CIS 786 Homework 2 (Due: Oct 7, 2004)

Problem 1. (20 points)
Do as instructed in
`http://www.cs.njit.edu/~alexg/courses/cis786/handouts/phw2.ps`
`http://www.cs.njit.edu/~alexg/courses/cis786/handouts/phw2.pdf`

Problem 2. (20 points)
There is an obvious way to represent a positive integer n in binary. For example, 82 in binary with 8 bits is 01010010. A question in computer architecture is how to represent negative numbers; one way to represent for example -82 is 2's complement representation. An example on how to do 2's complement follows.

```
 2's complement (X,n)                    2's complement (-82, 8)
//n-bit 2's complement for X; X negative
1. Form n 1's (i.e. 2**n -1)            1.  11111111  // 2**8 -1 = 255
2. Subtract |X| (absolute value of $X$    2. -01010010  // this is 82
3. Add 1                                   ---------
4. return result                             10101101  // bits inverted
                                        3. +       1  // add 1
                                           ---------
                                             10101110  // done
```

A mnemonic to do 2's complement is the following

```
                                                             <-------
1. Scan number from right to left                           01010010
                                                                   *stays same
2. If a   bit is zero leave it zero.                        01010010
                                                                  *stays same
3. If you see the first  one leave it,but invert  all bits   01010010
   past that one  until you reach the left side.            ****** invert others
                                                            10101110
```

Say you have a n-bit number and n processors. How can you find 2's complement in $O(\lg n)$ time with $O(n/\lg n)$ processors with a EREW PRAM (1-bit operation takes 1 step)? Explain.

Problem 3. (20 points)
Let $X = \langle x_1, x_2, \ldots, x_n \rangle$ be a sequence of n distinct keys. The rank of a key x in the sequence X or $r(x, X)$ is the number of keys less than x in S. Sorting can be viewed as finding the rank of each one of the input keys of the input sequence.

1. If you have a CRCW PRAM, determine the rank of all keys of X in $O(\lg n)$. How many processors did you use?
2. Sort the n keys within the same time bound in a CRCW PRAM. How many processors did you use?
3. What if you only have an EREW PRAM? What would the time and processor bounds be? Explain.

Problem 4. (20 points)
Suppose you have a CRCW PRAM and n keys. Find the MAX of the n keys in $O(1/t)$ time with $O(n^{1+t})$ processors for t an arbitrarily small and positive constant (eg. $t = 0.1$).

Problem 5. (20 points)
Ranking based techniques apply not only to sorting but also to merging. Design a parallel algorithms that merges two sorted arrays each of size $n/2$ into one sorted array in time O(1) using a polynomial number of processors (eg. $O(n^9)$) on a CRCW PRAM. (Your answer should minimize the number of processors used.)

Bonus question. What if you have an EREW PRAM? How fast can you do it? How many processors did you use?