CIS 435: Homework 0 (Due: January 27, 2004)

Solve ALL the problems; Collaboration is prohibited

Handouts 4-9 can obtained in electronic form from the Course Web page. You may need to consult your math/calculus books before you do this Homework. Some source of additional help may be Handout 4 which is an evaluation quiz along with its solutions. A brief overview of induction apppears in Handout 5, and some kind of a Discrete Math Review is in Handout 5. The appendices of the textbook is a source of additional reference material.

Let $\log_2 n$ be the logarithm of n base two i.e. the power we need to raise two to get n. Thus, $2^{\log_2 n} = n$. We frequently use the alternative notation $\lg n$ instead of $\log_2 n$. For example, $\lg 8$ is 3 and $\lg 65536$ is 16. The ceiling function is defined in the textbook on page 51; $\lceil x \rceil$ is the smallest integer which is greater than or equal to x. For example $\lceil 3.5 \rceil$ is 4. The floor function $\lfloor y \rfloor$ is the largest integer which is less than or equal to y.

Problem 1. (-15 points if you don't solve it)
Go to the course Web-page and submit through the form in the Homeworks and Exams section of the Handout link some information about you as indicated in that form. Make sure you remember the transaction number you enter in case your info is lost for any reason.

Problem 2. (-10 points if you don't solve it)
(a) How many bits (minimum number) do we need to represent 31? 127? 63? 256?
(b) How many bits do we need to represent all integers between 16 and 31? between 32 and 63? between 64 and 127?
(c) How many bits do we need to represent 2^{m-1}? How many for $2^m - 1$? How many for all integers between 2^{m-1} and $2^m - 1$? Express the answer as a function of m.
(d) Show that any positive integer M can be represented by $\lceil \lg (M+1) \rceil$ bits.
(e) How many non-negative integers can we represent with 16 bits? How many with m bits? How many with $\lg m$ bits, assuming $\lg m$ is integer? How many with $2 \lg m$ bits?

Problem 3. (-13 points if you don't solve it)
(a) $n^{(2/\lg n)}$ or 4 is larger ? Explain. (3 points).
(b) Show by induction that

$$\sum_{i=0}^{n} (6i) = 3n(n+1)$$

(c) Find the limits $\lim n \to \infty$ of (i) $n^2/3^n$, (ii) $2^{\lg n}/n^2$, (iii) $n!/2^n$, where $n! = 1 \cdot 2 \cdot \ldots \cdot n$. For $n!$ (pronounced "n factorial") go to page 55 of the the textbook (referred to as CLRS from now on). It may help you to use Eq. (3.19). (3 points)
(d) Show that a closed form for the sum $\frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^{\lg n - 1}}$ is $1 - 2/n$. (4 points)

Problem 4. (-6 points if you don't solve it)

```
1. int Loop (int n) // Search for key in A.
2. int i,j;
3. j=0;
4. for (i=0;i<n;i++)
5.   j=j+2*i;
6. return(j);
```

(a) How many times is line 4 executed? How many times is line 5 executed? Express your answer as a function of n. (Be very very careful with the first question!)
(b) What is the value returned by j as a function of n? Express the answer in closed form. Pages 1059-1060 may help you in deciding the answer.

Problem 5. (-6 points if you don't solve it)

```
int CodeA (int n)
int i,j,t;
t=0;
for(i=0;i<n;i++)
  for(j=0;j<n;j++)
    t= t +i;
return(t);
```

```
int CodeB( int n)
int i,t;
t=0;
for(i=1;i<n;i*= 2)
  t=t+i;
return((t+1));
```

Let n be a power of two, i.e. $n = 2^k$ for some integer k. What is the value returned in CodeA and CodeB as a function of n? Explain.