MEMORANDUM

To: ChE 486 Class -- Spring 1997          From: Prof. R. Barat
Date: January 21, 1997       Re: Gas Rotameter Corrections

Theory shows us that, for variable area rotameters, the mass flow rate is approximately proportional to the square root of the metered fluid mass density according to

\[ w = C_R S_2 \left( \frac{2 \, g \, V_f \, (\rho_f - \rho) \, \rho}{S_f} \right)^{0.5} \]  

(1)

where \( w \) = mass flow rate, \( C_R \) = constant coefficient, \( S_2 \) = minimum cross sectional area for flow between the float and the tube wall, \( g \) = gravitational constant, \( V_f \) = volume of the float, \( \rho_f \) = mass density of the float, \( \rho \) = mass density of the fluid, and \( S_f \) = approximately the cross sectional area of the float at its maximum diameter. For all practical purposes, \( (\rho_f - \rho) = \rho_f \). Equation (1) becomes

\[ w = C \, S_2 \, \rho^{0.5} \]  

(2)

where \( C \) = lumped constant. Now, as the flow rate increases and the float rises, \( S_2 \) increases to allow for the fluid to pass.

For a fluid metered at a fixed density \( \rho \), the rotameter calibration basically depends on \( w \) and \( S_2 \); hence, almost all variable area rotameters have a linear calibration curve. However, if the operating fluid density of the rotameter is significantly different from the calibration density, then the flow rate for a given reading must be corrected. In our lab, this would be irrelevant for liquids since they are essentially incompressible and metered at room temperature.

The corrections must be made for gas flows. Now, in your experiment, take note where the rotameter flow control valve is located. If located on the inlet, and if the rotameter outlet flow is essentially at the calibration pressure (usually atmospheric), then no correction is needed. However, if the control valve is on the outlet, then the rotameter is likely operating at an elevated pressure relative to the calibration pressure.
For a given rotameter reading (i.e. $S_2$ value), we begin with Equation 2 in which we can say

$$w_2 = w_1 \left( \frac{\rho_2}{\rho_1} \right)^{0.5}$$

Let state 1 = state of the gas (i.e. temperature, pressure, and identity of the gas) at which the rotameter is calibrated. Let state 2 = state of the gas at which the rotameter is operated. We will be able to correct the calibration curve of any rotameter for temperature, pressure, and type of gas.

It is desirable to convert from mass flow rates to volumetric flow rates at STP conditions. For a given mass rate $w$, we can say, using the ideal gas law, that

$$w = \rho \ V = \rho^* \ V^* = \left( \frac{P}{R \ T} \right)^* \ W \ V^*$$

where $V$ = volumetric flow rate, $^*$ = STP conditions, $W$ = molecular weight, $P$ = pressure, and $R$ = gas constant. Substituting Equation 4 into Equation 3, we get

$$\left( \frac{P}{R \ T} \right)^* W_2 \ V_2^* = \left( \frac{P}{R \ T} \right)^* W_1 \ V_1^* \left( \frac{\rho_2}{\rho_1} \right)^{0.5}$$

This reduces to our working relationship for calibration correction

$$V_2^* = V_1^* \left( \frac{W_1}{W_2} \right) \left( \frac{\rho_2}{\rho_1} \right)^{0.5}$$

We will consider three cases.

**Case I: Isothermal, Same Gas, Different Pressure**

We must evaluate the density ratio.

$$\frac{\rho_2}{\rho_1} = \left( \frac{P_2}{P_1} \right) \left( \frac{R \ T}{P_1 \ W} \right) = \frac{P_2}{P_1}$$

Equation 6 becomes
\[ V_2^* = V_1^* \left( \frac{P_2}{P_1} \right)^{0.5} \] (8)

**Case II: Isothermal, Different Gas, Same Pressure**

We must evaluate the density ratio.

\[ \frac{\rho_2}{\rho_1} = \left( \frac{P}{R} \right) \left( \frac{T_2}{T_1} \right) = \frac{W_2}{W_1} \] (9)

Equation 6 becomes

\[ V_2^* = V_1^* \left( \frac{W_1}{W_2} \right)^{0.5} \] (10)

**Case III: Different Temperature, Same Gas, Same Pressure**

We must evaluate the density ratio.

\[ \frac{\rho_2}{\rho_1} = \left( \frac{P}{R} \right) \left( \frac{T_1}{T_2} \right) = \frac{T_1}{T_2} \] (11)

Equation 6 becomes

\[ V_2^* = V_1^* \left( \frac{T_1}{T_2} \right)^{0.5} \] (12)

**General Case: Different Temperature, Gas, and Pressure**

We must evaluate the density ratio.

\[ \frac{\rho_2}{\rho_1} = \left( \frac{P_2}{R} \right) \left( \frac{T_1}{P_1} \right) = \frac{P_2}{T_2} \frac{T_1}{P_1} \frac{W_2}{W_1} \] (11)

Equation 6 becomes

\[ V_2^* = V_1^* \left( \frac{P_2}{T_2} \frac{T_1}{P_1} \frac{W_1}{W_2} \right)^{0.5} \] (12)