Quiz 2 (Due: Mar 8, 1999)

Problem 1.
Give an algorithm (word model) for broadcasting an one-word message on the following networks. What is its running time? Use $O(.)$ notation to express time bounds.

(i) $n$-cell linear array. Message is held by leftmost processor.
(ii) $n$-leaf complete binary tree. Message is held by root.
(iii) $(n \times n)$-cell 2d-array. Message is held by top-leftmost processor.

Problem 2.
On the bit model, how long does it take to broadcast an $n$-bit message on the following networks? Present your algorithm. What is its running time? You may assume that each processor has sufficient storage to store $m$ bits. Since your algorithm will be designed for the bit model, communication lines can carry one bit at a step. Use $O(.)$ notation to express time bounds.

(i) $n$-cell linear array. Message is held by leftmost processor.
(ii) $n$-leaf complete binary tree. Message is held by root.
(iii) $(n \times n)$-cell 2d-array. Message is held by top-leftmost processor.

Problem 3.
For the following interconnection networks,

1. linear array,
2. complete binary tree,
3. 3-d mesh,
4. hypercube,
5. butterfly,

express the following properties in terms of the number of processors of each network.

- Number of edges (use $O(.)$ notation).
- Minimum and maximum degree of vertices.
- Diameter of the network (use $O(.)$ notation).

Problem 4.
Show how the comparison of two $n$-bit numbers can be computed by a parallel prefix computation.

Problem 5.
Show how to sum $n$ $n$-bit numbers in $O(n)$ steps on an $O(n)$-cell linear array. You are allowed to input bits directly to each cell of the linear array.

Problem 6 (extra credit).
Design an algorithm for sorting $n$ numbers on an $O(\log n)$-processor complete binary tree that has $\Theta(1)$ efficiency. (You may assume that each processor can process and store $O(n/\log n)$ numbers, and you can take advantage of the fact that a single sequential processor can sort $N$ numbers in $O(N \log N)$ steps. You may also allow I/O at each processor of the network).