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Quiz 2

Quiz 2 (Due : Mar 8, 1999)

Problem 1.

Give an algorithm (word model) for broadcasting an one-word message on the following networks. What is its running time? Use $O(.)$ notation to express time bounds.

(i) n-cell linear array. Message is held by leftmost processor.

(ii) n-leaf complete binary tree. Message is held by root.

(iii) $(n \times n)$-cell 2d-array. Message is held by top-leftmost processor.

Problem 2.

On the bit model, how long does it take to broadcast an m-bit message on the following networks? Present your algorithm. What is its running time? You may assume that each processor has sufficient storage to store m bits. Since your algorithm will be designed for the bit model, communication lines can carry one bit at a step. Use $O(.)$ notation to express time bounds.

(i) n-cell linear array. Message is held by leftmost processor.

(ii) n-leaf complete binary tree. Message is held by root.

(iii) $(n \times n)$-cell 2d-array. Message is held by top-leftmost processor.

Problem 3.

For the following interconnection networks,

(1) linear array,

(2) complete binary tree,

(3) 3-d mesh,

(4) hypercube,

(5) butterfly,

express the following properties in terms of the number of **processors** of each network.

- Number of edges (use $O(.)$ notation).
- Minimum and maximum degree of vertices.
- Diameter of the network (use $O(.)$ notation).

Problem 4.

Show how the comparison of two n-bit numbers can be computed by a parallel prefix computation.

Problem 5.

Show how to sum n n-bit numbers in $O(n)$ steps on an $O(n)$-cell linear array. You are allowed to input bits directly to each cell of the linear array.

Problem 6 (extra credit).

Design an algorithm for sorting n numbers on an $O(\log n)$-processor complete binary tree that has $\Theta(1)$ efficiency. (You may assume that each processor can process and store $O(n/\log n)$ numbers, and you can take advantage of the fact that a single sequential processor can sort N numbers in $O(N \log N)$ steps. You may also allow I/O at each processor of the network).