CIS 435: Homework 4 (Due: Mar 2, 2003)

Solve problems 1-3 (Group 1) and either Problem 4 (Group 2) or Problem 5 (Group 3).

Problem 1. (8 points)

Compress the text "ABRACADABRACADABRA" (only the alphabetic characters not the surrounding delimiters) using Huffman coding. What is the number of bits of the compressed text (ignore table storage requirements)?

Problem 2. (8 points)

(a) What is the running time of QUICKSORT (of the textbook) when all elements are distinct and the input is a decreasing sequence? Explain. (5 points)

(b) Solve the recurrence $T(n) = T(n-1) + \Theta(n)$ of the worst-case running time of quicksort to deduce that $T(n)$ is indeed $\Theta(n^2)$.

Problem 3. (4 points)

What is the running time of QUICKSORT (of the textbook) when all elements of the input array have the same value 25? Explain.

Problem 4. (30 points)

(a) Suppose that the splits at evey level of quicksort are in the proportion 0.9 to 0.1, i.e. at every level 90 percent of the keys go to the left set and 10 percent to the right. Show that the minimum depth of a leaf in the recursion tree of quicksort is approximately $-\lg n/\lg 0.1$ and the maximum depth is approximately $-\lg n/\lg(0.9)$. Don't worry about integer round-off. (4 points)

(b) Dr I. M. RIGHT developed his own Build-MAX-Heap outlined below and also introduced in class. (8 points)

```
  RIGHT-BuildMAX-Heap(A,n)
1.    heap-size(A)=n;
2.    BuildMAX-Heap(A,1);
```

```
BuildMAX-Heap(A,i)
1. if (2*i <= heap-size(A))
2.   BuildMAX-Heap(A,2i);
3. if ((2*i+1) <= heap-size(A))
4.   BuildMAX-Heap(A,2i+1);
5. Max-Heapify(A,i); // as in the textbook
```

Show that this is a correct Build-Max-Heap method, and find the running time of BuildMAX-Heap. Justify your answer.

(c) On page 159 of CLRS, Problem 7-1, HOARE-PARTION is another way to partition keys in quicksort. Demonstrate its working on sequence $\langle 10,\ 18,\ 9,\ 14,\ 5,\ 3,\ 30\rangle$. What is the value of j returned at the completion of HOARE-PARTITION? Demonstrate its working on sequence $\langle 10,\ 10,\ 10,\ 10,\ 10\rangle$. What is the value of j returned at the completion of HOARE-PARTITION? What does this tell us about the performance of a HOARE-PARTITION-based quicksort when all keys are equal? Explain. (12 points)

(d) Give a divide and conquer algorithm that finds both the MAXIMUM and the MINIMUM of n keys, and the number of comparisons it performs in the worst case is at most $3n/2 - 1$, where n is a power of two. (6 points)

Problem 5. (30 points)

Do the Programming Module that is outlined in the electronic handout available at the course web page

`http://www.cs.njit.edu/~alexg/courses/cis435/handouts/phw4.ps`

or

`http://www.cs.njit.edu/~alexg/courses/cis435/handouts/phw4.pdf`