

## Evaluation Quiz

Answer all questions in 30minutes  
The answers are on the next page.

1) How much is the following sum if  $x \neq 1$ ?

$$1 + x + x^2 + x^3 + \dots + x^n = ?$$

2) What if  $x = 1$  below?

$$1 + x + x^2 + x^3 + \dots + x^n = ?$$

3) How much is the following sum?

$$\sum_{i=1}^n i = ?$$

4) How much is the following sum?

$$\sum_{i=1}^n i^2 = ?$$

5) Let  $a + b = 100000$ , for some  $0 \leq a, b \leq 100000$ . What are the values  $a, b$  that maximize the product  $a \cdot b$ ?

6) For  $x \in R$ , we define  $y = \lg(x)$  such that  $2^y = x$ . Answer the following.

- $\lg(1) = ?$
- $\lg(128) = ?$
- $\lg(2^{(x+y)}) = ?$
- $\lg(32 \cdot 2^{(x+y)}) = ?$

7) For  $x \neq 1$ , let  $1 + x + x^2 + \dots + x^n = f(x)$ . Let us assume that the first derivative of  $f$  with respect to  $x$  is known, ie  $f'(x)$  is known. Express the following sum in terms of  $x, f(x)$  and  $f'(x)$ .

$$x + 2x^2 + 3x^3 + \dots + nx^n = ?$$

8) How many times is the statement  $x = x + 1$  of line 3 executed? Express the result in terms of  $n$ . Operator  $/$  divides two numbers  $x$  and  $y$  so that  $x/y$  is the quotient of the division, i.e. the result of  $10/4$  is 2.

```
1. x=0;
2. for i=1 to n increment i by 1
   begin
3.   if (i-(i/5)*5) is equal to 0 then x=x+1;
   end
```

9) Answer this question if you are familiar with ANSI C. Function `sort` is the prototype of a function that sorts an array `base` of `number` data elements. Each data element is `width` bytes long. `compare` is a pointer to a function that takes two arguments `left` and `right` and returns a -1, 0 or 1 depending on whether `left` is smaller, equal or larger than `right`. Therefore for `int intbase[11]`, then `width= sizeof(int)` and `number=11`.

```
sort(void *base, int number, int width, int (*compare)(void *left, void *right));
```

Define a function `intcompare` by completing the following piece of code

```
int intcompare ( void * left, void *right)
{

}
}
```

so that the following call will sort `intbase`.

```
sort(intbase, 11, sizeof(int), intcompare);
```

### Evaluation Quiz Solutions

1) How much is the following sum if  $x \neq 1$ ?

$$1 + x + x^2 + x^3 + \dots + x^n = (x^{n+1} - 1)/(x - 1)$$

2) What if  $x = 1$  below?

$$1 + x + x^2 + x^3 + \dots + x^n = n + 1$$

3) How much is the following sum?

$$\sum_{i=1}^n i = n(n+1)/2$$

4) How much is the following sum?

$$\sum_{i=1}^n i^2 = n(n+1)(2n+1)/6$$

5) The product  $ab$  is maximized for  $a = b = 100000/2 = 50000$ . One way to prove it is solve for  $b$  in  $a + b = 100000$ , ie  $b = 100000 - a$ , substitute for  $b$  in  $ab$  to get a function  $f(a) = a(100000 - a)$ . The first derivative  $f'(a) = 100000 - 2a$  and has a root for  $a = 100000/2 = 50000$ . This root is maximum as the second derivative at  $a = 50000$  of  $f(a)$  is negative.

6) For  $x \in R$ , we define  $y = \lg(x)$  such that  $2^y = x$ . Answer the following.

- $\lg(1) = 0$
- $\lg(128) = 7$
- $\lg(2^{(x+y)}) = (x + y)$
- $\lg(32 \cdot 2^{(x+y)}) = (x + y + 5)$

7) Start from  $1 + x + x^2 + \dots + x^n = f(x)$ . Take the first derivative of both sides. We get  $1 + 2x + \dots + nx^{n-1} = f'(x)$ . Multiply both sides by  $x$  and the answer follows.

8) The test term  $(i - (i/5) * 5)$  computes the remainder of the division of  $i$  by 5. It is zero for  $i = 5, 10, 15, \dots$ . Therefore the number of times  $x = x + 1$  will be executed is  $\text{floor}(n/5)$ .

9)

```
int intcompare ( void * left, void *right)
{
    if (*((int*)left) > *((int*)right))
        return (1);
    if (*((int*)left) < *((int*)right))
        return (-1);
    return (0);
}
```

#### READING LIST

If you had problems with Questions 1-4, Review Section 3.2 and appendix A (e.g. questions related to equations A.1 and A.5 and A.6). For problem 7, read material related to equation A.6 of appendix A. Induction is covered in Appendix A.2. Go through section 3.2 (up to Fibonacci numbers on page 56) to review properties of floor and ceiling functions, exponentials etc. Appendix B covers material related to sets, relations and functions respectively. An introductory book in C programming such as the one by B. Kernighan and D. Ritchie, 'The C Programming Language', covers all that is needed for the programming part of this course.