

Name: .SOLUTIONS by INSTRUCTOR.....

Problem 1. (35 POINTS)

TYPE A

- (1) InsertionSort does not sort in-place. **FALSE**
- (2) SelectionSort is stable. **TRUE**
- (3) On the sorted sequence $\langle 1, 2, \dots, n \rangle$, SelectionSort is asymptotically faster than MergeSort. **FALSE**
- (4) On the sorted sequence $\langle 1, 2, \dots, n \rangle$, InsertionSort has running time that is $O(n \lg n)$. **TRUE** as $\Theta(n)$ is also $O(n \lg n)$.
- (5) The asymptotic solution of $T(n) = 2T(n/2) + n$ is $T(n) = \Theta(n \lg n)$. **TRUE**
- (6) The asymptotic solution of $T(2n) = 2T(n) + n$ is $T(n) = \Theta(n \lg n)$. **TRUE**
- (7) The asymptotic solution of $T(n) = T(n/2) + n^2$ is $T(n) = \Theta(n^2 \lg n)$ **FALSE**

TYPE B

- (1) MergeSort is stable. **TRUE**
- (2) Odd-even transposition sort sorts in-place. **TRUE**
- (3) On the reverse-sorted sequence $\langle n, n-1, \dots, 1 \rangle$, SelectionSort is asymptotically faster than InsertionSort. **FALSE** they have the same performance.
- (4) On the reverse-sorted sequence $\langle n, n-1, \dots, 1 \rangle$, InsertionSort has running time that is $\Omega(n \lg n)$. **TRUE** as $\Theta(n^2)$ is $\Omega(n \lg n)$.
- (5) The asymptotic solution of $T(n) = T(n/2) + n$ is $T(n) = \Theta(n \lg n)$. **FALSE**
- (6) The asymptotic solution of $T(4n) = T(2n) + n$ is $T(n) = \Theta(n \lg n)$. **FALSE**
- (7) The asymptotic solution of $T(n) = T(n/2) + n^2$ is $T(n) = \Theta(n^2)$. **TRUE**

Problem 2. (50 POINTS)

Most of this problem comes from PS1.Problem 6(b).

TYPE A

$$n!, \quad (n-3)!, \quad 3^n, \quad \left\{ n^3, \quad 2^{3 \lg n} \right\}, \quad n/\lg n, \quad 10000,$$

TYPE B

$$(n-2)!, \quad \left\{ n^2, \quad 2^{2 \lg n} \right\}, \quad \{ n \lg n, \quad \lg(n!) \}, \quad \sqrt{n}, \quad \lg n,$$

Problem 3. (40 POINTS)

TYPE A. Similar to problem PS1.Problem 6(a). Show that $n^3 - 2n^2 = \Theta(n^3)$ by providing the n_0, c_1, c_2 .

Big-Oh first. $n^3 - 2n^2 \leq n^3 \leq 1n^3$ for all $n \geq 1$. Thus choose $n_0 = 1$ and $c_2 = 1$ for the definition of Big-Oh.

Big-Omega next. $n^3 - 2n^2 \geq n^3 - n^3/2 \geq n^3/2$ for all $n \geq 4$. The bound $-2n^2 \geq -n^3/2$ is true for $n \geq 4$, and so is the combined bound. Thus $n^3 - 2n^2 \geq n^3/2$. Thus choose $c_1 = 1/2$ and $n_0 = 4$ for the definition of Ω .

Combining O and Ω choices: $c_1 = 1/2$, $c_2 = 1$ and $n_0 = 4$ for Θ .

TYPE B. Similar to problem PS1.Problem 6(a). Show that $n^3 - n^2 - 2 = \Theta(n^3)$ by providing the n_0, c_1, c_2 .

Big-Oh first. $n^3 - n^2 - 2 \leq n^3$ for all $n \geq 1$. Thus choose $n_0 = 1$ and $c_1 = 2$ for the definition of Big-Oh.

Big-Omega next. $n^3 - n^2 - 2 \geq n^3 - n^3/4 - 2n^3/4 \geq n^3/4$ for all $n \geq 4$. The bound $-n^2 \geq -n^3/4$ is true for $n \geq 4$. The bound $-2 \geq -2n^3/4$ is true for $n \geq 2$. Thus the combined bound above is true for $n \geq 4$. Thus $n^3 - n^2 - 2 \geq n^3/4$. Pick $c_1 = 1/4$ and $n_0 = 4$ for the definition of Ω .

Combining O and Ω choices: $c_1 = 1/4$, $c_2 = 1$ and $n_0 = 4$ for Θ .

Problem 4. (41 POINTS)

TYPE A	TYPE B
$[n, 2n, n/2, n/2]$	$[3n, n/4, n/4, n/2]$
$X = \text{merge}(C, D);$	$Q = \text{merge}(B, C);$
$Y = \text{merge}(X, A);$	$R = \text{merge}(Q, D);$
$Z = \text{merge}(Y, B);$	$S = \text{merge}(R, A);$

TYPE A. Combine the smaller length sequences. C and D can be merged using $n/2 + n/2 - 1 = n - 1$ comparisons (maximum) into a sequence X of length n . Sequence X and A can be merged using at most $n + n - 1 = 2n - 1$ comparisons into sequence Y of $2n$ keys. Finally Y and B can be merged using at most $2n + 2n - 1 = 4n - 1$ comparisons into Z of $4n$ keys. Total comparison count is $n - 1 + 2n - 1 + 4n - 1 = 7n - 3$.

TYPE B. Combine the smaller length sequences. B and C can be merged using $n/4 + n/4 - 1 = n/2 - 1$ comparisons (maximum) into a sequence Q of length $n/2$. Sequence Q and D can be merged using at most $n/2 + n/2 - 1 = n - 1$ comparisons into sequence R of n keys. Finally R and A can be merged using at most $3n + n - 1 = 4n - 1$ comparisons into S of $4n$ keys. Total comparison count is $n/2 - 1 + n - 1 + 4n - 1 = 11n/2 - 3$.

Problem 5. (34 POINTS)

	RA	RLL	SDL
Ins(x)	n	n	n
Del(z)	n	n	1
Sea(x)	$\lg n$	n	n
FindMin	1	n	1
ExMax	n	1	n
Succ(z)	1	n	1

	SA	SLL	SDL
Ins(x)	n	n	n
Del(z)	n	n	1
Sea(x)	$\lg n$	n	n
FindMin	1	1	1
ExMin	n	1	1
Succ(z)	1	1	1