A Computational Model for Estimating Blocking Probabilities of Multifiber WDM Optical Networks

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***Abstract*—In this letter, we propose a computational model for calculating blocking probabilities of multifiber wavelength division multiplexing (WDM) optical networks. We first derive the blocking probability of a fiber based on a Markov chain, from which the blocking probability of a link is derived by means of conditional probabilities. The blocking probability of a lightpath can be computed by a recursive formula. Finally, the network-wide blocking probability can be expressed as the ratio of the total blocked load versus the total offered load. Simulation results for different fiber-wavelength configurations conform closely to the numerical results based on our proposed model, thus demonstrating the feasibility of our proposed model for estimating the blocking performance of multifiber WDM optical networks.**

***Index Terms*—Blocking probability, multifiber wavelength division multiplexing (WDM) optical networks, wavelength assignment.**

I. Introduction

THE common practice of installing bundles of multiple fibers motivates the research of multifiber wavelength division multiplexing (WDM) optical networks. In such a network, each link includes multiple fibers, and each fiber contains multiple wavelengths. On a lightpath in multifiber WDM networks, a wavelength that cannot continue on the next hop can be switched to another fiber if the same wavelength is unoccupied in at least one of the other fibers. Multifiber WDM optical networks are attractive alternatives to single fiber WDM optical networks with wavelength converters. An F-fiber W-wavelength network (i.e., each link has F fibers, and each fiber supports W wavelengths) is functionally equivalent to an FW-wavelength network with wavelength conversion of degree F. The research in [1] showed that it is possible to use fewer wavelengths in each fiber with multiple fibers than with a single fiber.

Many studies on the blocking probability analysis of WDM optical networks have been reported. The analytical model for a static-routing WDM network was proposed in [2]. For single fiber networks, the deployment of wavelength converters was shown to reduce the network-wide blocking probability. The blocking probability of a lightpath under the assumption of circuit-switched network traffic models was formulated in [3], and the lightpath length, the node degree, and the interference length were identified to be the three most important parameters. The upper and lower bounds of the loss probability were analyzed in optical burst-switched WDM networks with fiber delay lines (FDLs) [4]. In [5], the authors proposed the multifiber link-load correlation (MLLC) model to study the blocking performance for multifiber networks. The MLLC model considers the status of wavelength trunks and the link-load correlation. According to their simulations, a limited number of fibers were sufficient to guarantee high network performance.

In this letter, we analyze blocking probabilities in multifiber WDM optical networks from a bottom up manner. The wavelength occupancy states of a fiber are analyzed by a Markov chain, from which the link blocking performance is evaluated. The lightpath model is derived based on the wavelength occupancies of the intermediate links in a lightpath, and the network-wide blocking probability is obtained by combining the lightpath blocking probabilities. In Section II, we propose our computational model. In Section III, the simulation and computational results are presented and compared. Section IV provides the conclusions.

II. Analytical Model

In this section, we derive the computational model of the multifiber WDM networks. First, we develop the fiber model as a Markov chain; then we extend it to the link model, which includes multiple fibers; the lightpath model is based on the idle wavelength availability in all of the intermediate links; the network-wide blocking probability is obtained at the final step.

A. Assumptions and Notations

The following assumptions are applied. *1)* The network $G(f, w, N, E)$ is an even multifiber WDM network without wavelength converters. $n = |N|$ is the number of nodes, while $e = |E|$ is the number of links. Each link consists of a bundle of f fibers, and each fiber has w wavelengths. *2)* Connections arrive according to a Poisson distribution. The average holding time of a connection is exponentially distributed with a mean of one unit. *3)* An entire wavelength channel is allocated to a single connection. *4)* An incoming connection can be switched to any fiber at the next hop as long as the same wavelength is free. The employed fiber is selected randomly. *5)* A connection is dropped immediately if it is blocked. *6)* Loads of individual fibers and links are independent. Wavelength occupancies in fibers and links are independent.

In addition, we also assume that static shortest lightpath routing is employed as the routing policy, and wavelengths are assigned randomly. Since no wavelength converters are in the

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network, each lightpath must satisfy the wavelength continuity constraint.

The following notations are defined:

- E_m link m,
- E_m^n fiber n in link E_m,
- P the network-wide blocking probability,
- P_r the blocking probability of lightpath r,
- r_i a lightpath in the network for node pair i,
- T^i the offered load for node pair i,
- $t_{E_m^n}$ the offered load on fiber E_m^n,
- $A_{E_m^n}$ the number of idle wavelengths on fiber E_m^n,
- A_{E_m} the number of idle wavelengths on link E_m,
- $A_{r_i}^h$ the number of idle wavelegnths on the h-hop lightpath r_i.

B. Analysis of a Fiber

The offered load on fiber E_m^n is the sum of the offered loads on all lightpaths, in which fiber E_m^n is a component, i.e.,

$$t_{E_m^n} = \sum_{i=1}^{n(n-1)} \sum_{E_m^n \in r_i} T^i. \tag{1}$$

The connection arrival of fiber E_m^n is a Poisson process with rate $t_{E_m^n}$. The states of the number of idle wavelengths on fiber E_m^n, i.e., $A_{E_m^n}$, can be depicted by the Markov chain in Fig. 1. The arrival rate is the aggregated traffic of fiber E_m^n while the holding time of each connection is exponentially distributed with a mean of one unit. Consequently, the arriving and servicing behavior forms an $M/M/w/w$ Markov system.

The idle wavelength distribution $P(A_{E_m^n} = k)$ is:

$$P\left(A_{E_m^n} = k\right) = \frac{\prod_{i=1}^{k}(w-i+1)}{t_{E_m^n}^k} P\left(A_{E_m^n} = 0\right) \tag{2}$$

where

$$P\left(A_{E_m^n} = 0\right) = \left[1 + \sum_{x=1}^{w} \frac{\prod_{i=1}^{x}(w-i+1)}{t_{E_m^n}^x}\right]^{-1}. \tag{3}$$

Equation (3) is the probability of fiber E_m^n being blocked.

C. Analysis of a Link

Link E_m consists of f fibers denoted as $E_m^1, E_m^2, \ldots, E_m^f$. The conditional probability that there are k different idle wavelengths given that fiber E_m^i has k_i idle wavelengths is $P(A_{E_m} = k | A_{E_m^1} = k_1, A_{E_m^2} = k_2, \ldots, A_{E_m^f} = k_f)$. First,

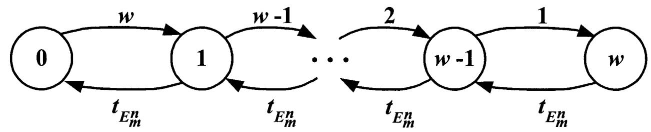

Fig. 1. Markov chain for the distribution of idle wavelengths on a fiber.

consider the wavelength occupancies in the first two fibers, E_m^1 and E_m^2. Assume E_m^1 has k_1 idle wavelengths and E_m^2 has k_2 idle wavelengths; the conditional probability that there are k different idle wavelengths in these two fibers given the above conditions is:

$$P\left(A_{E_m^{1,2}} = k | A_{E_m^1} = k_1, A_{E_m^2} = k_2\right) = \frac{\binom{k_2}{k_1+k_2-k}}{\binom{k}{k_1}} \tag{4}$$

where $\max(0, k_1,\, k_2) \le \min(w, k_1 + k_2)$, and $A_{E_m^{1,2}}$ denotes the total number of different idle wavelengths in fibers E_m^1 and E_m^2. When $k = 0$, no idle wavelength is available in the first two fibers. Extending (4) to the first three fibers, given that E_m^1 has k_1, E_m^2 has k_2, and E_m^3 has k_3 idle wavelengths, the conditional probability that there are k different idle wavelengths available is

$$\begin{aligned} &P\left(A_{E_m^{1,3}} = k | A_{E_m^1} = k_1, A_{E_m^2} = k_2, A_{E_m^3} = k_3\right) \\ &= \sum_{x=\max(0,k_1,k_2)}^{\min(w,k_1+k_2)} P\left(A_{E_m^{1,2}} = x | A_{E_m^1} = k_1, A_{E_m^2} = k_2\right) \\ &\cdot P\left(A_{E_m^{1,3}} = k | A_{E_m^{1,2}} = x, A_{E_m^3} = k_3\right) \end{aligned} \tag{5}$$

where $\max(0, k_1, k_2, k_3) \le k \le \min(w, k_1 + k_2 + k_3)$. When $k = 0$, no idle wavelength is available in the first three fibers. For all the f fibers in link E_m, the corresponding conditional probability that there are k different idle wavelengths in link E_m given that fiber E_m^i has k_i idle wavelengths is shown in (6), at the bottom of the page, where $\max(0, k_1, \ldots, k_f) \le k \le \min(w, \sum_{i=1}^f k_i)$, and

$$\begin{aligned} &P(A_{E_m}=k) \\ &= \sum_{k_1=0}^{w} \cdots \sum_{k_f=0}^{w} P\left(A_{E_m}=k | A_{E_m^1}=k_1, A_{E_m^2}=k_2, \ldots, A_{E_m^f}=k_f\right) \\ &\cdot \prod_{i=1}^{f} P\left(A_{E_m^i}=k_i\right). \end{aligned} \tag{7}$$

Since the wavelength occupancies in all of the f fibers are independent, the probability that there are k idle wavelengths in link E_m can be obtained as (7). The idle wavelength distribution of

$$\begin{aligned} &P\left(A_{E_m}=k | A_{E_m^1}=k_1, A_{E_m^2}=k_2, \ldots, A_{E_m^f}=k_f\right) \\ &= \sum_{x_{1,2}=\max(0,k_1,k_2)}^{\min(w,k_1+k_2)} \cdots \sum_{x_{1,f-1}=\max(0,x_{1,f-2},k_{f-1})}^{\min(w,x_{1,f-1}+k_{f-1})} P\left(A_{E_m^{1,2}}=x_{1,2} | A_{E_m^1} = k_1, A_{E_m^2} = k_2\right) \\ &\cdot \prod_{y=3}^{f-1} P\left(A_{E_m^{1,y}} = x_{1,y} | A_{E_m^{1,y-1}} = x_{1,y-1}, A_{E_m^y} = k_y\right) P\left(A_{E_m} = k | A_{E_m^{1,f-1}} = x_{1,f-1}, A_{E_m^f} = k_f\right) \end{aligned} \tag{6}$$

a fiber, $P(A_{E_m^i} = k_i)$, is from (2). When $k = 0$, $P(A_{E_m} = 0)$ is the blocking probability of link E_m.

D. Analysis of a Lightpath

We first consider a two-hop lightpath. Assume lightpath r has links E_i and E_j, and the conditional probability that there are k different idle wavelengths in lightpath r given E_i has k_i idle wavelengths and E_j has k_j idle wavelengths is

$$P\left(\frac{A_r^2 = k}{A_{E_i} = k_i}, A_{E_j} = k_j\right) = \binom{k_j}{k}\binom{w-k_j}{k_i-k}|\binom{w}{k_i} \quad (8)$$

where $\max(0, k_i + k_j - w) \le k \le \min(k_i, k_j)$. Under the assumption that the wavelength occupancies in links are independent, the probability that there are k different idle wavelengths in the two-hop lightpath $P(A_r^2 = k)$ is

$$P\left(A_r^2 = k\right) = \sum_{k_i=0}^{w}\sum_{k_j=0}^{w} P\left(A_r^2 = k | A_{E_i} = k_i, A_{E_j} = k_j\right) \cdot P\left(A_{E_i} = k_i\right) P\left(A_{E_j} = k_j\right) \quad (9)$$

For an h-hop lightpath where $h > 2$, the probability that there are k different idle wavelengths in lightpath r can be derived recursively

$$P\left(A_r^h = k\right) = \sum_{k_i=0}^{w}\sum_{k_j=0}^{w} P\left(A_r^h = k | A_r^{h-1} = k_i, A_{E_m} = k_j\right) \cdot P\left(A_r^{h-1} = k_i\right) P\left(A_{E_m} = k_j\right). \quad (10)$$

The initial condition for (10) is (9), and $P(A_{E_m} = k_j)$ can be calculated by (7). The blocking probability for lightpath r is $P(A_r^h = 0)$, where h is the length of lightpath r.

E. Analysis of a Network

Based on the lightpath model, the network-wide blocking probability can be obtained by the ratio of the total blocked load versus the total offered load, i.e.,

$$P = \frac{\sum_{i=1}^{n(n-1)} T^i P\left(A_{r_i}^h = 0\right)}{\sum_{i=1}^{n(n-1)} T^i}. \quad (11)$$

III. Simulations and Discussions

Simulations are conducted on the NSFNET with different fiber-wavelength combinations of the same link capacity: 2-fiber 8-wavelength, 4-fiber 4-wavelength, and 8-fiber 2-wavelength. The network traffic load varies from 10 to 100 erlangs. The numerical results are computed based on (1)–(11). Initially, the probabilities in (4), (5), (6) and (8) are calculated independently based on w and f, and reused for different scenarios. For a specific routing policy, the offered load for node pair i, i.e., T^i, is known, and the offered load of each fiber is determined by (1) with $O(n^2)$ complexity. The idle wavelength distribution of each fiber is then derived by (2) with $O(w^3)$ complexity. Next, the idle wavelength distribution of each link is computed by (7) with $O(w^f f)$ complexity. The idle wavelength distribution of a lightpath is obtained by (10) with $O(w^4 h)$ complexity. Finally, the network-wide blocking probability is achieved by (11) with $O(n^2)$ complexity. Compared to the MLLC model [5], our model has less dependent terms, and is a more efficient computational model. Fig. 2 shows the numerical results which conform closely to the simulation results. The network-wide blocking probability increases as the network load becomes heavier, and it decreases as the number of fibers increases.

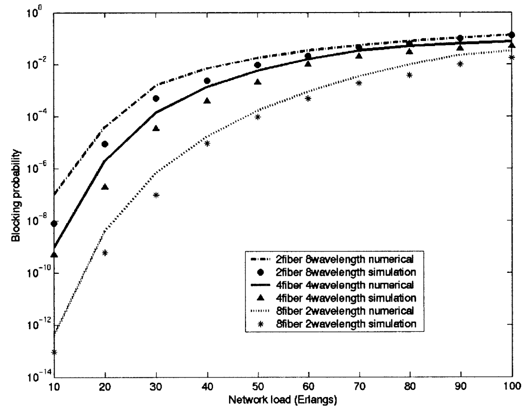

Fig. 2. Network-wide blocking probability versus network load.

IV. Conclusions

We have proposed the computational model for calculating the blocking probabilities of multifiber WDM optical networks. The network-wide blocking probability is derived from the fiber model, the link model, and the lightpath model. The comparison between numerical and simulation results indicates that such a computational model is adequate in estimating the blocking performance of multifiber WDM optical networks.

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