

Quiz 2 (Due : Mar 8, 1999)

**Problem 1.**

Give an algorithm (word model) for broadcasting an one-word message on the following networks. What is its running time? Use  $O(\cdot)$  notation to express time bounds.

- (i)  $n$ -cell linear array. Message is held by leftmost processor.
- (ii)  $n$ -leaf complete binary tree. Message is held by root.
- (iii)  $(n \times n)$ -cell 2d-array. Message is held by top-leftmost processor.

**Problem 2.**

On the bit model, how long does it take to broadcast an  $m$ -bit message on the following networks? Present your algorithm. What is its running time? You may assume that each processor has sufficient storage to store  $m$  bits. Since your algorithm will be designed for the bit model, communication lines can carry one bit at a step. Use  $O(\cdot)$  notation to express time bounds.

- (i)  $n$ -cell linear array. Message is held by leftmost processor.
- (ii)  $n$ -leaf complete binary tree. Message is held by root.
- (iii)  $(n \times n)$ -cell 2d-array. Message is held by top-leftmost processor.

**Problem 3.**

For the following interconnection networks,

- (1) linear array,
- (2) complete binary tree,
- (3) 3-d mesh,
- (4) hypercube,
- (5) butterfly,

express the following properties in terms of the number of **processors** of each network.

- Number of edges (use  $O(\cdot)$  notation).
- Minimum and maximum degree of vertices.
- Diameter of the network (use  $O(\cdot)$  notation).

**Problem 4.**

Show how the comparison of two  $n$ -bit numbers can be computed by a parallel prefix computation.

**Problem 5.**

Show how to sum  $n$   $n$ -bit numbers in  $O(n)$  steps on an  $O(n)$ -cell linear array. You are allowed to input bits directly to each cell of the linear array.

**Problem 6 (extra credit).**

Design an algorithm for sorting  $n$  numbers on an  $O(\log n)$ -processor complete binary tree that has  $\Theta(1)$  efficiency. (You may assume that each processor can process and store  $O(n/\log n)$  numbers, and you can take advantage of the fact that a single sequential processor can sort  $N$  numbers in  $O(N \log N)$  steps. You may also allow I/O at each processor of the network).