CIS 667 and CIS 467H: Homework 1 (Due: Feb 8, 2005)

Problem 1. (20 points)

Go to the course Web-page and submit through the form in the Homeworks and Exams section of the Handout link some information about you as indicated in that form. Make sure you remember the transaction number you enter in case your info is lost for any reason.

Problem 2. (20 points)

You are given six polynomials $f_1, \ldots, f_6$ of degrees $1, 3, 2, 3, 4, 5$ respectively. We are interested in finding the product $f = f_1 f_2 f_3 f_4 f_5 f_6$ by performing pairwise multiplications. Assume that the cost of multiplying two polynomials of degree a and b is $a \cdot b$. Find a schedule for multiplying the six polynomials that is of the lowest possible total cost (total cost is the sum of the costs of all multiplications performed to determine f).

Show (i.e. give a proof) that the schedule you have chosen can not be improved in any way.

Problem 3. (20 points)

Evaluating a polynomial A(x) of degree-bound n at a given point x_0 can also be done by dividing $A(x)$ by the polynomial $(x - x_0)$ to obtain the quotient polynomial $q(x)$ of degree-bound $n - 1$ and a remainder r, such that

$$A(x) = q(x)(x - x_0) + r$$

Clearly $A(x_0) = q(x_0)(x_0 - x_0) + r = r$. Show how to compute the remainder r and the coefficients of $q(x)$ in time $\Theta(n)$ from x_0 and the coefficients $a_0, a_1, \ldots,$ of A.

Problem 4. (20 points)

Consider two sets A and B each having n integers in the range from 0 to $10n$. We wish to compute the Cartesian sum of A and B defined by

$$C = \{x + y : x \in A \ \ and \ \ y \in B\}$$

Note that the integers in C are in the range from 0 to 20n. We want to find the elements of C and the number of times each element of C is realized as a sum of elements in A and B. Show that if the product of two degree bound n polynomials can be computed in $O(n \lg n)$ time, then this problem can also be solved in $O(n \lg n)$ time. **Hint.** Represent A and B as polynomials of degree at most $10n$.

Problem 5. (20 points)

Evaluate the following polynomial of degree n, at $x = c$ using $o(n)$ additions/subtractions and $o(n)$ multiplications (you are not allowed to use divisions or any operations other than $+, -, *$).

$$f(x) = \sum_{k=0}^{n} \binom{n}{k} x^k$$

[Note:

$f(n) = o(g(n))$ if and only if $\qquad \lim_{n \to \infty} f(n)/g(n) = 0$.
$\binom{n}{k} = n!/(k!(n-k)!)$
$n! = n \cdot (n-1) \cdot \ldots \cdot 2 \cdot 1$].