Name: .....

ID Number: ..... **Section Number:** .....

Grade: 1: ..... 2: ..... 3: ..... 4: ..... 5: ..... Total: .....

Quiz 1

Solve all the problems in the space provided.
Duration: 1hour20minutes

Read the Problems CAREFULLY!
300 points total

There are 5 (FIVE) pages this page included

NOTE: You may refer to an algorithm in the notes without repeating the code in this quiz

ASSUMPTIONS: Throughout the exam, you may assume that n is a multiple of p, and p is a power of two. You may assume that it takes one operation to compute $a+b$ and one operation to compute $a\,b$ (product of a and b), given a and b.

Problem 1 (25 points)
Input: A p processor EREW PRAM, p numbers $x_1, x_2, \ldots, x_p$, and an associative operator $+$.
Output: The parallel sum $x = x_1 + x_2 + \ldots + x_p$.

(a) Give or cite an efficient EREW PRAM algorithm that finds the parallel sum x. (10 points)

(b) What is the parallel running time, speedup and efficiency of your proposed algorithm? (15 points)

Problem 2 (75 points)
Input: A $p < n$ processor EREW PRAM, n numbers $x_1, x_2, \ldots, x_n$, and an associative operator $+$.
Output: The parallel sum $x = x_1 + x_2 + \ldots + x_n$.

(a) Give an EREW PRAM algorithm that finds the parallel sum x in at most $n/p + \lg p$ steps. (25 points)

(b) What is the speedup and efficiency of your algorithm? (25 points)

(c) For which values of p (in terms of n) is the speedup of the proposed algorithm $\Theta(p)$? (25 points)

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Problem 3 (90 points)
Input: A $p < n$ processor EREW PRAM, n numbers $x_1, x_2, \ldots, x_n$, and an associative operator $+$.
Output: The parallel prefix sums $x_1, x_1 + x_2, x_1 + x_2 + x_3, \ldots, x_1 + x_2 + \ldots + x_n$.

(a) Give a time efficient EREW PRAM algorithm that finds all parallel prefix sums in $O(n/p + \lg p)$ operations. (20 points)

(b) What are the constants hidden in the term $O(n/p + \lg p)$? Explain. (20 points)

(c) What is the speedup and efficiency of your algorithm? (10 points)

(d) Show that for $p = O(n/\lg n)$ the speedup of the EREW PRAM parallel prefix algorithm is $\Theta(p)$? (20 points)

(e) Under which conditions is asymptotic efficiency higher than 50% ? Under which conditions is asymptotic efficiency higher than 25% ? (20points)

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Problem 4 (60 points)
Input: A $p < n$ processor EREW PRAM, and two n-element vectors $x = \langle x_1, x_2, \ldots, x_n \rangle$ and $y = \langle y_1, y_2, \ldots, y_n \rangle$.
Output: The parallel vector product

$$z = x \cdot y = x_1\ y_1 + x_2\ y_2 + \ldots + x_n\ y_n$$

(a) Give an efficient EREW PRAM algorithm that finds the vector parallel vector product of two n-element vectors. (25 points)
(b) What is the the parallel running time of your algorithm (include constants), its speedup and efficiency ? (35 points)

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Problem 5 (50 points)
Input: A $n^{1+1/4} = n^{5/4}$ processor CRCW PRAM, and n Boolean variables (ie 0 or 1) $x_1, \ldots, x_n$.
Output: The logical AND operation on the input variables, ie, $x = x_1$ **AND** x_2 **AND** $\ldots$ **AND** x_n.

Give a **constant** time CRCW PRAM algorithm that finds the logical AND of n boolean variables using $n^{5/4}$ processor only (ie significantly fewer processors than the algorithm presented in class).

Hint: Note that $n^{3/4} \cdot (n^{1/4})^2 = n^{5/4}$.

End of Quiz 1