CIS 435: Homework 3 (Due: Feb 24, 2004)

Solve problems 1-3 (Group 1) and either Problem 4 (Group 2) or Problem 5 (Group 3).

Problem 1. (9 points)

(a) Solve exactly using the recursion tree method or the iteration method the following recurrence. You may assume that n is a power of 2.

$$T(n) = 2T(n/2) + 16n, \text{ where } T(4) = 16.$$

(b) Solve exactly using the recursion tree method or the iteration method the following recurrence. You may assume that n is a power of 4.

$$T(n) = 8T(n/2) + n, \text{ where } T(1) = 1.$$

(c) Solve the following recurrence. Make your bounds as tight as possible. Use asymptotic notation to express your solution.

$T(n) = T(n/2) + T(n/3) + 32n$, where $T(1) = 100$.

Problem 2. (7 points)

(a) What is the minimum and maximum number of elements in a heap of height h?

(b) Show that an n element heap has height $\lfloor \lg n \rfloor$.

(c) Show that in any subtree of a MAX-HEAP, the root of the subtree contains the largest value occurring anywhere in that subtree.

(d) Is a reverse-sorted (non-increasing order) array a MIN-HEAP?

(e) Is the following sequence a MAX-HEAP? Explain. $\langle 200,\ 80,\ 150,\ 30,\ 10,\ 150\ 20 \rangle$

(f) What happens if I perform an Extract-MAX operation on the heap of question (e)? Show the heap adjustment steps.

(g) In the resulting heap of part (f) I perform an Insertion of key 90. Show the steps.

Problem 3. (4 points)

Give pseudocode for MAX-HEAPIFY that is non-recursive.

Problem 4. (30 points)

(a) Explain how to implement two stacks in one array A[1..n] in such as way that neither stack overflows unless the total number of elements in both stacks together is n. The PUSH and POP operations must still run in $O(1)$ time. (10 points)

(b) What does the following recursive program do? Explain. (10 points)

```
int  Findme ( int x , int y) {
 if ( y == 0)  return (0);
   else
    return( Findme( 2*x, floor(y/2)) + x * (y % 2 ) );
}
```

(c) **Checking Legal Parenthesizations.**. You are given a string of $n > 0$ left and right parentheses. Give an algorithm (eg. pseudocode) that checks in $O(n)$ worst case running time whether the parenthesization implied by the string is a legal one. Justify your answer. For example (())) is NOT a legal parenthesization as we have more right than left parentheses.))((is not legal either as it does not make sense. (()), ((())()()), ()()(), are legal ones. (5 points)

(d)This is Problem 10-1 on page 237. For each one of (1) an unsorted singly linked list L, (2) sorted singly linked list L, (3) unsorted doubly linked list L, (4) sorted doubly linked list L, what is the asymptotic worst-case running time for each dynamic-set operation such as (i) Search(L,k), (ii) Insert(L,x), (iii) Delete(L,x), (iv) Successor(L,x), (v) Predecessor(L,x), (vi) Minimum(L), (vii) Maximum(L). The operations are defined on page 198. (5 points)

Problem 5. (30 points)

Do the Programming Module that is outlined in the electronic handout available at the course web page
`http://www.cs.njit.edu/~alexg/courses/cis435/handouts/phw3.ps`
or
`http://www.cs.njit.edu/~alexg/courses/cis435/handouts/phw3.pdf`