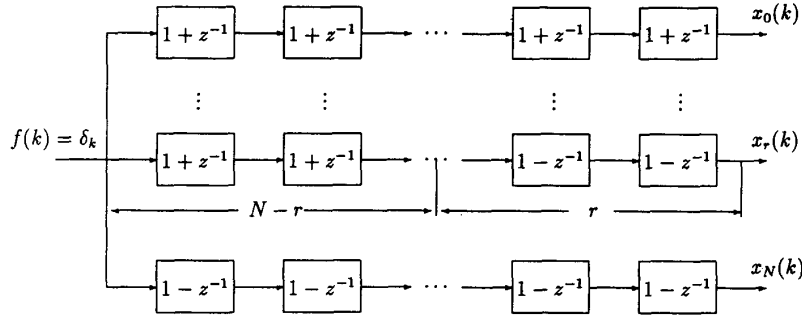
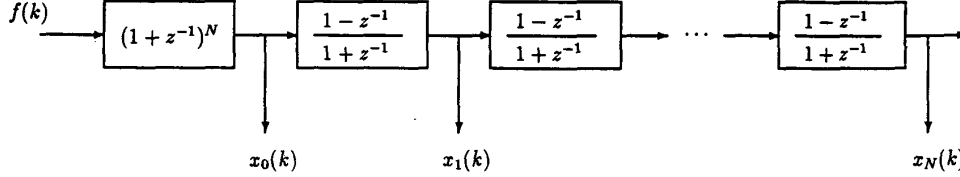
where  $k^{(\nu)}$  is a polynomial in  $k$  of degree  $\nu$

$$k^{(\nu)} = \begin{cases} k(k-1) \cdots (k-\nu+1), & \nu \geq 1 \\ 1, & \nu = 0. \end{cases} \quad (4)$$

A network realization of this family of filters is shown in Fig. 1. This structure represents an interconnection of add and difference operators, in a purely nonrecursive FIR form. Yet, another configuration arises from the representation

$$\begin{aligned} X_r(z) &= \left( \frac{1-z^{-1}}{1+z^{-1}} \right) X_{r-1}(z) \\ &= \left( \frac{1-z^{-1}}{1+z^{-1}} \right)^r X_0(z) = G_r(z) X_0(z). \end{aligned} \quad (5)$$

This form, (5) suggests the bank of filters shown in Fig. 2. The advantage of this structure is evident—the entire family is obtained by simply tapping off the appropriate point in Fig. 2. Since each  $(1-z^{-1})/(1+z^{-1})$  block can be synthesized with one delay element, the pole-zero cancellation structure of Fig. 2 can be synthesized with  $2N$  delay elements as compared with  $N^2$  delays in Fig. 1.

Fig. 1. Bank of binomial-Hermite filters realized using  $N^2$  delay elements.Fig. 2. Bank of binomial-Hermite filters using pole-zero cancellation, and only  $2N$  delay elements.

The pole-zero cancellation implicit in (5) can be achieved exactly since all coefficients are  $\pm 1$ . However, care must be taken to clear all registers before data is inputted to the front end of the filter. That is to say, the initial state must be zero to ensure stability. At any rate, either realization is achieved without multiply operations.

For a given  $N$ , we define the cross correlation of the sequences  $x_r(n)$ , and  $x_s(n)$  by

$$\rho_{rs}(n) = x_r(n) * x_s(-n) = \sum_{k=0}^N x_r(k)x_s(n+k) \leftrightarrow R_{rs}(z) \quad (6)$$

and

$$R_{rs}(z) = X_r(z^{-1})X_s(z). \quad (7)$$

Now for any real cross correlation

$$\rho_{rs}(-n) = \rho_{sr}(n) \quad \forall s, r. \quad (8)$$

Furthermore,

$$\begin{aligned} \rho_{rs}(n) &= -\rho_{sr}(n) & (s-r) \text{ is odd} \\ \rho_{rs}(n) &= \rho_{sr}(n) & (s-r) \text{ is even.} \end{aligned} \quad (9)$$

We can build up higher order correlation matrices from lower order ones. Using superscript notation, we can easily show that

$$R_{rs}^{(N+1)}(z) = (z + 2 + z^{-1})R_{rs}^{(N)}(z)$$

or

$$\rho_{rs}^{(N+1)}(k) = (\delta_{k+1} + 2\delta_k + \delta_{k-1}) * \rho_{rs}^{(N)}(k). \quad (10)$$

These cross correlations will be used later in the design of binomial QMF.

### III. TWO-CHANNEL PR-QMF BANK

The conditions for perfect reconstruction in the prototype two-channel FIR filter bank of Fig. 3 have been determined by several authors [1], [4]. Tracing the signals through top and bottom branches gives the reconstructed signal as

$$\hat{X}(z) = T(z)X(z) + S(z)X(-z) \quad (11)$$

where

$$T(z) = \frac{1}{2}[H_1(z)K_1(z) + H_2(z)K_2(z)]$$

$$S(z) = \frac{1}{2}[H_1(-z)K_1(z) + H_2(-z)K_2(z)]. \quad (12)$$

Perfect reconstruction requires

$$\text{i) } S(z) = 0 \quad \text{for all } z \quad (13)$$

$$\text{ii) } T(z) = cz^{-n_0} \quad c \text{ a constant.} \quad (14)$$

The choice of

$$K_1(z) = -H_2(-z)$$

$$K_2(z) = H_1(-z)$$

satisfies the first requirement  $S(z) = 0$  and eliminates the aliasing. Next, with  $N$  odd, one can choose

$$H_2(z) = z^{-N}H_1(-z^{-1}) \quad (15)$$

leaving us with the familiar

$$T(z) = \frac{1}{2}z^{-N}[H_1(z)H_1(z^{-1}) + H_1(-z)H_1(-z^{-1})]. \quad (16)$$

Therefore, with these constraints, this perfect reconstruction requirement reduces to finding an  $H(z) = H_1(z)$  such

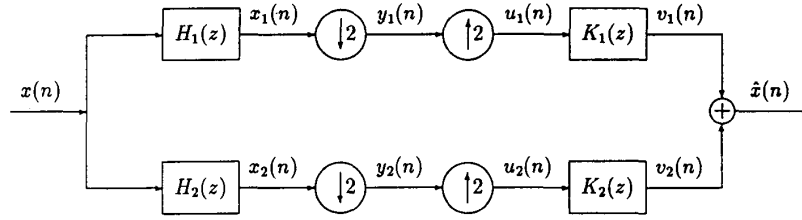


Fig. 3. Two-channel QMF bank.

that

$$\begin{aligned} Q(z) &= H(z)H(z^{-1}) + H(-z)H(-z^{-1}) = \text{constant} \\ &= R(z) + R(-z). \end{aligned} \quad (17)$$

This selection implies that all four filters are causal whenever  $H_1(z)$  is causal.

The PR requirement (17) can be readily recast in an alternate time domain form. First, one notes that  $R(z)$  is a spectral density function and hence is representable by a finite series of the form

$$\begin{aligned} R(z) &= \gamma_N z^N + \gamma_{N-1} z^{N-1} + \cdots + \gamma_0 z^0 \\ &\quad + \cdots + \gamma_N z^{-N} \end{aligned} \quad (18)$$

Then

$$\begin{aligned} R(-z) &= -\gamma_N z^N + \gamma_{N-1} z^{N-1} - \cdots - \gamma_0 z^0 \\ &\quad - \gamma_1 z^{-1} \cdots - \gamma_N z^{-N}. \end{aligned} \quad (19)$$

Therefore,  $Q(z)$  consists only of even-powered  $z$ . To force  $Q(z) = \text{constant}$ , it suffices to make all even-indexed coefficients in  $R(z)$  equal to zero except  $\gamma_0$ .

However, the  $\gamma_n$  coefficients in  $R(z)$  are simply the samples of the autocorrelation  $\rho(n)$  given by

$$\begin{aligned} \rho(n) &= \sum_{k=0}^N h(k)h(k+n) = \rho(-n) \\ &\stackrel{\text{def}}{=} h(n) \odot h(n) \end{aligned} \quad (20)$$

where  $\odot$  indicates a correlation operation. This follows from the  $z$ -transform relationships

$$R(z) = H(z)H(z^{-1}) \leftrightarrow h(n) * h(-n) = \rho(n) \quad (21)$$

where  $\rho(n)$  is the convolution of  $h(n)$  with  $h(-n)$ , or equivalently, the time autocorrelation (20).

Hence, we need to set  $\rho(n) = 0$  for  $n$  even, and  $n \neq 0$ . Therefore

$$\rho(2n) = \sum_{k=0}^N h(k)h(k+2n) = 0, \quad n \neq 0. \quad (22)$$

If the normalization is imposed

$$\sum_{k=0}^N |h(k)|^2 = 1 \quad (23)$$

one obtains the PR requirement as

$$\sum_{k=0}^N h(k)h(k+2n) = \delta_n. \quad (24)$$

#### IV. THE BINOMIAL QMF

It is now a straightforward matter to impose PR condition of (24) on the binomial family. First, we take as the low-pass filter

$$h(n) = \sum_{r=0}^{(N-1)/2} \theta_r x_r(n)$$

or

$$\begin{aligned} H(z) &= \sum_{r=0}^{(N-1)/2} \theta_r (1+z^{-1})^{N-r} (1-z^{-1})^r \\ &= (1+z^{-1})^{(N+1)/2} F(z) \end{aligned} \quad (25)$$

where  $F(z)$  is FIR filter of order  $(N-1)/2$ . For convenience, we take  $\theta_0 = 1$ , and later impose the normalization of (23). Substituting (25) into (20) gives

$$\begin{aligned} \rho(n) &= \left( \sum_{r=0}^{(N-1)/2} \theta_r x_r(n) \right) \odot \sum_{s=0}^{(N-1)/2} \theta_s x_s(n) \\ &= \sum_{r=0}^{(N-1)/2} \sum_{s=0}^{(N-1)/2} \theta_r \theta_s [x_r(n) \odot x_s(n)] \\ &= \sum_{r=0}^{(N-1)/2} \sum_{s=0}^{(N-1)/2} \theta_r \theta_s \rho_{rs}(n) \\ &= \sum_{r=0}^{(N-1)/2} \theta_r^2 \rho_{rr}(n) + \sum_{\substack{r=0 \\ r \neq s}}^{(N-1)/2} \sum_{s=0}^{(N-1)/2} \theta_r \theta_s \rho_{rs}(n) \end{aligned} \quad (26)$$

where  $\rho_{rs}(n)$  is given by (6) and (8). Equation (9) implies that the second summation in (26) has only terms where the indices differ by an even integer. Therefore, the autocorrelation for the binomial half-bandwidth low-pass filter is

$$\begin{aligned} \rho(n) &= \sum_{r=0}^{(N-1)/2} \theta_r^2 \rho_{rr}(n) \\ &\quad + 2 \sum_{l=1}^{(N-3)/2} \sum_{\nu=0}^{\lfloor (N-1)/2 \rfloor - 2l} \theta_\nu \theta_{\nu+2l} \rho_{\nu, \nu+2l}(n). \end{aligned} \quad (27)$$

Finally, the PR requirement is

$$\rho(n) = 0, \quad n = 2, 4, \dots, N-1. \quad (28)$$

This condition gives a set of  $(N-1)/2$  nonlinear algebraic equations, in the  $(N-1)/2$  unknowns  $\theta_1, \theta_2, \dots, \theta_{(N-1)/2}$ . These equations were solved using Macsyma.

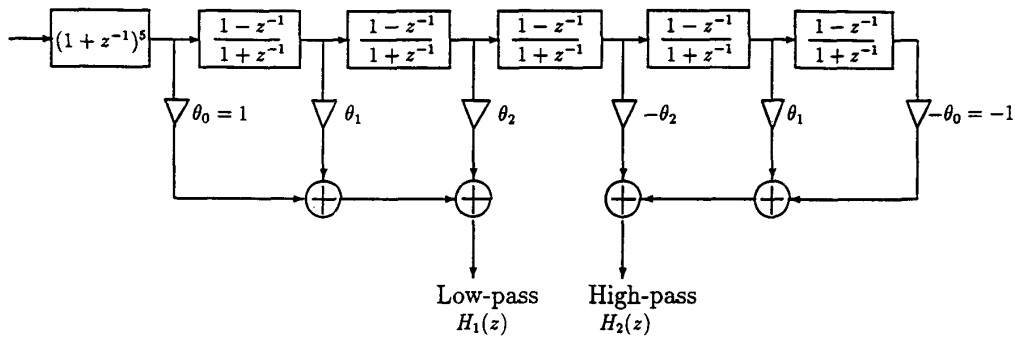


Fig. 4. Low-pass and high-pass QMF filters from binomial network.

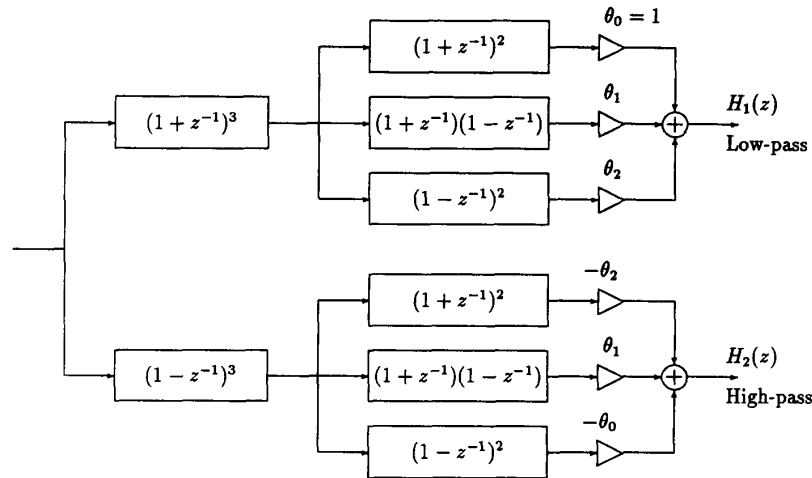


Fig. 5. Low-pass and high-pass QMF's using direct form binomial structure of Fig. 1.

The implementation of these half-bandwidth filters is trivially simple and efficient using either the purely FIR structure, or the pole-zero cancellation configuration. The latter is shown in Fig. 4 for  $N = 5$ , wherein both low-pass and high-pass filters are simultaneously realized. Fig. 5 shows the QMF bank using the direct form. Coefficient  $\theta_0$  can be taken equal to unity, leaving only  $\theta_1$  and  $\theta_2$  as tap weights. These are the only multiplications needed when using the binomial network as the half-bandwidth QMF rather than the six  $h(n)$  weights in a transversal structure.

The values of  $\theta_r$ , for  $N = 3, 5, 7$ , (corresponding to 4, 6, 8 tap filters, respectively) are given in Table I (where  $\theta_0 = 1$ ). As seen, there is more than one filter solution for a given  $N$ . For example, with  $N = 3$ , one obtains  $\theta_1 = \sqrt{3}$ , and also  $\theta_1 = -\sqrt{3}$ . The positive  $\theta_1$  corresponds to a minimum phase solution, while the negative  $\theta_1$  provides a nonminimum phase filter. The magnitude responses of both filters are identical. Although in our derivation, no linear phase constraint on  $h(n)$  was imposed; it is noteworthy that the phase responses are almost linear, the nonminimum phase filters even more so. The magnitude and phase responses of these minimum phase binomial QMF's are given in Fig. 6 for the cases  $N = 3, 5, 7$ .

TABLE I  
 $\theta_r$  VALUES FOR  $N = 3, 5, 7$ 

$\theta_r$	Set 1	Set 2	Set 3	Set 4
$N = 3$				
$\theta_0$	1	1		
$\theta_1$	$\sqrt{3}$	$-\sqrt{3}$		
$N = 5$				
$\theta_0$	1	1		
$\theta_1$	$\sqrt{2\sqrt{10}+5}$	$-\sqrt{2\sqrt{10}+5}$		
$\theta_2$	$\sqrt{10}$	$\sqrt{10}$		
$N = 7$				
$\theta_0$	1	1	1	1
$\theta_1$	4.9892	-4.9892	1.0290	-1.0290
$\theta_2$	8.9461	8.9461	-2.9705	-2.9705
$\theta_3$	5.9160	5.9160	-5.9160	5.9160

Table II provides the normalized 4, 6, 8 tap filter coefficients  $h(n)$  for both minimum and nonminimum phase cases.

We may recognize that these filters are the unique max-

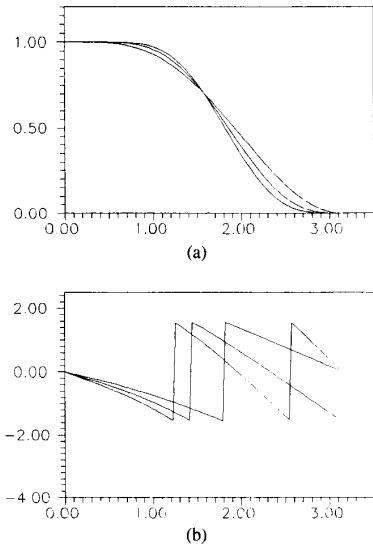


Fig. 6. (a) Amplitude and (b) phase responses of minimum phase binomial QMF's for  $N = 3, 5, 7$ .

TABLE II  
BINOMIAL QMF-WAVELET FILTERS  $h(n)$  FOR  $N = 3, 5, 7$

$n$	$h(n)$		
	4 tap	6 tap	8 tap
Miniphase			
0	0.48296291314453	0.33267055439701	0.23037781098452
1	0.83651630373780	0.80689151040469	0.71484656725691
2	0.22414386804201	0.45987749838630	0.63088077185926
3	-0.12940952255126	-0.13501102329922	-0.02798376387108
4		-0.08544127212359	-0.18703481339693
5		0.03522629355424	0.03084138344957
6			0.03288301895913
7			-0.01059739842942
Nonminimum Phase			
0	-0.1294095225512	0.0352262935542	-0.0105973984294
1	0.2241438680420	-0.0854412721235	0.0328830189591
2	0.8365163037378	-0.1350110232992	0.0308413834495
3	0.4829629131445	0.4598774983863	-0.1870348133969
4		0.8068915104046	-0.0279837638710
5		0.3326705543970	0.6308807718592
6			0.7148465672569
7			0.2303778109845
0			-0.0757657137833
1			-0.0296355292117
2			0.4976186593836
3			0.8037387521124
4			0.2978578127957
5			-0.0992195317257
6			-0.0126039690937
7			0.0322230981272
0			0.0322230981272
1			-0.0126039690937
2			-0.0992195317257
3			0.2978578127957
4			0.8037387521124
5			0.4976186593836
6			-0.0296355292117
7			-0.0757657137833

imally flat PR QMF solutions. In fact, it can be shown that the PR requirements of (17) are satisfied if we choose the  $\theta_r$  coefficients to satisfy maximally flat requirements at  $\omega = 0$ , and  $\omega = \pi$ . Explicitly, with  $R(\omega) = |H(e^{j\omega})|^2$ , we can set  $\theta_r$  to satisfy

$$R(0) = 1, \quad R(\pi) = 0$$

$$\left. \frac{d^k R(\omega)}{d\omega^k} \right|_{\omega=0} = 0, \quad k = 1, 2, \dots, N.$$

Herrmann [11] provides the unique maximally flat function on the interval  $[0, 1]$ . This function can be easily mapped onto the  $Z$  plane to obtain the maximally flat magnitude square function  $R(z)$  [12], [19]. Now, one can obtain the corresponding  $H(z)$  from  $R(z)$  via factorization. This approach extends Herrmann's solution to the PR QMF case. The explicit form of  $R(z)$  is given later in (31).

V. ORTHONORMAL WAVELET TRANSFORMS AND THE BINOMIAL QMF

The orthonormality condition on wavelet transforms leads to the wavelet filters that are PR QMF's themselves. Therefore, the theory of orthonormal wavelet transforms is strongly associated with the theory of orthonormal two-band PR QMF filter banks. We have demonstrated that the binomial QMF's are identical to the wavelet filters proposed by Daubechies [7]. Since wavelet approximations are made in the continuous domain, some regularity on the wavelet function is desired. This regularity actually implies the degree of differentiability of the wavelet basis functions. It imposes conditions on the corresponding wavelet filters. Since the design of wavelet bases starts with the design of the wavelet filters, one should define the connection between the PR QMF design and the behavior of the corresponding continuous-time wavelet functions. Daubechies showed that the number of zeros of the wavelet filters at  $\omega = \pi$  is related to the regularity of the corresponding wavelet function [7].

The regularity concept is unique to wavelet filters. Conventional PR QMF design does not invoke this requirement explicitly, except that the zero-mean condition on the high-pass QMF implies some degree of regularity. The binomial QMF has this feature inherent. The regularity tool suggested in [7] assumes a low-pass interscale sequence or filter of length  $N + 1$

$$H(z) = (1 + z^{-1})^k F(z) \quad 1 \leq k \leq \frac{N + 1}{2}.$$

Here,  $F(z)$  is a polynomial of degree  $(N + 1)/2 \leq l \leq N$ , such that  $k + l = N$ .

If  $k = (N + 1)/2$ , the maximum number of zeros of  $H(z)$  are located at  $\omega = \pi$ . Therefore,  $F(z)$  is of degree  $(N - 1)/2$ . But the binomial QMF,  $H(z)$  in (25), can now be written as

$$H(z) = (1 + z^{-1})^{(N+1)/2} \cdot \sum_{r=0}^{(N-1)/2} \theta_r (1 + z^{-1})^{(N-1)/2-r} (1 - z^{-1})^r \quad (29)$$

hence

$$F(z) = \sum_{r=0}^{(N-1)/2} \theta_r (1+z^{-1})^{[(N-1)/2]-r} (1-z^{-1})^r. \quad (30)$$

Combining this regular nature of  $H(z)$  with the PR requirement leads to the unique maximally flat magnitude square function [11], [12], [18]

$$\begin{aligned} R(z) &= H(z)H(z^{-1}) \\ &= \frac{z^N(1+z^{-1})^{N+1}}{4^{N+1}} \sum_{l=0}^{(N-1)/2} (-1)^l \binom{N}{l} \\ &\quad \cdot (1+z^{-1})^{N-1-2l} (1-z^{-1})^{2l} \end{aligned} \quad (31)$$

therefore

$$\begin{aligned} V(z) &= F(z)F(z^{-1}) \\ &= z^{(N-1)/2} \sum_{l=0}^{(N-1)/2} (-1)^l \binom{N}{l} \\ &\quad \cdot (1+z^{-1})^{N-1-2l} (1-z^{-1})^{2l}. \end{aligned} \quad (32)$$

$V(z)$  in (32) is identical to the polynomial used in [7]. The magnitude square function  $R(z)$  is a linear combination of the lower-half, even-indexed binomial sequences with length  $2N+1$ .  $H(z)$  is now obtained via factorization.

## VI. PERFORMANCE OF BINOMIAL QMF-WAVELET TRANSFORM

The performance of the binomial QMF signal decomposition scheme is compared with the industry standard—the discrete cosine transform (DCT) in this section.

The energy compaction power of any unitary transform is a commonly used performance criterion in the literature. The gain of transform coding over PCM at the same bit rate is defined as [13]

$$G_{TC} = \frac{\frac{1}{M} \sum_{k=0}^{M-1} \sigma_k^2}{\left[ \prod_{k=0}^{M-1} \sigma_k^2 \right]^{1/M}} \quad (33)$$

where  $\sigma_k^2$  are transform coefficient variances. This measure assumes that all coefficients, as well as the original signal, have the *same type* probability density function. This assumption is clearly correct only for Gaussian sources. Nevertheless, it is known in the literature that this measure is consistent with the observed experimental coding performance for block transforms.

Similarly, the gain of subband coding over PCM is defined as

$$G_{SBC} = \frac{\frac{1}{M} \sum_{l=0}^{M-1} \sigma_l^2}{\left[ \prod_{l=0}^{M-1} \sigma_l^2 \right]^{1/M}}. \quad (34)$$

Here  $\sigma_l^2$  is the variance of the signal in the  $l$ th subband.

TABLE III  
ENERGY COMPACTION COMPARISON: DCT VERSUS BINOMIAL QMF FOR SEVERAL AR(1) SOURCES

	$\rho$	$G_{SBC}$				
		$G_{TC}$	4-tap	6-tap	8-tap	16-tap
4 × 4 trans. or Four-band QMF (two levels)	0.95 0.85 0.75 0.65 0.5	5.71 2.59 1.84 1.49 1.23	6.43 2.82 1.95 1.56 1.26	6.77 2.95 2.02 1.60 1.28	6.91 3.01 2.05 1.62 1.29	7.08 3.07 2.09 1.64 1.30
8 × 8 trans. or Eight-band QMF (three level)	0.95 0.85 0.75 0.65 0.5	7.63 3.03 2.03 1.59 1.27	8.01 3.11 2.06 1.60 1.28	8.53 3.27 2.14 1.65 1.30	8.74 3.34 2.17 1.67 1.31	8.99 3.42 2.22 1.69 1.32

TABLE IV  
ENERGY COMPACTION COMPARISON: DCT VERSUS BINOMIAL QMF FOR SEVERAL TEST IMAGES

		$G_{SBC}$			
		$G_{TC}$	4-tap	6-tap	8-tap
4 × 4 2-D trans. or 16-band regular tree	LENA BUILDING CAMERAMAN BRAIN	16.002 14.107 14.232 3.295	16.70 15.37 15.45 3.25	18.99 16.94 16.91 3.32	20.37 18.17 17.98 3.42
8 × 8 2-D trans. or 64-band regular tree	LENA BUILDING CAMERAMAN BRAIN	21.988 20.083 19.099 3.788	19.38 18.82 18.43 3.73	22.12 21.09 20.34 3.82	24.03 22.71 21.45 3.93

This formula holds for a regular tree structure, implying equal bandwidths. It should be emphasized that this measure is valid only for unitary transforms or filter banks. It is properly modified for nonunitary transforms or unequal bandwidth filter banks, which are beyond the focus of this paper [14], [18].

We assume a Markov 1 source model with autocorrelation

$$R(k) = \rho^{|k|}, \quad k = 0, \pm 1, \dots, \quad (35)$$

and calculated  $G_{TC}$  and  $G_{SBC}$  for different cases. These results are displayed in Table III. Equations (33) and (34) are easily extended to the two-dimensional case for separable transforms and separable QMF's.

The energy compaction performance of the two techniques is also tested for several standard images. These results are given in Table IV.

The results demonstrate that the six-tap binomial QMF compacts the input signal energy better than the comparable sized DCT for both theoretical source models as well as for the standard test images considered.

The objective performance of binomial QMF and the optimal PR QMF designed based on energy compaction are very close for AR(1) sources [15].

In [16], binomial QMF's have been successfully employed for subband compression of high-definition television (HDTV). They compared the performance of several well-known filter banks at bit rates 0.5, 0.75, 1, 1.25

and higher bits/pixel. It is reported that six- and eight-tap binomial QMF's rated subjectively the best along with Johnston filters of lengths 12, 16, and 32 [17].

## VII. CONCLUSIONS

An efficient perfect reconstruction binomial QMF structure is developed. The new configuration utilizes the binomial network that has only addition operations. This approach provides a set of unique filter solutions with the maximally flat magnitude square functions. The phase responses of these filters are almost linear. These filters are the same as the orthonormal wavelet filters derived by Daubechies [7].

The binomial QMF-wavelet signal decomposition structures have better energy compaction than the industry standard DCT for Markov sources and the standard test images considered. Their good subjective performance for subband coding of HDTV was reported in [16]. These QMF's have a very simple algorithm to implement on VLSI, and may be a good competitor to the existing tools for signal decomposition and coding applications.

## REFERENCES

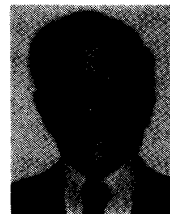
- [1] M. Smith and T. P. Barnwell, "Exact reconstruction techniques for tree-structured subband coders," *IEEE Trans. Acoust., Speech, Signal Processing*, pp. 434-441, 1986.
- [2] J. W. Woods and S. D. O'Neil, "Subband coding of images," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-34, no. 5, Oct. 1986.
- [3] H. Gharavi and A. Tabatabai, "Subband coding of digital image using two-dimensional quadrature mirror filtering," *Proc. SPIE Int. Soc. Opt. Eng.*, vol. 707, pp. 51-61, Sept. 1986.
- [4] P. P. Vaidyanathan, "Quadrature mirror filter banks, M-band extensions and perfect reconstruction techniques," *IEEE Trans. Acoust., Speech, Signal Processing Mag.*, pp. 4-20, June 1987.
- [5] P. J. Burt and E. H. Adelson, "The Laplacian pyramid as a compact image code," *IEEE Trans. Commun.*, pp. 532-540, Apr. 1983.
- [6] S. G. Mallat, "A theory for multiresolution signal decomposition: the wavelet representation," Univ. of Pennsylvania, MS-CIS-87-22, GRASP Lab. 103, May 1987.
- [7] I. Daubechies, "Orthonormal bases of compactly supported wavelets," *Commun. Pure Appl. Math.*, vol. XLI, pp. 909-996, 1988.
- [8] S. G. Mallat, "Multifrequency channel decomposition of images and wavelet models," *IEEE Trans. Acoust., Speech, Signal Processing*, pp. 2091-2110, Dec. 1989.
- [9] R. A. Haddad, "A class of orthogonal nonrecursive binomial filters," *IEEE Trans. Audio Electroacoust.*, pp. 296-304, Dec. 1971.
- [10] R. A. Haddad and A. N. Akansu, "A new orthogonal transform for signal coding," *IEEE Trans. Acoust., Speech, Signal Processing*, pp. 1404-1411, Sept. 1988.
- [11] O. Herrmann, "On the approximation problem in nonrecursive digital filter design," *IEEE Trans. Circuit Theory*, vol. CT-18, no. 3, pp. 411-413, May 1971.
- [12] J. A. Miller, "Maximally flat nonrecursive digital filters," *Electronics Lett.*, vol. 8, no. 6, pp. 157-158, Mar. 1972.
- [13] N. S. Jayant and P. Noll, *Digital Coding of Waveforms*. Englewood Cliffs, NJ: Prentice-Hall, 1984.
- [14] A. N. Akansu and Y. Liu, "On signal decomposition techniques," *Optical Eng.*, pp. 912-920, July 1991.
- [15] H. Caglar, Y. Liu, and A. N. Akansu, "Statistically optimized PR-QMF design," *Proc. SPIE Visual Communication and Image Processing*, pp. 86-94, Nov. 1991.
- [16] J. W. Woods and T. Naveen, "Subband compression of HDTV," in *Proc. 4th Int. Workshop on HDTV*, Turin, Italy, Feb. 1991.
- [17] J. D. Johnston, "A filter family designed for use in quadrature mirror filter banks," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, Apr. 1980, pp. 291-294.
- [18] A. N. Akansu and R. A. Haddad, *Multiresolution Signal Decomposition: Transforms, Subbands and Wavelets*. New York: Academic, 1992.
- [19] H. Caglar and A. N. Akansu, "A generalized, parametric PR-QMF design technique based on Bernstein polynomial approximation," *IEEE Trans. Signal Processing*, to be published, July 1993.



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