$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}.$$

For $a \neq 1$, and $\mid b \mid < 1$ we have that

$$\sum_{i=0}^{n} a^i = \frac{a^{n+1}-1}{a-1}, \qquad \sum_{i=0}^{n-1} i a^i = \frac{(n-1)a^{n+1} - na^n + a}{(1-a)^2},$$

$$\sum_{i=0}^{\infty} b^i = \frac{1}{1-b}, \qquad \sum_{i=1}^{\infty} b^i = \frac{b}{1-b}, \qquad \sum_{i=0}^{\infty} i b^i = \frac{b}{(1-b)^2}.$$

$$H_n = \sum_{i=1}^{n} \frac{1}{i}, \quad \sum_{i=1}^{n} i H_i = \frac{n(n+1)}{2} H_n - \frac{n(n-1)}{4}.$$

$$n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \left(1 + \Theta\left(\frac{1}{n}\right)\right), \qquad n! \approx \left(\frac{n}{e}\right)^n, \qquad a^{\log_b n} = n^{\log_b a},$$

$$e \approx 2.718281, \qquad \pi \approx 3.14159, \qquad \gamma \approx 0.57721, \qquad \phi = \frac{1+\sqrt{5}}{2} \approx 1.61803, \qquad \hat{\phi} = \frac{1-\sqrt{5}}{2} \approx -.61803.$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \qquad \sum_{k=0}^{n} \binom{n}{k} = 2^n, \qquad \binom{n}{k} = \binom{n}{n-k}, \qquad \binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}.$$

A1. $f(n) = o(g(n))$, iff $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$.
A2. $f(n) = \omega(g(n))$, iff $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$.
A3. If $f(n) = o(g(n))$, then $f(n) = O(g(n))$.
A4. If $f(n) = \omega(g(n))$, then $f(n) = \Omega(g(n))$.
A5. $f(n) = \Theta(g(n))$, iff $f(n) = \Omega(g(n))$ and $f(n) = O(g(n))$.
B1. $f(n) = \Theta(g(n))$ iff $\exists$ positive constants $c_1, c_2, n_0: \ 0 \le c_1 g(n) \le f(n) \le c_2 g(n) \ \forall\, n \ge n_0$.
B2. $f(n) = \Omega(g(n))$ iff $\exists$ positive constants $c_1, n_0: \ 0 \le c_1 g(n) \le f(n) \ \forall\, n \ge n_0$.
B3. $f(n) = O(g(n))$ iff $\exists$ positive constants $c_2, n_0: \ 0 \le f(n) \le c_2 g(n) \ \forall\, n \ge n_0$.
L1.

$$\lg(ab) = \lg a + \lg b, \quad \lg(a/b) = \lg a - \lg b, \quad \lg(a^b) = b \lg a, \quad 2^{\lg(a)} = a, \quad a^x\, a^y = a^{x+y}, \quad a^x / a^y = a^{x-y}, \quad (a^x)^y = a^{xy}.$$

Master Method. $T(n) = aT(n/b) + f(n)$, such that $a \ge 1$, $b > 1$.

- M1 If $f(n) = O(n^{\lg_b a - \epsilon})$ for some constant $\epsilon > 0$, then $T(n) = \Theta(n^{\lg_b a})$.
- M2 If $f(n) = \Theta(n^{\lg_b a})$, then $T(n) = \Theta(n^{\lg_b a} \lg n)$.
- M3 If $f(n) = \Omega(n^{\lg_b a + \epsilon})$ for some constant $\epsilon > 0$, and if $af(n/b) \le cf(n)$ for some constant $0 < c < 1$ and for large n, then $T(n) = \Theta(f(n))$.

D1.

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x), \qquad \left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}, \qquad (c^x)' = \ln(c)\, c^x.$$

S1.

$$\frac{1}{1-x} = 1 + x + x^2 + \ldots + x^i + \ldots = \sum_{i=0}^{\infty} x^i, \qquad \frac{x}{(1-x)^2} = x + 2x^2 + \ldots + ix^i + \ldots = \sum_{i=0}^{\infty} ix^i, \qquad e^x = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^i}{i!} + \ldots = \sum_{i=0}^{\infty} \frac{x^i}{i!}.$$