

ALLOWED IN EXAM ROOM IF NO OTHER MARKINGS ON IT

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}.$$

For $a \neq 1$, and $|b| < 1$ we have that

$$\sum_{i=0}^n a^i = \frac{a^{n+1} - 1}{a - 1}, \quad \sum_{i=0}^{n-1} ia^i = \frac{(n-1)a^{n+1} - na^n + a}{(1-a)^2},$$

$$\sum_{i=0}^{\infty} b^i = \frac{1}{1-b}, \quad \sum_{i=1}^{\infty} b^i = \frac{b}{1-b}, \quad \sum_{i=0}^{\infty} ib^i = \frac{b}{(1-b)^2}.$$

$$H_n = \sum_{i=1}^n \frac{1}{i}, \quad \sum_{i=1}^n iH_i = \frac{n(n+1)}{2}H_n - \frac{n(n-1)}{4}.$$

$$n! = \sqrt{2\pi n} \left(\frac{n}{e}\right)^n \left(1 + \Theta\left(\frac{1}{n}\right)\right), \quad n! \approx \left(\frac{n}{e}\right)^n, \quad a^{\log_b n} = n^{\log_b a},$$

$$e \approx 2.718281, \quad \pi \approx 3.14159, \quad \gamma \approx 0.57721, \quad \phi = \frac{1+\sqrt{5}}{2} \approx 1.61803, \quad \hat{\phi} = \frac{1-\sqrt{5}}{2} \approx -.61803.$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \quad \sum_{k=0}^n \binom{n}{k} = 2^n, \quad \binom{n}{k} = \binom{n}{n-k}, \quad \binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}.$$

A1. $f(n) = o(g(n))$, iff $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0$.

A2. $f(n) = \omega(g(n))$, iff $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \infty$.

A3. If $f(n) = o(g(n))$, then $f(n) = O(g(n))$.

A4. If $f(n) = \omega(g(n))$, then $f(n) = \Omega(g(n))$.

A5. If $f(n) = \Theta(g(n))$, iff $f(n) = \Omega(g(n))$ and $f(n) = O(g(n))$.

B1. $f(n) = \Theta(g(n))$ iff \exists positive constants c_1, c_2, n_0 : $0 \leq c_1g(n) \leq f(n) \leq c_2g(n) \quad \forall n \geq n_0$.

B2. $f(n) = \Omega(g(n))$ iff \exists positive constants c_1, n_0 : $0 \leq c_1g(n) \leq f(n) \quad \forall n \geq n_0$.

B3. $f(n) = O(g(n))$ iff \exists positive constants c_2, n_0 : $0 \leq f(n) \leq c_2g(n) \quad \forall n \geq n_0$.

L1.

$$\lg(ab) = \lg a + \lg b, \quad \lg(a/b) = \lg a - \lg b, \quad \lg(a^b) = b \lg a, \quad 2^{\lg(a)} = a, \quad a^x a^y = a^{x+y}, \quad a^x/a^y = a^{x-y}, \quad (a^x)^y = a^{xy}.$$

Master Method. $T(n) = aT(n/b) + f(n)$, such that $a \geq 1, b > 1$.

M1 If $f(n) = O(n^{\lg_b a - \epsilon})$ for some constant $\epsilon > 0$, then $T(n) = \Theta(n^{\lg_b a})$.

M2 If $f(n) = \Theta(n^{\lg_b a})$, then $T(n) = \Theta(n^{\lg_b a} \lg n)$.

M3 If $f(n) = \Omega(n^{\lg_b a + \epsilon})$ for some constant $\epsilon > 0$, and if $af(n/b) \leq cf(n)$ for some constant $c < 1$ and for large n , then $T(n) = \Theta(f(n))$.

D1.

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x), \quad \left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}, \quad (c^x)' = \ln(c) c^x.$$

S1.

$$\frac{1}{1-x} = 1+x+x^2+\dots+x^i+\dots = \sum_{i=0}^{\infty} x^i, \quad \frac{x}{(1-x)^2} = x+2x^2+\dots+ix^i+\dots = \sum_{i=0}^{\infty} ix^i, \quad e^x = 1+x+\frac{x^2}{2!}+\dots+\frac{x^i}{i!}+\dots = \sum_{i=0}^{\infty} \frac{x^i}{i!}.$$