An Information Theory Based Framework for Optimal Link State Update

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***Abstract*—In this letter, based on information theory, we present a theoretical framework for the optimal link-state update, upon which efficient link-state update policies may be developed.**

***Index Terms*—Information theory, link-state update, rate-distortion function.**

I. Introduction

CURRENT link-state routing protocols, such as Open Shortest Path First (OSPF) [1], recommend that the link state is updated periodically with large intervals. For instance, a link disseminates its state information every 30 min in OSPF. Consequently, because of the highly dynamic nature of link state parameters, the link state information is often outdated. As a result, the effectiveness of the quality-of-service (QoS) routing algorithms may be degraded significantly. To overcome this problem, many trigger based link-state update policies (e.g., [2]) have been proposed in the literature. Instead of using the link capacities or instantaneous available bandwidth values, Li *et al.* [3] used a stochastic metric, Available Bandwidth Index (ABI), and extended BGP to perform the bandwidth advertising.

Intuitively, from the perspective of QoS routing, it is preferable that accurate state information is available throughout the whole network. On the other hand, achieving this goal may impose an intolerable protocol overhead on network resources that is practically impossible. Therefore, our task in this letter is to provision as accurate link state information as possible with a minimum overhead. Without loss of generality, we only focus on one link state metric. Here, we adopt the bandwidth. In this letter, we measure the consumed network resource for updating state information by the average bandwidth used. Note that link state information is distributed by packets in the network. We can also measure the consumed network resource by the average number of packets used for link-state update in a unit time.

II. Proposed Framework

Since a link can be shared by many connections and a connection may traverse many links, the state metrics on different links are not independent from each other. However, most proposed update policies, if not all, reported in the literature are **per-link based**, i.e., the state information of a single link is updated without considering the status of other links. As a result, they are not efficient enough. For example, given a network shown in Fig. 1, in which links e_1 and e_2 are the only two links connected to node n, and have the same link capacity. If no connection joins or leaves the network on node n, links e_1 and e_2 always have the same available bandwidth, i.e., their state metrics are always the same. Therefore, for a per-link based update policy, whenever an update for link e_1 is triggered, an update for e_2 is also triggered. As a result, per-link based update schemes inevitably waste network resources on distributing almost duplicated state information from the two links throughout the network. Assume that when a connection request arrives at an node, the node makes the decision if the connection should be accepted according to its available (inaccurate) link state information. If, from the perspective of the node, there is enough network resource (bandwidth) to accommodate this connection, it starts a setup process for the connection. Otherwise, it rejects the connection request immediately. Ideally, the connection is accepted if there is actually enough network resource, and rejected otherwise. However, due to inaccurate link state information, there are two possible undesirable cases.

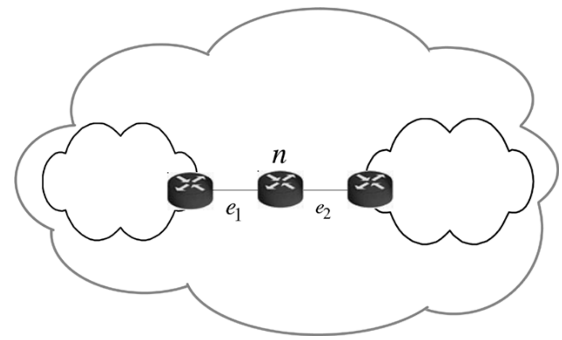

Fig. 1. Links e_1 and e_2 are the only two links connected to core node n and have the same link capacity.

- *False positive:* There is actually not enough bandwidth to accommodate a connection, but is indicated otherwise by its link state information. Since a setup process will be initialized by the node, network resource is wasted.
- *False negative:* A connection can actually be accepted by the network, but is rejected by the node because of inaccurate link state information.

Collectively, the two cases are all referred to as routing failures in this letter. We define the optimal link-state update scheme as follows:

Definition 1—Optimal Link State Update Scheme: Given an upperbound B on the average bandwidth used for updating link

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state information, an update scheme is called optimal if the cost due to routing failures in the network is minimized.

First, we shall numerically define the cost. Note that a connection can traverse many links. Hence, it is possible that for a per-link-based update scheme, many links update their state information due to the joining or the leaving of a single connection. However, since the amounts of the changed available bandwidths on the links, along which the connection traverses the network are the same, network resources can be saved if only one update is triggered to inform other nodes the acceptance or the removal of the connection (including its requested bandwidth and route, where a route is defined as the set of links by which a connection traverses the network). Hence, we recommend **route-based** link-state update in this letter and present a corresponding information theory based theoretical framework. Accordingly, we also define the cost based on routes. Denote n and m as the numbers of nodes and links, respectively, and $e_1, e_2, \dots, e_m$ as the links. Assume there are totally u possible routes, $\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_u$, in the network, and the amount of the requested bandwidth of a connection on any route is a random variable z with a same probability density function $p(z)$. Denote $\pi_k, 1 \le k \le u$, as the normalized number of connection requests that tries to use route r_k. For example, denote the average number of connection requests in a unit of time on using route r_k as $\alpha_k, \pi_k = ((\alpha_k)/(\sum_{i=1}^{u} \alpha_i))$. Hence, $\sum_{k=1}^{u} \pi_k = 1$. In particular, if we envision a uniform distribution of connection requests among the routes, then $\pi_k = (1/u)$.

Assume a route r_k consists of links $e_{k(1)}, e_{k(2)}, \dots, e_{k(h_k)}$, where h_k is its hop count. Note that we do not consider the one hop route because a node always has accurate state information of the directly connected links. Hence, $h_k \ge 2$. Since bandwidth is a concave metric, its actual maximum available bandwidth at time t is

$$b_k(t) = \min\{x_{k(1)}(t), x_{k(2)}(t), \dots, x_{k(h_k)}(t)\} \quad (1)$$

where $x_i(t)$ is the available bandwidth of e_i at moment t. At the same time, from the perspective of a node, the available bandwidth (from link state information; not actual) on r_k is

$$\bar{b}_k(t) = \min\{y_{k(1)}(t), y_{k(2)}(t), \dots, y_{k(h_k)}(t)\} \quad (2)$$

where $y_i(t)$ is, from the perspective of a node that is not directly connected to e_i, the instantaneous available bandwidth of e_i at t . Hence, if $\bar{b}_k(t) > b_k(t)$ and a connection request with bandwidth of $b(\bar{b}_k(t) > b > b_k(t))$ arrives, this is a false positive. On the other hand, if $\bar{b}_k(t) < b_k(t)$ and the requested bandwidth satisfies that $\bar{b}_k(t) < b < b_k(t)$, this is a false negative. Define $f(r_k, b)$ and $g(r_k, b)$ as the costs of an occurrence of the false positive and false negative routing failure on route r_k, respectively, where b is the requested bandwidth. Therefore, by letting

$$\zeta_k(t,b) = \left[1 - \frac{b_k(t) - \bar{b}_k(t)}{|b_k(t) - \bar{b}_k(t)|}\right] f(r_k, b) + \left[1 + \frac{b_k(t) - \bar{b}_k(t)}{|b_k(t) - \bar{b}_k(t)|}\right] g(r_k, b) \quad (3)$$

the cost due to inaccurate link state information at time t on route r_k is defined as

$$c(k,t) = \frac{1}{2}\int_{\min\{b_k(t),\bar{b}_k(t)\}}^{\max\{b_k(t),\bar{b}_k(t)\}} p(z)\zeta_k(t,z)\,dz. \quad (4)$$

Note that when $b_k(t) > \bar{b}_k(t), \zeta_k(t,b) = 2g(r_k, b)$, and when $b_k(t) < \bar{b}_k(t), \zeta_k(t,b) = 2f(r_k, b)$. The cost at time t for the whole network can be defined as

$$c(t) = \sum_{k=1}^{u} \pi_k c(k,t). \quad (5)$$

By letting $f(r_k, b) = g(r_k, b) = 1$ for all k, the cost $c(k,t)$ and $c(t)$ are actually the probabilities of routing failures on route r_k and in the network, respectively.

Many studies [4] have shown the Long-Range Dependence or multifractal behaviors of the Internet traffic. Therefore, we cannot assume that the source signal (available bandwidth) is memoryless. Hence, each link state metric is viewed as a random **process** with memory that is independent of time in this letter, and by applying the sampling theorem, it can be treated as a time independent continuous random **sequence** with memory. Denote $x_i^1, x_i^2, \dots, x_i^N$ as N consecutive samples of the actual available bandwidth of link e_i. Similarly, the state information of e_i from the perspective of a single node can also be treated as a time independent continuous random **sequence** with memory, denoted by $y_i^1, y_i^2, \dots, y_i^N$. Note that a route may be shared by many connections and consists of several links, implying that link state metrics on different links are not statistically independent from each other. Hence, the link state metrics are correlated. Denote $\mathbf{I}(\mathbf{A};\mathbf{B})$ as the mutual information between $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{H}(\mathbf{A})$ as self-entropy of $\mathbf{A}$, and $\mathbf{H}(\mathrm{A} \mid \mathbf{B})$ as the conditional entropy between $\mathbf{A}$ and $\mathbf{B}$. By information theory, the mutual information between $\mathbf{X_1^N}, \mathbf{X_2^N}, \dots, \mathbf{X_m^N}$, and $\mathbf{Y_1^N}, \mathbf{Y_2^N}, \dots, \mathbf{Y_m^N}$ is

$$\begin{aligned}\mathbf{I}\left(\mathbf{X_1^N X_2^N \dots X_m^N}; \mathbf{Y_1^N Y_2^N \dots Y_m^N}\right) &= \mathbf{H}\left(\mathbf{X_1^N X_2^N \dots X_m^N}\right) + \mathbf{H}\left(\mathbf{Y_1^N Y_2^N \dots Y_m^N}\right) \\ &\quad - \mathbf{H}\left(\mathbf{X_1^N X_2^N \dots X_m^N Y_1^N Y_2^N \dots Y_m^N}\right)\end{aligned} \quad (6)$$

where $\mathbf{X_i^N}$ and $\mathbf{Y_i^N}$ represent the sequences $x_i^1, x_i^2, \dots, x_i^N$, and $y_i^1, y_i^2, \dots, y_i^N$, respectively. In the rest of this letter, for simplicity, we denote $\underline{\mathbf{X}}^{\mathbf{N}}$ as $\mathbf{X_1^N X_2^N \dots X_m^N}$ and $\underline{\mathbf{Y}}^{\mathbf{N}}$ as $\mathbf{Y_1^N Y_2^N \dots Y_m^N}$. If we treat the network cost defined by (5) as the source distortion, by rate-distortion analysis, the optimal link-state update satisfies that

$$\begin{aligned}R(D) &= \min_{\xi_D}\left\{\lim_{N->\infty}\frac{1}{N}\mathbf{I}(\underline{\mathbf{X}}^{\mathbf{N}};\underline{\mathbf{Y}}^{\mathbf{N}})\right\} \\ &= \min_{\xi_D}\left\{\lim_{N\to\infty}\frac{1}{N}[\mathbf{H}(\underline{\mathbf{X}}^{\mathbf{N}}) - \mathbf{H}(\underline{\mathbf{X}}^{\mathbf{N}} \mid \underline{\mathbf{Y}}^{\mathbf{N}})]\right\} \\ &= \min_{\xi_D}\left\{\lim_{N\to\infty}\frac{1}{N}[\mathbf{H}(\underline{\mathbf{X}}^{\mathbf{N}}) + \mathbf{H}(\underline{\mathbf{Y}}^{\mathbf{N}}) - \mathbf{H}(\underline{\mathbf{X}}^{\mathbf{N}}\underline{\mathbf{Y}}^{\mathbf{N}})]\right\}\end{aligned} \quad (7)$$

where $R(D)$ is the transmission rate (a single link) used by the optimal link-state update when the cost is upper bounded by D, and ξ_D is the set of transition probabilities from $\underline{\mathbf{X}}^{\mathbf{N}}$ to $\underline{\mathbf{Y}}^{\mathbf{N}}$ subject to the distortion D, i.e., ξ_D satisfies the following condition:

$$\sum_{k=1}^{u}\pi_k\left\{\lim_{N\to\infty}\frac{1}{2N}\sum_{i=1}^{N}\int_{\min\{\bar{b}_k^i, b_k^i\}}^{\max\{\bar{b}_k^i, b_k^i\}} p(z)\varphi_k\left(\bar{b}_k^i, b_k^i, z\right)\,dz\right\} \le D \quad (8)$$

where

$$\bar{b}_k^i = \min\left\{y_{k(1)}^i, y_{k(2)}^i, \ldots, y_{k(h_k)}^i\right\} \quad (9)$$

$$b_k^i = \min\left\{x_{k(1)}^i, x_{k(2)}^i, \ldots, x_{k(h_k)}^i\right\} \quad (10)$$

$$\varphi_k\left(\bar{b}_k^i, b_k^i, b\right) = \left[1 - \frac{b_k^i - \bar{b}_k^i}{\left|b_k^i - \bar{b}_k^i\right|}\right] f(r_k, b) + \left[1 + \frac{b_k^i - \bar{b}_k^i}{\left|b_k^i - \bar{b}_k^i\right|}\right] g(r_k, b). \quad (11)$$

Note that

$$\lim_{N\to\infty} \frac{1}{N}\mathbf{H}(\underline{\mathbf{X}}^{\mathbf{N}}) = \lim_{N\to\infty} \mathbf{H}\left(x_1^N x_2^N \ldots x_m^N \middle| \underline{\mathbf{X}}^{\mathbf{N-1}}\right). \quad (12)$$

Similarly

$$\lim_{N\to\infty} \frac{1}{N}\mathbf{H}(\underline{\mathbf{Y}}^{\mathbf{N}}) = \lim_{N\to\infty} \mathbf{H}\left(y_1^N y_2^N \ldots y_m^N \middle| \underline{\mathbf{Y}}^{\mathbf{N-1}}\right) \quad (13)$$

and

$$\lim_{N\to\infty} \frac{1}{N}\mathbf{H}(\underline{\mathbf{X}}^{\mathbf{N}}\underline{\mathbf{Y}}^{\mathbf{N}}) = \lim_{N\to\infty} \mathbf{H}\left(x_1^N x_2^N \ldots x_m^N y_1^N y_2^N \ldots y_m^N \middle| \underline{\mathbf{X}}^{\mathbf{N-1}}\underline{\mathbf{Y}}^{\mathbf{N-1}}\right). \quad (14)$$

Hence

$$R(D) = \min_{\xi_D}\left\{\lim_{N->\infty}\left[\mathbf{H}\left(x_1^N x_2^N \ldots x_m^N \middle| \underline{\mathbf{X}}^{\mathbf{N-1}}\right) + \mathbf{H}\left(y_1^N y_2^N \ldots y_m^N \middle| \underline{\mathbf{Y}}^{\mathbf{N-1}}\right) - \mathbf{H}\left(x_1^N x_2^N \ldots x_m^N y_1^N y_2^N \ldots y_m^N \middle| \underline{\mathbf{X}}^{\mathbf{N-1}}\underline{\mathbf{Y}}^{\mathbf{N-1}}\right)\right]\right\}. \quad (15)$$

If the memory lengths of all the sequences $\mathbf{X_1^N}, \mathbf{X_2^N}, \ldots, \mathbf{X_m^N}$ and $\mathbf{Y_1^N}, \mathbf{Y_2^N}, \ldots, \mathbf{Y_m^N}$ are $\alpha+1$, i.e., $\forall k > \alpha$, for all $1 \leq i \leq m$

$$p\left(x_i^k \middle| x_i^1 x_i^2 \ldots x_i^{k-1}\right) = p\left(x_i^k \middle| x_i^{k-1} x_i^{k-2} \ldots x_i^{k-\alpha}\right) \quad (16)$$

$$p\left(y_i^k \middle| y_i^1 y_i^2 \ldots y_i^{k-1}\right) = p\left(y_i^k \middle| y_i^{k-1} y_i^{k-2} \ldots y_i^{k-\alpha}\right) \quad (17)$$

we have

$$R(D) = \min_{\xi_D}\left\{\mathbf{H}\left(x_1^{\alpha+1} x_2^{\alpha+1} \ldots x_m^{\alpha+1} \middle| \underline{\mathbf{X}}^{\alpha}\right) + \mathbf{H}\left(y_1^{\alpha+1} y_2^{\alpha+1} \ldots y_m^{\alpha+1} \middle| \underline{\mathbf{Y}}^{\alpha}\right) - \mathbf{H}\left(x_1^{\alpha+1} x_2^{\alpha+1} \ldots x_m^{\alpha+1} y_1^{\alpha+1} y_2^{\alpha+1} \ldots y_m^{\alpha+1} \middle| \underline{\mathbf{X}}^{\alpha}\underline{\mathbf{Y}}^{\alpha}\right)\right\} \quad (18)$$

and ξ_D satisfies that

$$\sum_{k=1}^{u} \pi_k \left\{\frac{1}{2\alpha}\sum_{i=1}^{\alpha}\int_{\min\{\bar{b}_k^i, b_k^i\}}^{\max\{\bar{b}_k^i, b_k^i\}} p(z)\varphi_k\left(\bar{b}_k^i, b_k^i, z\right) dz\right\} \leq D. \quad (19)$$

By (7) and (8), a lower bound on the transmission rate of the link-state update under the constraint of a given source distortion may be computed. For the cases that sample sequences have memory length of $\alpha + 1$, the computation of the lower bound can be simplified by solving (18) and (19). In this letter, since no assumption on the distributions of $\underline{\mathbf{X}}^{\mathbf{N}}$ and requested bandwidth of connections $(p(z))$ is made, we cannot compute $R(D)$. Note that $R(D)$ is the lower bound on the transmission rate used for updating link state information under the constraint that the cost due to the routing failures is no larger than D. In turn, given an upper bound B on the transmission rate used for updating link state information, we can find out the the corresponding lower bound on the cost by solving the equation

$$R(D) = B. \quad (20)$$

Observe that if we set $\varphi_k(\bar{b}_k^i, b_k^i, b) = 2, R(D)$ is the minimized transmission rate under the constraint that the probability of routing failures is no larger than D. Hence, given an upper bound on the transmission rate, we can also compute the corresponding minimum probability of routing failures for the link-state update policies. Moreover, define

$$c(k) = \lim_{N\to\infty} \frac{1}{2N}\sum_{i=1}^{N}\int_{\min\{\bar{b}_k^i, b_k^i\}}^{\max\{\bar{b}_k^i, b_k^i\}} p(z)\varphi_k\left(\bar{b}_k^i, b_k^i, z\right) dz \quad (21)$$

as the cost due to routing failures on using route r_k. By letting $\varphi_k(\bar{b}_k^i, b_k^i, b) = 2, p_k = c(k)$ is the probability of routing failures of r_k. Denote $\theta(s,d) = \{r_{\kappa(s,d,1)}, r_{\kappa(s,d,2)}, \ldots, r_{\kappa(s,d,\epsilon(s,d))}\}$ as the set of routes that starts at edge node s and ends at d, where $\epsilon(s,d)$ represents the number of the routes. Hence, we can compute the probability of routing failures between nodes s and d as

$$p_{s,d} = \frac{1}{\sum_{i=1}^{\epsilon(s,d)} \pi_{\kappa(s,d,i)}} \sum_{i=1}^{\epsilon(s,d)} \pi_{\kappa(s,d,i)} p_{\kappa(s,d,i)}. \quad (22)$$

III. Conclusion

In this letter, after showing that per-link-based link-state update schemes cannot be optimal and efficient enough, we have proposed a framework based on information theory for optimizing link-state update, upon which efficient route-based link-state update schemes may be designed. We have also suggested, in this letter, that in order to improve the efficiency of link-state update schemes, the distribution of requested bandwidth of connections and the correlation between the state metrics of links should be considered.

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