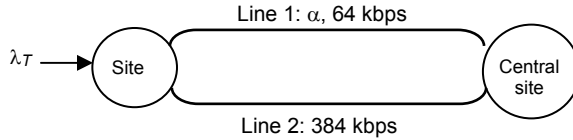


7.19



(a) Find the value of α that minimizes the total average delay.

$$E[L] = 8000 \text{ bits}$$

$$\lambda_1 = \alpha \lambda_T, \lambda_2 = (1 - \alpha) \lambda_T$$

$$E[X_1] = 8000 \text{ bits} / 64 \text{ kbps} = 0.125 \text{ sec}$$

$$E[X_2] = 8000 \text{ bits} / 384 \text{ kbps} = 0.02083 \text{ sec}$$

$$E[D_T] = E[D_1] \alpha + E[D_2] (1 - \alpha), \alpha < 1$$

$$= \frac{1}{\frac{1}{E[X_1]} - \lambda_1} \alpha + \frac{1}{\frac{1}{E[X_2]} - \lambda_2} (1 - \alpha)$$

$$= \frac{1}{8 - \lambda_1} \alpha + \frac{1}{48 - \lambda_2} (1 - \alpha)$$

$$= \frac{1}{8 - \lambda_T \alpha} \alpha + \frac{1}{48 - \lambda_T (1 - \alpha)} (1 - \alpha)$$

Before doing the math to find the minimum in the above expression, it is worthwhile to consider a couple of extreme cases. The maximum capacity available to transfer packets across the combined links is 56 packets per second, 8 p/s across the first link and 48 p/s across the second link. Clearly when the arrival rate approaches capacity, we will send traffic across the two links. However consider the case when the arrival rate is very small. The delay across the first link is $1/8$ and the delay across the second link is $1/48$. Clearly at low traffic we want to send all the traffic on the fast link. As we increase the arrival rate, we eventually reach a point where it is worthwhile to route some packets along the slow link. Thus we expect α to be zero for some range of arrival rates, and then become nonzero after a specific value of arrival rate is reached.

Now take derivatives to find the minimum:

Let A be the first term in the delay expression and let B be the second term.

$$\frac{dE[D_T]}{d\alpha} = \frac{dA}{d\alpha} + \frac{dB}{d\alpha}$$

$$\frac{dA}{d\alpha} = \frac{d}{d\alpha} \frac{1}{8 - \lambda_T \alpha} \alpha = (-1)(8 - \lambda_T \alpha)^{-2} (-\lambda_T) \alpha + \frac{1}{8 - \lambda_T \alpha}$$

$$= \frac{\lambda_T \alpha}{(8 - \lambda_T \alpha)^2} + \frac{1}{8 - \lambda_T \alpha} = \frac{\lambda_T \alpha + 8 - \lambda_T \alpha}{(8 - \lambda_T \alpha)^2} = \frac{8}{(8 - \lambda_T \alpha)^2}$$

Similarly, we have

$$\begin{aligned}\frac{dB}{d\alpha} &= \frac{d}{d\alpha} \left[\frac{1}{48 - \lambda_T(1 - \alpha)} (1 - \alpha) \right] = (-1) [48 - \lambda_T + \lambda_T \alpha]^{-2} (\lambda_T)(1 - \alpha) + \frac{-1}{48 - \lambda_T(1 - \alpha)} \\ &= \frac{-\lambda_T(1 - \alpha)}{(48 - \lambda_T + \lambda_T \alpha)^2} + \frac{-1}{48 - \lambda_T + \lambda_T \alpha} = \frac{\lambda_T(1 - \alpha) - (48 - \lambda_T + \lambda_T \alpha)}{(48 - \lambda_T + \lambda_T \alpha)^2} = \frac{-48}{(48 - \lambda_T + \lambda_T \alpha)^2}\end{aligned}$$

To find the α that minimizes the delay, we set $\frac{dA}{d\alpha} + \frac{dB}{d\alpha} = 0$. We then obtain:

$$\frac{8}{(8 - \lambda_T \alpha)^2} + \frac{-48}{(48 - \lambda_T + \lambda_T \alpha)^2} = 0$$

$$\frac{1}{(8 - \lambda_T \alpha)^2} = \frac{6}{(48 - \lambda_T + \lambda_T \alpha)^2}$$

$$6(8 - \lambda_T \alpha)^2 = (48 - \lambda_T + \lambda_T \alpha)^2$$

$$\sqrt{6}(8 - \lambda_T \alpha) = 48 - \lambda_T + \lambda_T \alpha$$

$$8\sqrt{6} - \sqrt{6}\lambda_T \alpha = 48 - \lambda_T + \lambda_T \alpha$$

$$8\sqrt{6} - 48 + \lambda_T = \lambda_T \alpha + \sqrt{6}\lambda_T \alpha$$

$$\alpha = \frac{8\sqrt{6} - 48 + \lambda_T}{(1 + \sqrt{6})\lambda_T} = \frac{\lambda_T - 28.4}{3.45\lambda_T}$$

Thus we see that α must be zero for arrival rates less than 28.4.

- (b) Compare the average delay in part (a) to the average delay in a single multiplexer that combines the two transmission lines into a single transmission line.

With a single transmission line the link speed will be $64 + 384 = 448$ kbps. Therefore,

$$E[X] = 8000 \text{ bits} / 448 \text{ kbps} = 0.0179 \text{ sec}$$

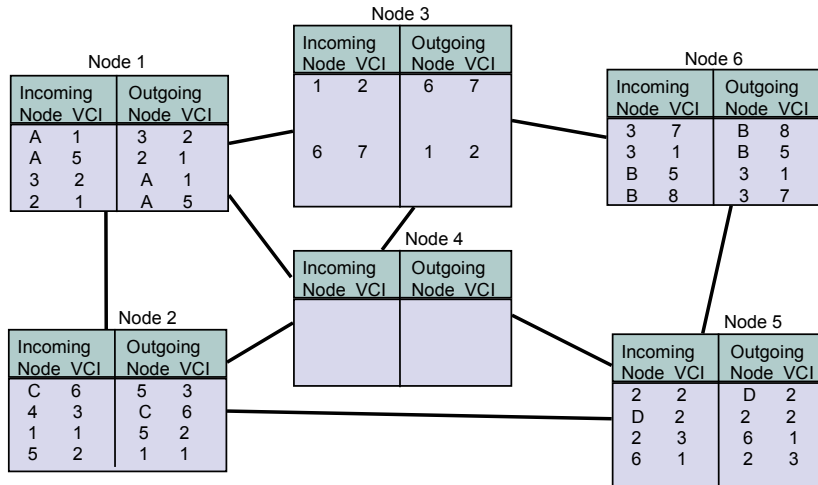
$$E[D] = \frac{1}{\frac{1}{E[X]} - \lambda_T} = \frac{1}{\frac{1}{0.0179} - \lambda_T} = \frac{1}{56 - \lambda_T}$$

The average delay in the single multiplexer is smaller than the average delay in the system served by two servers. This reduction is due to the fact that some of the capacity in the two-server system can be idle when there is only one packet in the system, whereas the single multiplexer system is always working at full rate.

7.28. Consider the virtual-circuit packet network in Figure 7.23. Suppose that node 4 in the network fails. Reroute the affected virtual circuits and show the new set of routing tables.

Solution:

When node 4 fails, all the affected connections need to be re-routed. In the example, the VC from A to D needs to be rerouted. One possible new route is A-1-2-5-D. The routing tables at nodes 1, 2, and 5 need to be modified. The VC from C to B also needs to be rerouted. A possible new route is C-2-5-6-D. The routing tables in 2, 5, and 6 need to be modified. The routing tables in node 3 have to be modified to remove the connections that no longer traverse it.



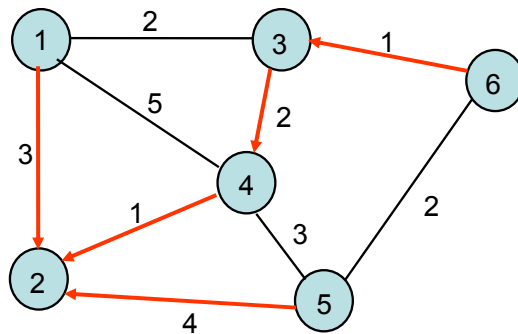
7.32. Consider the network in Figure 7.30.

Solutions follow questions:

- (a) Use the Bellman-Ford algorithm to find the set of shortest paths from all nodes to destination node 2.

Iteration	Node 1	Node 3	Node 4	Node 5	Node 6
Initial	$(-1, \infty)$	$(-1, \infty)$	$(-1, \infty)$	$(-1, \infty)$	$(-1, \infty)$
1	$(2, 3)$ $(3, \infty)$ $(4, \infty)$	$(1, \infty)$ $(4, \infty)$ $(6, \infty)$	$(1, \infty)$ $(2, 1)$ $(3, \infty)$ $(5, \infty)$	$(2, 4)$ $(4, \infty)$ $(6, \infty)$	$(3, \infty)$ $(5, \infty)$
2	$(2, 3)$ $(3, \infty)$ $(4, 6)$	$(1, 5)$ $(4, 3)$ $(6, \infty)$	$(1, 8)$ $(2, 1)$ $(3, \infty)$ $(5, 7)$	$(2, 4)$ $(4, 4)$ $(6, \infty)$	$(3, \infty)$ $(5, 6)$
3	$(2, 3)$ $(3, 5)$ $(4, 6)$	$(1, 5)$ $(4, 3)$ $(6, 7)$	$(1, 8)$ $(2, 1)$ $(3, 5)$ $(5, 7)$	$(2, 4)$ $(4, 4)$ $(6, 8)$	$(3, 4)$ $(5, 6)$
4	$(2, 3)$ $(3, 5)$ $(4, 6)$	$(1, 5)$ $(4, 3)$ $(6, 5)$	$(1, 8)$ $(2, 1)$ $(3, 5)$ $(5, 7)$	$(2, 4)$ $(4, 4)$ $(6, 6)$	$(3, 4)$ $(5, 6)$

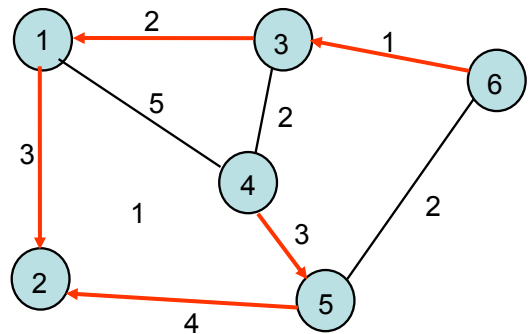
The set of paths to destination 2 are shown below:



Now continue the algorithm after the link between node 2 and 4 goes down.

Iteration	Node 1	Node 3	Node 4	Node 5	Node 6
Before break	(2 , 3)	(4 , 3)	(2 , 1)	(2 , 4)	(3 , 4)
1	(2 , 3) (3 , 5) (4 , 6)	(1 , 5) (4 , 3) (6 , 5)	(1 , 8) (3 , 5) (5 , 7)	(2 , 4) (4 , 4) (6 , 6)	(3 , 4) (5 , 6)
2	(2 , 3) (3 , 5) (4 , 10)	(1 , 5) (4 , 7) (6 , 5)	(1 , 8) (3 , 5) (5 , 7)	(2 , 4) (4 , 8) (6 , 6)	(3 , 4) (5 , 6)
3	(2 , 3) (3 , 7) (4 , 10)	(1 , 5) (4 , 7) (6 , 5)	(1 , 8) (3 , 7) (5 , 7)	(2 , 4) (4 , 8) (6 , 6)	(3 , 6) (5 , 6)
4	(2 , 3) (3 , 7) (4 , 12)	(1 , 5) (4 , 9) (6 , 7)	(1 , 8) (3 , 7) (5 , 7)	(2 , 4) (4 , 10) (6 , 8)	(3 , 6) (5 , 6)

The new set of paths are shown below:

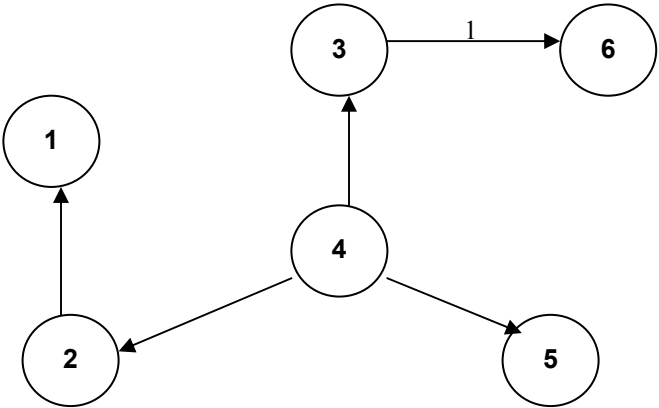


7.33. Consider the network in Figure 7.30.

Solutions follow questions:

(a) Use the Dijkstra algorithm to find the set of shortest paths from node 4 to other nodes.

Iteration	N	D ₁	D ₂	D ₃	D ₅	D ₆
Initial	{ 4 }	5	1	2	3	∞
1	{ 2 , 4 }	4	1	2	3	∞
2	{ 2 , 3 , 4 }	4	1	2	3	3
3	{ 2 , 3 , 4 , 5 }	4	1	2	3	3
4	{ 2 , 3 , 4 , 5 , 6 }	4	1	2	3	3
5	{ 1 , 2 , 3 , 4 , 5 , 6 }	4	1	2	3	3



(b) Find the set of associated routing table entries.

Destination	Next Node	Cost
1	2	4
2	2	1
3	3	2
5	5	3
6	3	3