Physics 111: Mechanics
Lecture 2

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Registering your iClicker in class

- Smart devices are not accepted.
- Turn on your iClicker.
- Make sure to use the channel ‘AB’.
- When you see your name, press the letters shown beside it.
- You are registered!
- If you make a mistake, press ‘DD’ to cancel.

Let's test it…

What is the Most Advanced Physics Course You Have Had?

A. High school AP Physics course

B. High school regular Physics course

C. College non-calculus-based course

D. College calculus-based course (or I am retaking Phys 111)

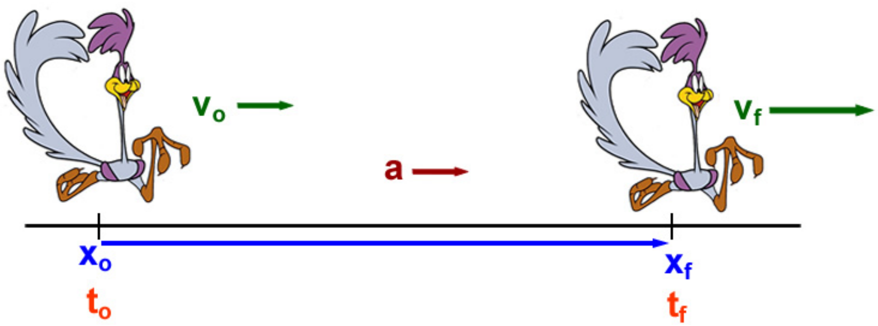
E. None, or none of the above

Announcements

- iClicker: please procure your iClicker. We'll start setting it up today. Don't lose your quiz credits (10% of your final grade).
- PHYS 111 tutoring schedule updated. See our course website https://web.njit.edu/~binchen/phys111/ for detailed information

Lecture 1 Review: Problem-Solving Hints

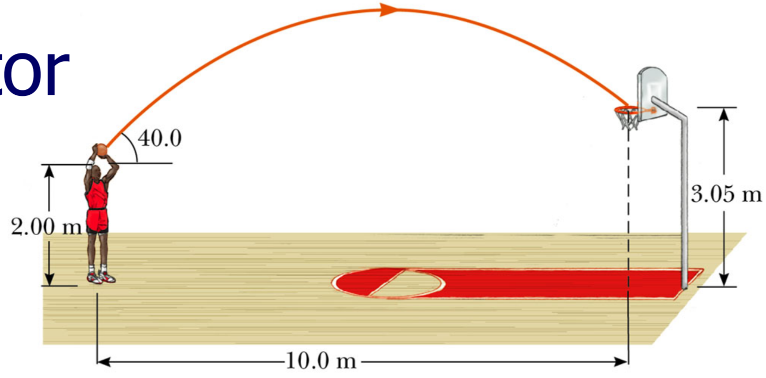
- Read the problem
- Draw a diagram
  - Choose a coordinate system, label initial and final points, indicate a positive direction for velocities and accelerations



- Label all quantities, be sure all the units are consistent
  - Convert if necessary
- Choose the appropriate kinematic equation
- Solve for the unknowns
  - You may have to solve two equations for two unknowns
- Check your results

Chapter 3 Motion in 2-D or 3-D

- Introduction to vectors
- 3.1 Position and Velocity Vectors
- 3.2 The Acceleration Vector
- 3.3 Projectile Motion
- 3.4 Motion in a Circle
- 3.5* Relative Velocity (self-study section)



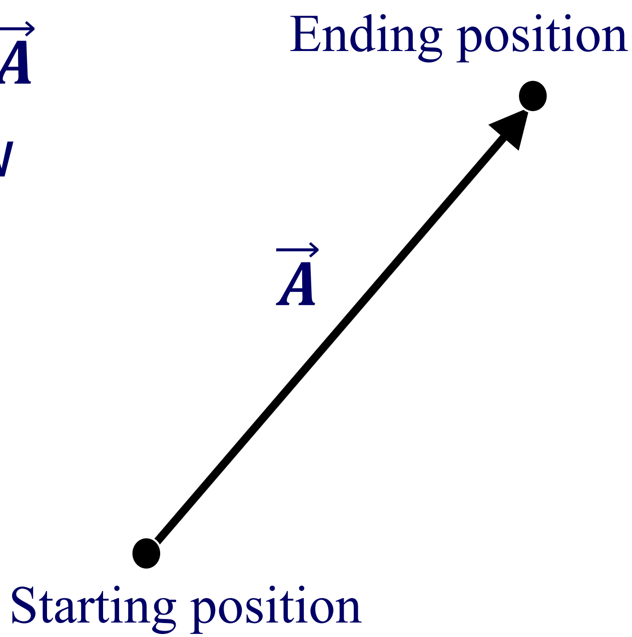
Vectors and Scalars

- Vectors
  - Displacement
  - Velocity (magnitude and direction!)
  - Acceleration
  - Force
  - Momentum
- Scalars:
  - Distance
  - Speed (magnitude of velocity)
  - Temperature
  - Mass
  - Energy
  - Time

- A **vector** quantity has both **magnitude** (value + unit) and **direction**
- A **scalar** is completely specified by only a **magnitude** (value + unit)

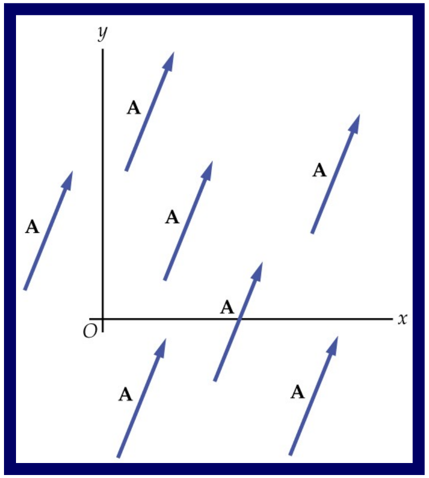
Vectors: Important Notation

- To **describe vectors** we will use:
  - The **bold font**: Vector A is **A**
  - And/or an arrow above the vector: $\vec{A}$
  - In the pictures, we will always show vectors as arrows
  - Arrows point the direction
  - To describe the magnitude of a vector we will use absolute value sign: $|\vec{A}|$ or just A
  - Magnitude is always positive, the magnitude of a vector is equal to the length of a vector.



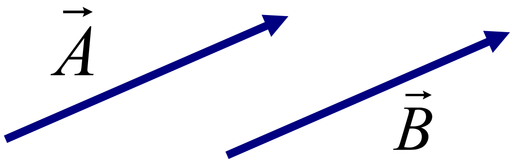
Properties of Vectors

- Equality of Two Vectors
  - Two vectors are **equal** if they have the same magnitude and the same direction
- Movement of vectors in a diagram
  - Any vector can be moved parallel to itself without being affected



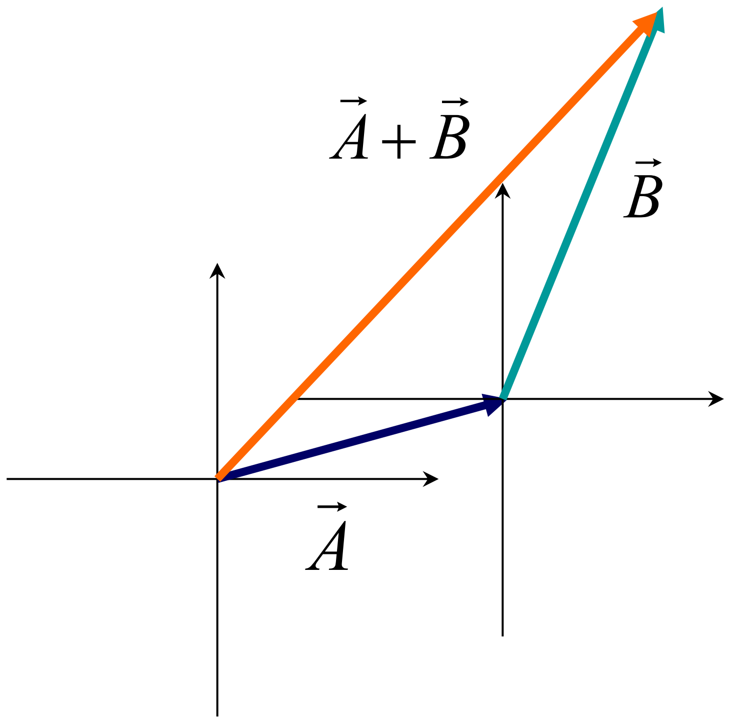
- Negative of a vector
  - A vector the **negative** of another if they have the same magnitude but are 180° apart (opposite directions)

$$\vec{A} = -\vec{B} \text{ or } \vec{B} = -\vec{A}$$



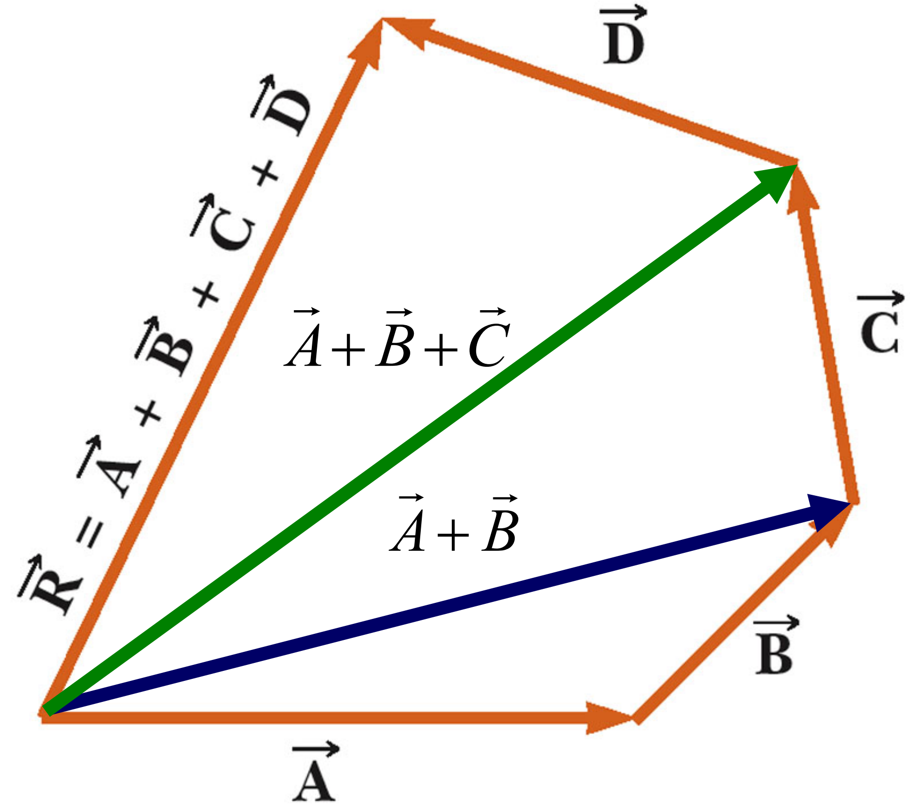
Adding Vectors Geometrically (Triangle Method)

- Draw the first vector $\vec{A}$ with the appropriate length and in the direction specified, with respect to a coordinate system
- Draw the next vector $\vec{B}$ with the appropriate length and in the direction specified, with respect to a coordinate system whose origin is the end of vector $\vec{A}$ and parallel to the coordinate system used for $\vec{A}$: "tip-to-tail".
- The resultant is drawn from the origin of $\vec{A}$ to the end of the last vector $\vec{B}$



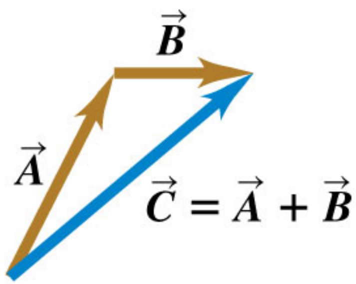
Adding Vectors Graphically

- When you have many vectors, just keep repeating the process until all are included
- The resultant is still drawn from the origin of the first vector to the end of the last vector

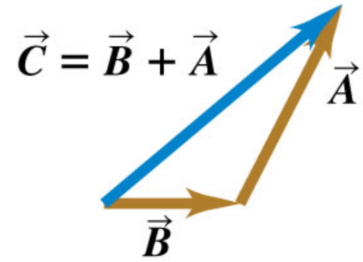


More on Vector Addition

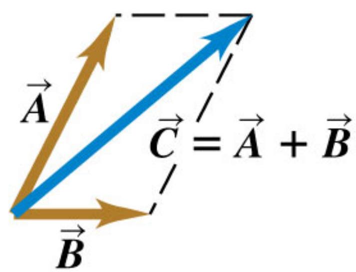
(a) We can add two vectors by placing them head to tail.



(b) Adding them in reverse order gives the same result.



(c) We can also add them by constructing a parallelogram.

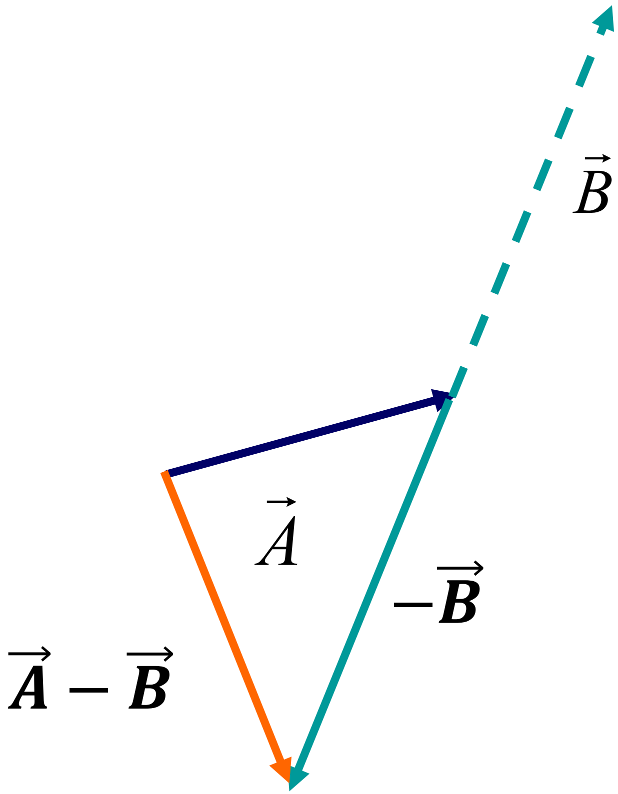


Vector Subtraction

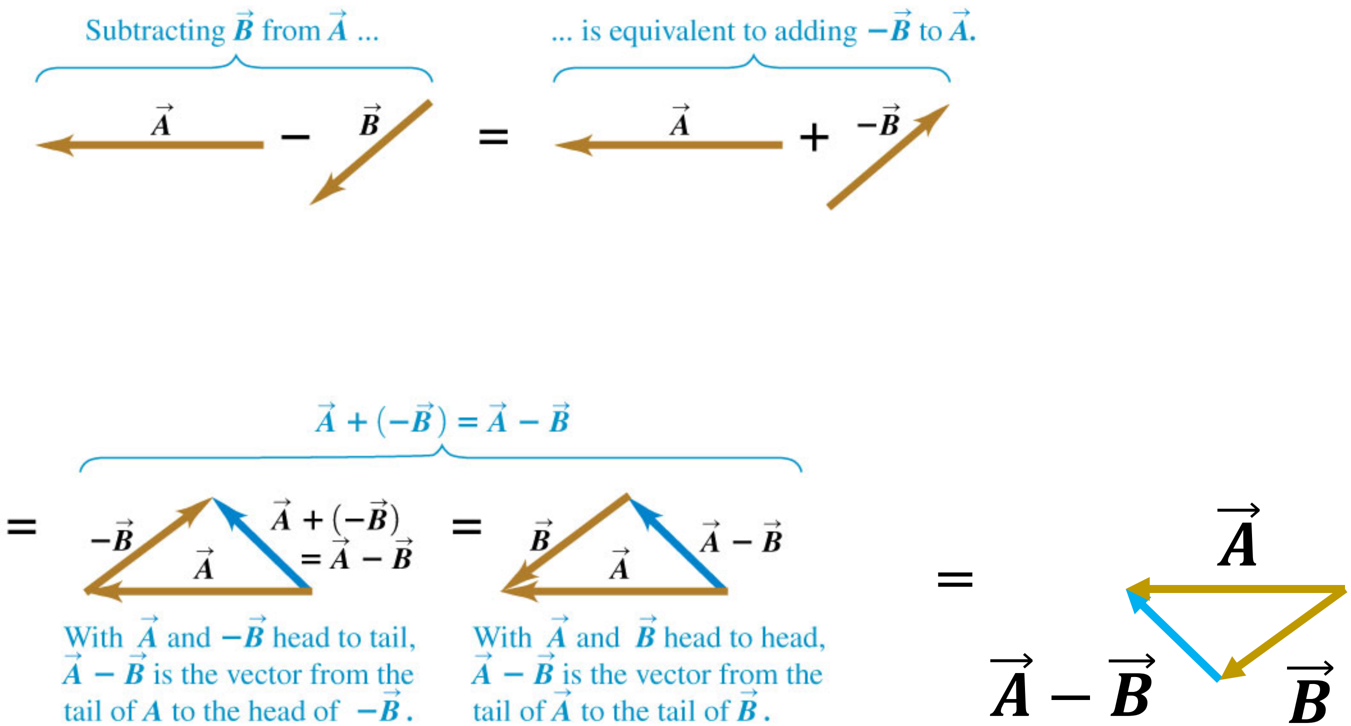
- Special case of vector addition
  - Add the negative of the subtracted vector

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

- Continue with standard vector addition procedure



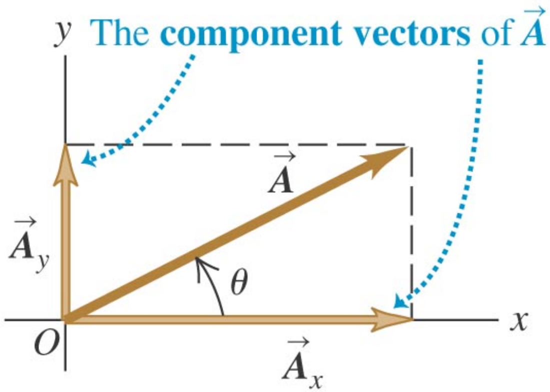
More on Vector Subtraction

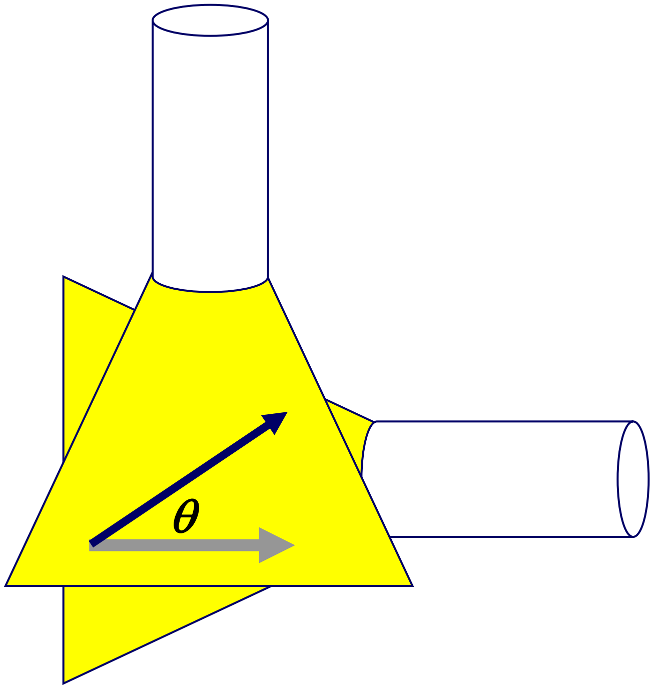


Components of a Vector

- **Components of a vector** are the projections of the vector along the x- and y-axes
- **Components** are **not** vectors, they are magnitudes of **component vectors**

$$\vec{A} = \vec{A_x} + \vec{A_y}$$





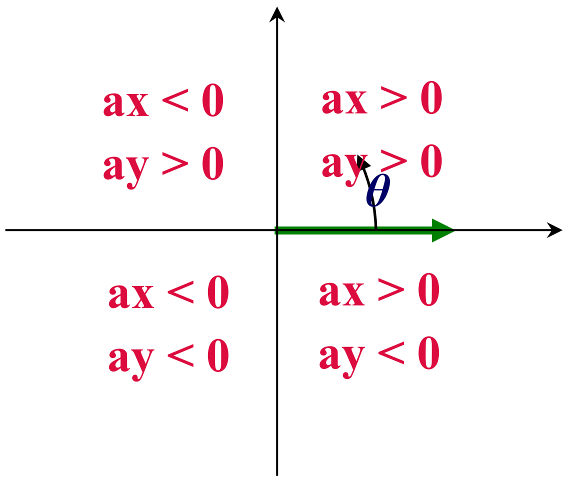
Components of a vector $\vec{A}$:

$A_x = A\cos\theta$

$A_y = A\cos(90^\circ - \theta) = A\sin\theta$

Components of a Vector

- The previous equations are valid ***only if θ is measured with respect to the x-axis***
- The components can be positive or negative and will have the same units as the original vector



$\theta=0$, $A_x=A>0$, $A_y=0$

$\theta=45°$, $A_x=A\cos 45°>0$, $A_y=A\sin 45°>0$

$\theta=90°$, $A_x=0$, $A_y=A>0$

$\theta=135°$, $A_x=A\cos 135°<0$, $A_y=A\sin 135°>0$

$\theta=180°$, $A_x=-A<0$, $A_y=0$

$\theta=225°$, $A_x=A\cos 225°<0$, $A_y=A\sin 225°<0$

$\theta=270°$, $A_x=0$, $A_y=-A<0$

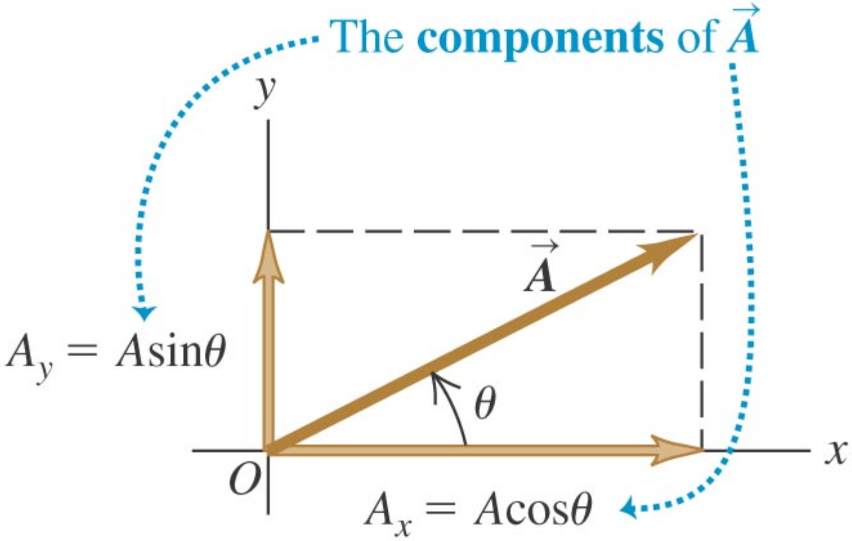
$\theta=315°$, $A_x=A\cos 315°<0$, $A_y=A\sin 315°<0$

Calculations Using Components: Magnitude and Direction

- We can use the components of a vector to find its magnitude a direction:

$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$\tan\theta = \frac{A_y}{A_x}, \text{ and } \theta = \arctan\frac{A_y}{A_x}$$



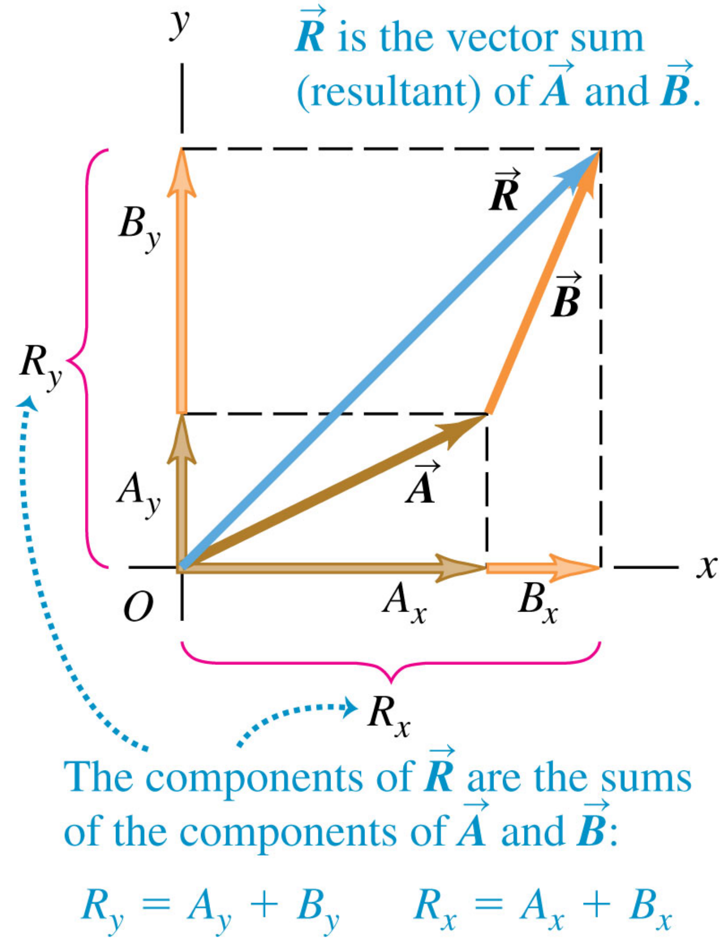
Calculations Using Components: Vector Addition

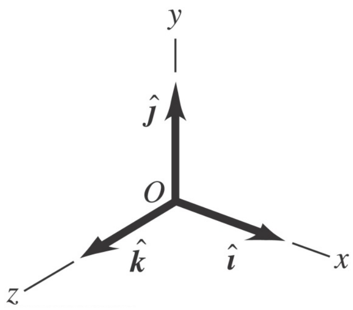
- We can use the components of a set of vectors to find the components of their sum:

$$\vec{\boldsymbol{R}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}}$$

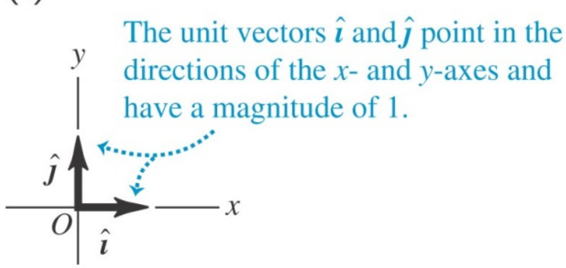
- Components of $\vec{\boldsymbol{A}}$: A_x, A_y
- Components of $\vec{\boldsymbol{B}}$: B_x, B_y
- Components of $\vec{\boldsymbol{R}}$:

$$R_x = A_x + B_x, R_y = A_y + B_y$$

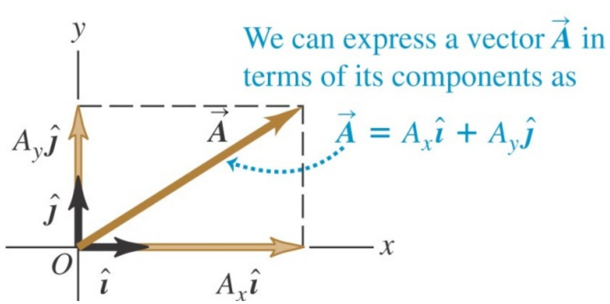




(a)



(b)



Unit Vectors

- Unit vectors $\hat{\imath}, \hat{\jmath}, \hat{k}$
- Unit vectors used to specify direction
- Unit vectors have a magnitude of 1
- Then $\overrightarrow{A_x} = A_x\hat{\imath}$, $\overrightarrow{A_y} = A_y\hat{\jmath}$

Magnitude **Magnitude**

Unit vector **Unit vector**

$$\vec{A} = \overrightarrow{A_x} + \overrightarrow{A_y} = A_x\hat{\imath} + A_y\hat{\jmath}$$

Adding Vectors Algebraically

- Consider two vectors

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$$

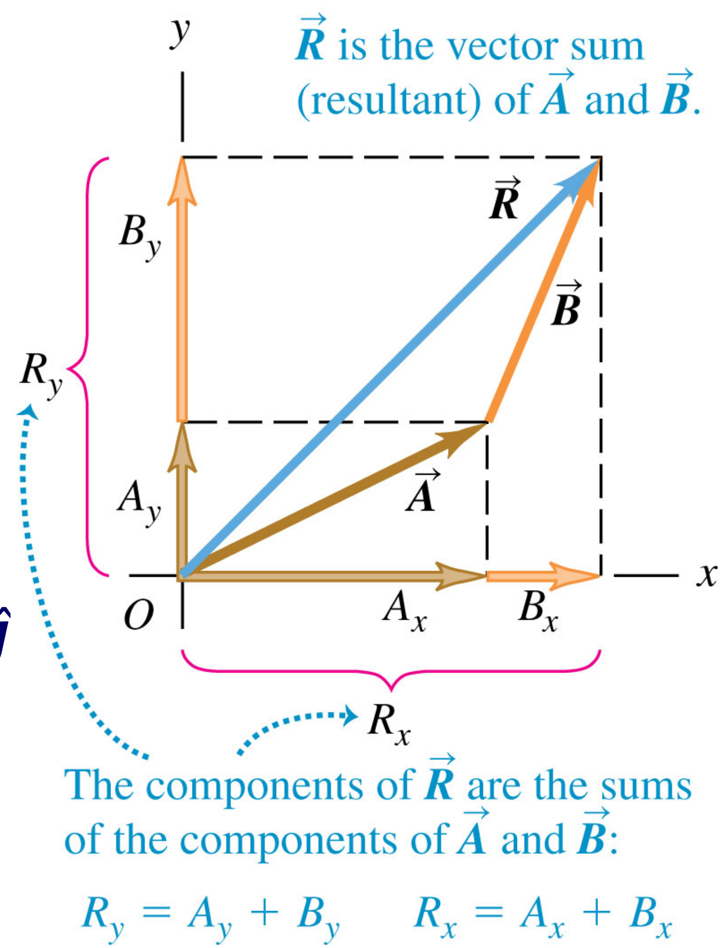
- Then

$$\vec{A} + \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath}) + (B_x\hat{\imath} + B_y\hat{\jmath})$$

$$=(A_x+B_x)\hat{\imath} + (A_y+B_y)\hat{\jmath}$$

- $\vec{R} = \vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + (A_y+B_y)\hat{\jmath}$
  $\vec{R} = C_x\hat{\imath} + C_y\hat{\jmath}$
- So we have reproduced

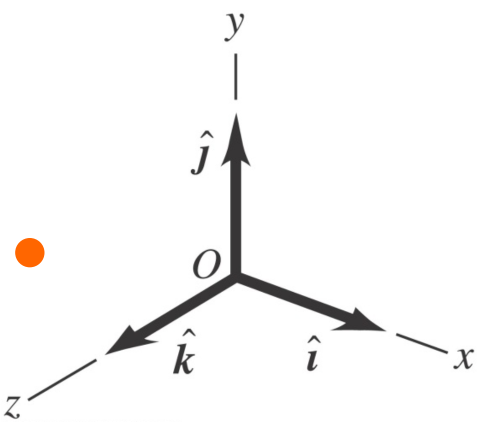
$$R_x = A_x + B_x,\ R_y = A_y + B_y$$



Motion in two dimensions

- Kinematic variables in one dimension
  - Position: $\vec{\boldsymbol{x}}(t)$ m
  - Velocity: $\vec{\boldsymbol{v}}(t)$ m/s
  - Acceleration: $\vec{\boldsymbol{a}}(t)$ m/s^2

- Kinematic variables in three dimensions
  - Position: $\vec{\boldsymbol{r}}(t) = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}} + z\hat{\boldsymbol{k}}$ m
  - Velocity: $\vec{\boldsymbol{v}}(t) = v_x\hat{\boldsymbol{\imath}} + v_y\hat{\boldsymbol{\jmath}} + v_z\hat{\boldsymbol{k}}$ m/s
  - Acceleration: $\vec{\boldsymbol{a}}(t) = a_x\hat{\boldsymbol{\imath}} + a_y\hat{\boldsymbol{\jmath}} + a_z\hat{\boldsymbol{k}}$ m/s^2
- All are vectors: have direction and magnitudes



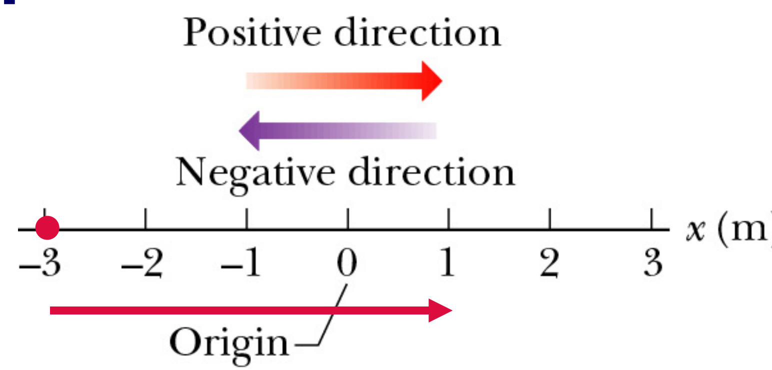
Position and Displacement

- In one dimension

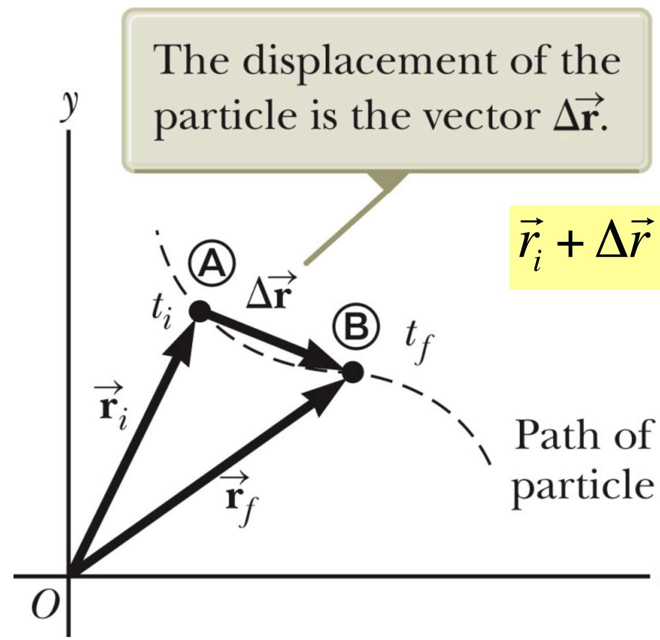
$$\Delta\vec{x} = \vec{x_f}(t_f) - \vec{x_i}(t_i)$$

$x_i\,(t_i) = -\,3.0$ m, $x_f\,(t_f) = +\,1.0$ m

$\Delta x = +1.0$ m $+ 3.0$ m $= +4.0$ m



- In two dimensions
  - Position: the position of an object is described by its position vector $\vec{r}(t)$ always points to particle from origin.
  - Displacement: $\Delta\vec{r} = \vec{r_f} - \vec{r_i}$



$$\vec{r}_i + \Delta\vec{r} = \vec{r}_f$$

Example: Walkman

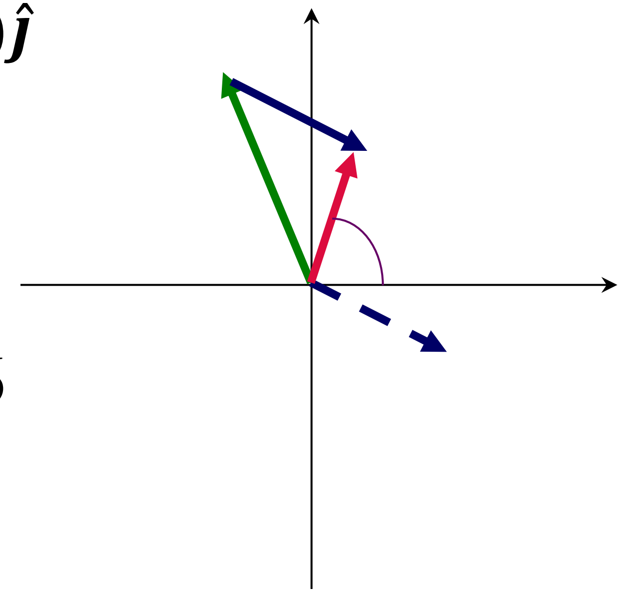
- A man walking (with a walkman) firstly takes one walk which can be described algebraically as $\vec{A} = -3\hat{\imath} + 5\hat{\jmath}$, followed by another $\vec{B} = 4\hat{\imath} - 2\hat{\jmath}$. Find the final displacement and direction of the sum of these motions

$$\vec{C} = \vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}$$
$$= (-3 + 4)\hat{\imath} + (5 - 2)\hat{\jmath} = 1\hat{\imath} + 3\hat{\jmath}$$

$$C_x = 1 \qquad C_y = 3$$

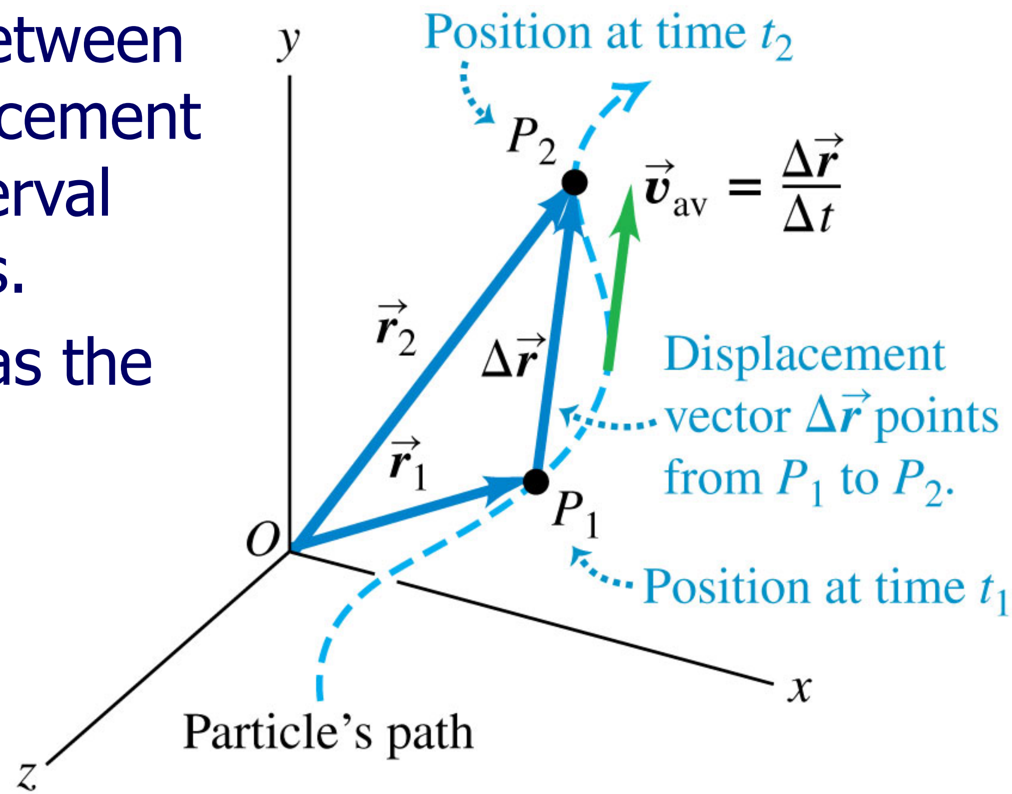
$$C = (C_x{}^2 + C_y{}^2)^{1/2} = (1^2 + 3^2)^{1/2} = 3.16$$

$$\theta = \arctan\left(\frac{C_y}{C_x}\right) = \arctan\frac{3}{1} = 71.5^{\circ}$$



Average Velocity

- The average velocity between two points is the displacement divided by the time interval between the two points.
- The average velocity has the same direction as the displacement.



$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

$$= \frac{\Delta x \hat{\imath} + \Delta y \hat{\jmath}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath} = v_{av,x}\hat{\imath} + v_{av,y}\hat{\jmath}$$

Instantaneous Velocity

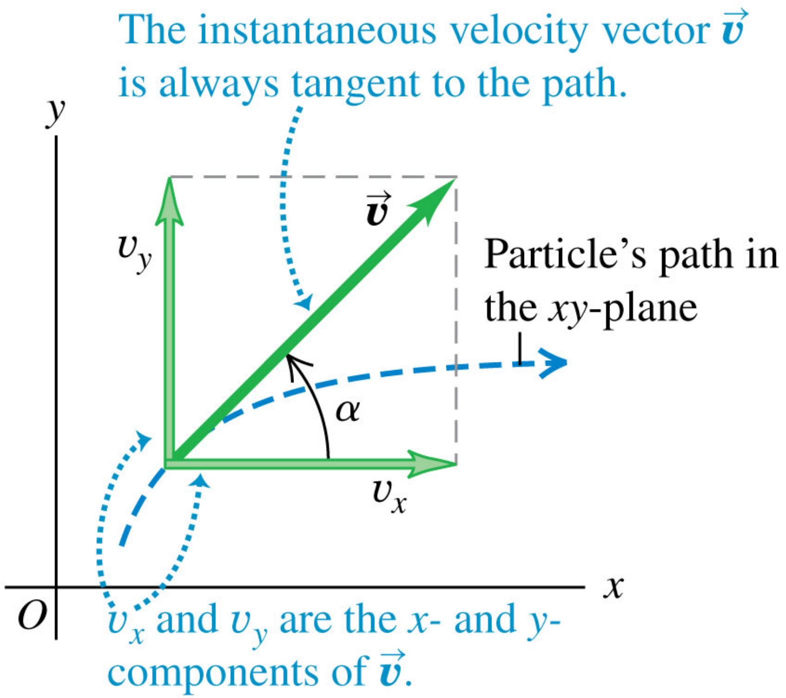
- The ***instantaneous velocity*** is the instantaneous rate of change of position vector with respect to time.

$$\vec{\boldsymbol{\upsilon}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\boldsymbol{r}}}{\Delta t} = \frac{d\vec{\boldsymbol{r}}}{dt}$$

- The components of the ***instantaneous velocity*** are

$$\upsilon_x = \frac{dx}{dt} \quad \upsilon_y = \frac{dy}{dt} \quad \upsilon_z = \frac{dz}{dt}$$

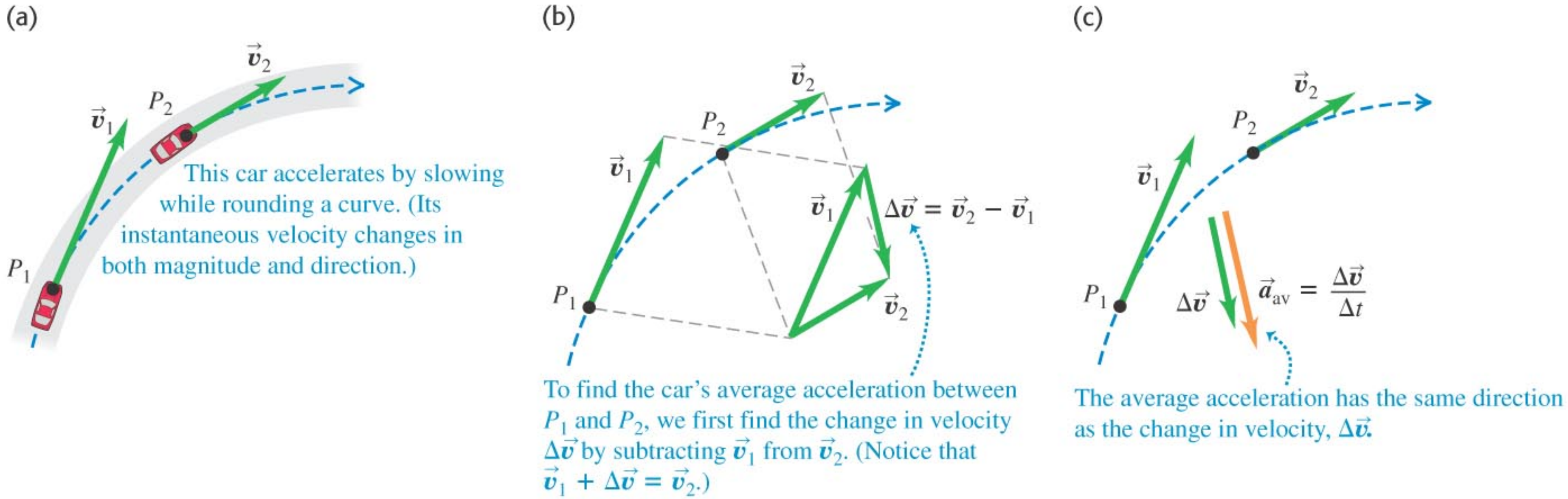
- The instantaneous velocity of a particle ***is always tangent to its path***.



Average Acceleration

- The ***average acceleration*** during a time interval Δt is defined as the velocity change $\Delta\vec{v}$ divided by Δt.

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}$$



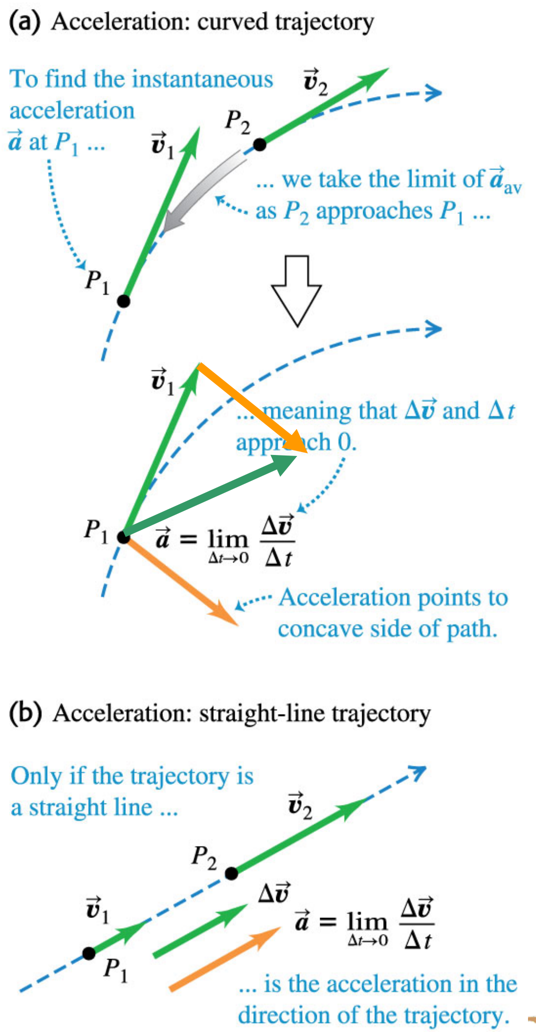
Instantaneous Acceleration

- The ***instantaneous acceleration*** is the instantaneous rate of change of the velocity with respect to time.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

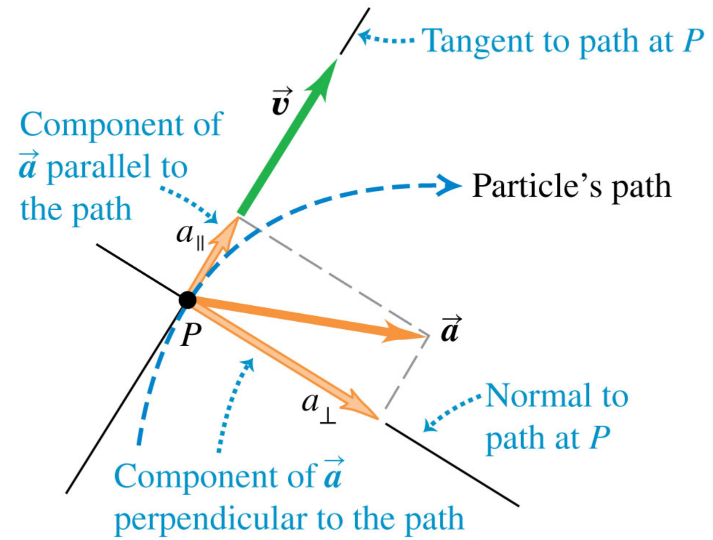
- Any particle following a curved path is accelerating, even if it has constant speed.
- The components of the instantaneous acceleration are

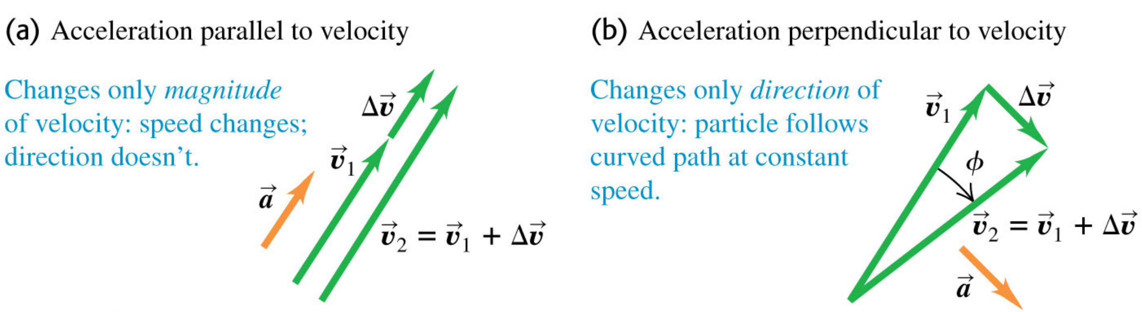
$$a_x = \frac{dv_x}{dt} \quad a_y = \frac{dv_y}{dt} \quad a_z = \frac{dv_z}{dt}$$



Acceleration Direction

- Another useful way to think about ***instantaneous acceleration*** is in terms of its component *parallel* or *perpendicular* to the velocity.
- Parallel component tells us about changes in the particle's speed.
- Perpendicular component tells us about changes in the particle's direction of motion.





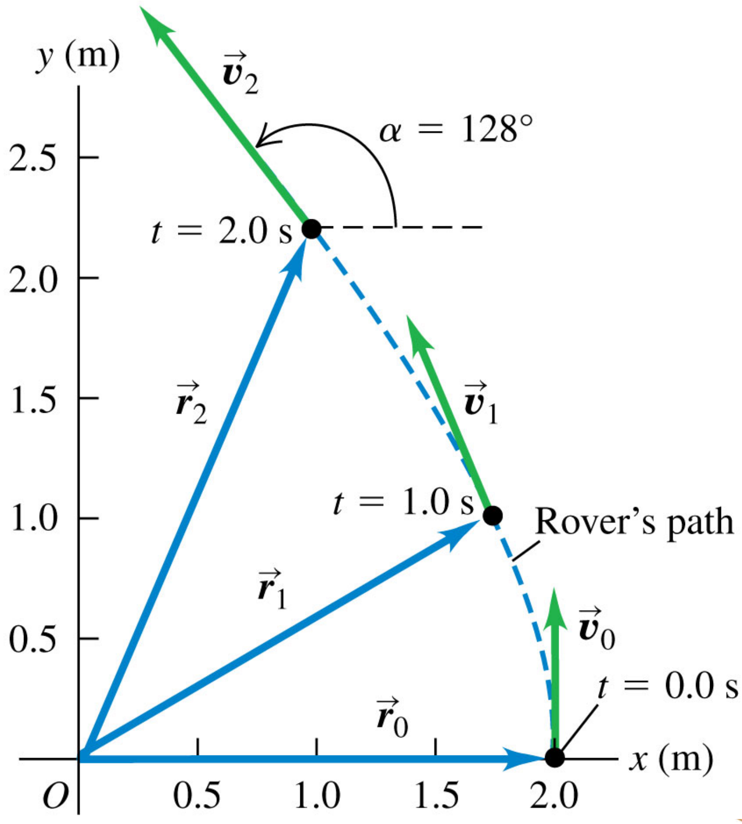
Calculating Displacement

- A rover is exploring the surface of Mars, which we represent as a point. It has x- and y-coordinates that vary with time:

$$x = 2.0\,\text{m} - (0.25\,\text{m/s}^2)t^2$$

$$y = (1.0\,\text{m/s})t + (0.025\,\text{m/s}^3)t^3$$

- Find the rover's coordinates and distance from the origin at $t = 2\ s$.



$$x = 2.0\,\text{m} - (0.25\,\text{m/s}^2)(2.0\,\text{s})^2 = 1.0\,\text{m}$$

$$y = (1.0\,\text{m/s})(2.0\,\text{s}) + (0.025\,\text{m/s}^3)(2.0\,\text{s})^3 = 2.2\,\text{m}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{1.0^2 + 2.2^2} = 2.4\,\text{m}$$

Calculating Average Velocity

- A rover is exploring the surface of Mars, which we represent as a point. It has x- and y-coordinates that vary with time:

$$x = 2.0\,\text{m} - (0.25\,\text{m/s}^2)t^2$$

$$y = (1.0\,\text{m/s})t + (0.025\,\text{m/s}^3)t^3$$

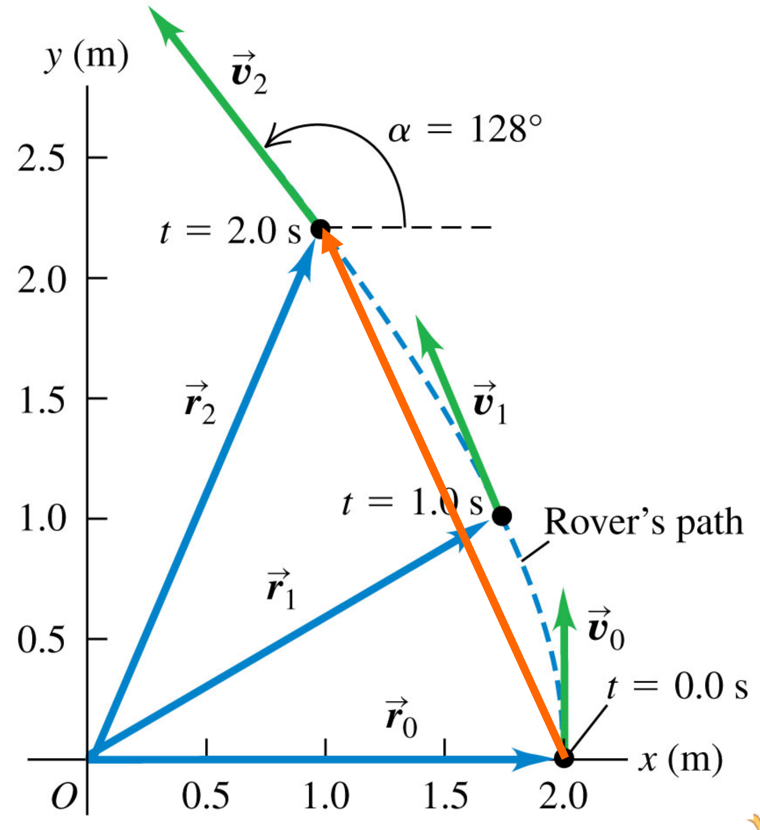
- Find the rover's displacement and average velocity vectors for the interval $t = 0.0$ s to $t = 2.0$ s.

$$\vec{r}_0 = (2.0\text{ m})\hat{\imath} + (0.0\text{ m})\hat{\jmath}$$

$$\vec{r}_2 = (1.0\text{ m})\hat{\imath} + (2.2\text{ m})\hat{\jmath}$$

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_0 = (-1.0\text{ m})\hat{\imath} + (2.2\text{ m})\hat{\jmath}$$

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(-1.0\text{ m})\hat{\imath} + (2.2\text{ m})\hat{\jmath}}{2\text{ s} - 0\text{ s}} = ?$$



Calculating Instantaneous Velocity

- A rover is exploring the surface of Mars, which we represent as a point. It has x- and y-coordinates that vary with time:

$$x = 2.0\,\text{m} - (0.25\,\text{m/s}^2)t^2$$

$$y = (1.0\,\text{m/s})t + (0.025\,\text{m/s}^3)t^3$$

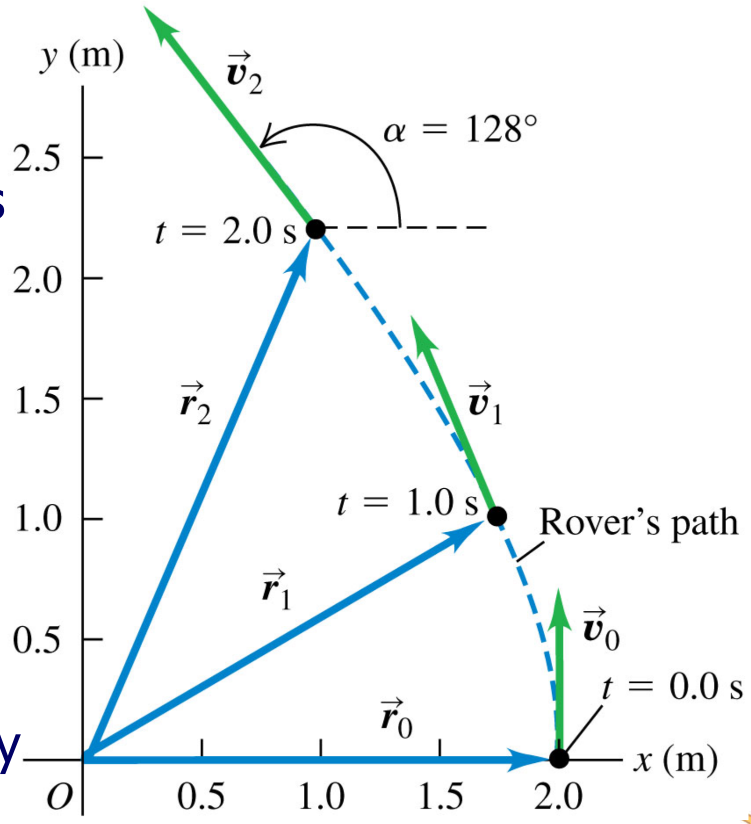
- Find a general expression for the rover's instantaneous velocity vector $\boldsymbol{V}$.

$$v_x = \frac{dx}{dt} = -(0.25\,\text{m/s}^2)(2t)$$

$$v_y = \frac{dy}{dt} = 1.0\,\text{m/s} + (0.025\,\text{m/s}^3)(3t^2)$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$

- What is the rover's instantaneous velocity at t = 2 s (magnitude and direction)?



Calculating Acceleration

- A rover is exploring the surface of Mars, which we represent as a point. It has x- and y-coordinates that vary with time:

$$x = 2.0\,\text{m} - (0.25\,\text{m/s}^2)t^2$$

$$y = (1.0\,\text{m/s})t + (0.025\,\text{m/s}^3)t^3$$

- Find a general expression for the rover's instantaneous acceleration **a**.

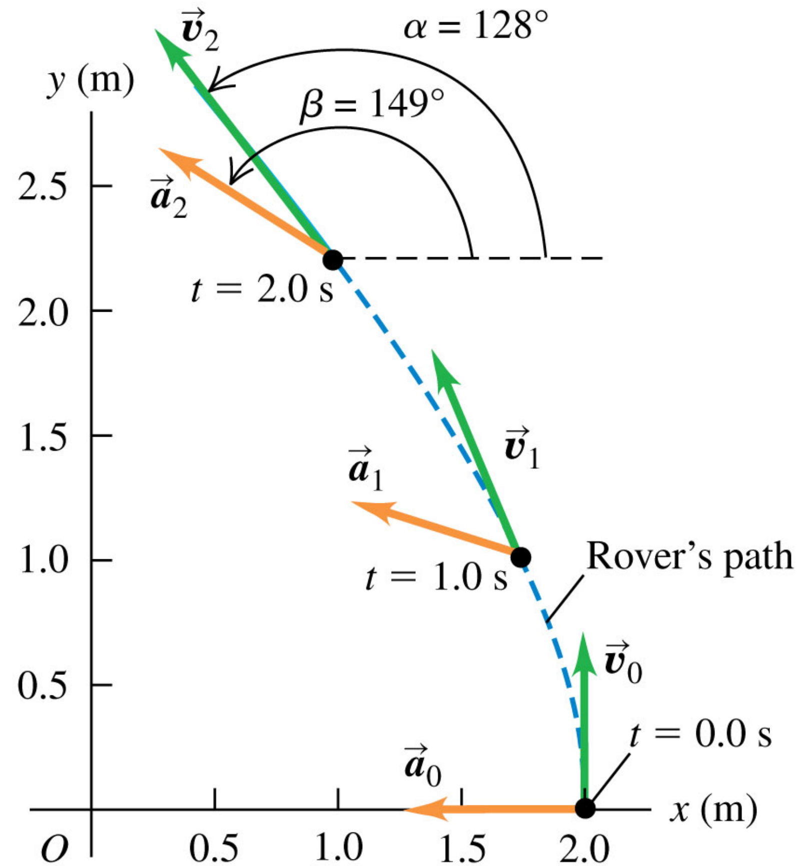
$$v_x = -(0.50\,\text{m/s}^2)t$$

$$v_y = \frac{dy}{dt} = 1.0\,\text{m/s} + (0.075\,\text{m/s}^3)(t^2)$$

$$a_x = \frac{dv_x}{dt} = -0.50\,\text{m/s}^2$$

$$a_y = \frac{dv_y}{dt} = (0.075\,\text{m/s}^2)(2t)$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

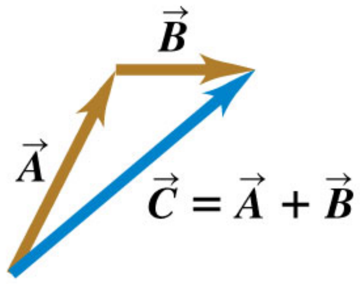


Summary I: Vectors

- Vector addition

$$\vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} = (A_x\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}}) + (B_x\hat{\boldsymbol{\imath}} + B_y\hat{\boldsymbol{\jmath}})$$

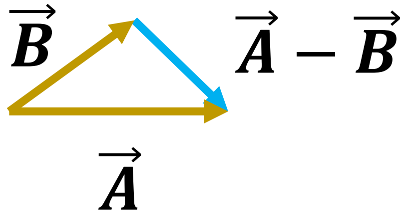
$$=(A_x+B_x)\hat{\boldsymbol{\imath}} + (A_y+B_y)\hat{\boldsymbol{\jmath}}$$



- Vector subtraction

$$\vec{\boldsymbol{A}} - \vec{\boldsymbol{B}} = (A_x\hat{\boldsymbol{\imath}} - A_y\hat{\boldsymbol{\jmath}}) + (B_x\hat{\boldsymbol{\imath}} - B_y\hat{\boldsymbol{\jmath}})$$

$$=(A_x-B_x)\hat{\boldsymbol{\imath}} + (A_y-B_y)\hat{\boldsymbol{\jmath}}$$



Summary II: Position, Velocity, Acceleration

- Position $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$
- Average velocity $\vec{\upsilon}_{\text{av}} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}$
- Instantaneous velocity $\vec{\upsilon} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

$$\upsilon_x = \frac{dx}{dt} \quad \upsilon_y = \frac{dy}{dt} \quad \upsilon_z = \frac{dz}{dt} \quad \text{(components of instantaneous velocity)}$$

- Acceleration $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\upsilon}}{\Delta t} = \frac{d\vec{\upsilon}}{dt}$

$$a_x = \frac{d\upsilon_x}{dt} \quad a_y = \frac{d\upsilon_y}{dt} \quad a_z = \frac{d\upsilon_z}{dt} \quad \text{(components of instantaneous acceleration)}$$

- $\vec{r}(t)$, $\vec{v}(t)$, and $\vec{a}(t)$ are not necessarily along the same direction.

Calculating displacement

- An ant moves in one direction with a displacement derived by **A**=3**i**+5**j**, then it turns to another direction describes as **B**=4**i**+10**j**. The units are in m. Find the angle in degrees of the final displacement of the ant relative to the X axis.

  (A) 79

  (B) 11

  (C) 65

  (D) 25

  (E) 36

Flying a plane

- A plane is trying to land in the Newark International Airport from south at a reduced speed of 300 mph. However there is a strong wind blowing **from west** at 50 mph. In order to keep the plane in the north direction for landing, how many degrees away from due north (+ for east and – for west) should the captain steers the plane?

(A) -11.3 (B) -9.6 (C) 0 (D) 9.6 (E) 11.3

2-D Motion under **Constant** Acceleration

- Motions in two dimensions are independent components
- Constant acceleration equations in vector form of

$$\vec{\boldsymbol{v}} = \vec{\boldsymbol{v}}_0 + \vec{\boldsymbol{a}}t \qquad \vec{\boldsymbol{r}} - \vec{\boldsymbol{r}}_0 = \vec{\boldsymbol{v}}_0 t + \frac{1}{2}\vec{\boldsymbol{a}}t^2 \qquad \vec{\boldsymbol{v}}^{\mathbf{2}} = \vec{\boldsymbol{v}}_0^2 + 2\vec{\boldsymbol{a}}(\vec{\boldsymbol{r}} - \vec{\boldsymbol{r}}_0)$$

- Constant acceleration equations hold in each dimension

$$v_x = v_{0x} + a_x t \qquad v_y = v_{0y} + a_y t$$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 \qquad y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$$

$$v_x^2 = {v_{0x}}^2 + 2a_x(x - x_0) \qquad v_y^2 = {v_{0y}}^2 + 2a_y(y - y_0)$$

  - $t = 0$ beginning of the process
  - $\vec{a} = a_x\hat{i} + a_y\hat{j}$ where a_x and a_y are constant
  - Initial velocity $\vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j}$ initial displacement $\vec{r}_0 = x_0\hat{i} + y_0\hat{j}$

Hints for solving problems

- Define coordinate system. Make sketch showing axes, origin.
- List known quantities. Find v_{0x}, v_{0y}, a_x, a_y, etc. Show initial conditions on sketch.
- List equations of motion to see which ones to use.
- Time t is the same for x and y directions.

  $x_0 = x(t = 0)$, $y_0 = y(t = 0)$, $v_{0x} = v_x(t = 0)$, $v_{0y} = v_y(t = 0)$.
- Have an axis point along the direction of $\vec{a}$ if it is constant.

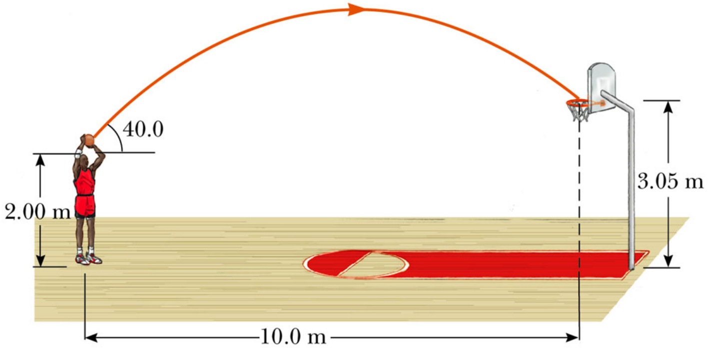
$$v_x = v_{0x} + a_x t \qquad v_y = v_{0y} + a_y t$$

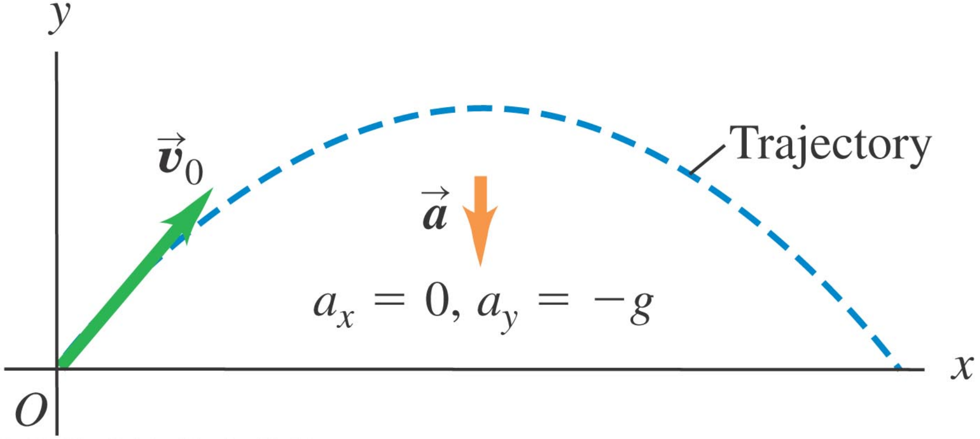
$$x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2 \qquad y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$$

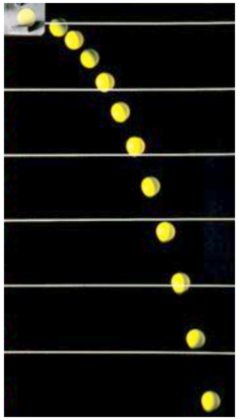
$$v_x^2 = {v_{0x}}^2 + 2a_x(x - x_0) \qquad v_y^2 = {v_{0y}}^2 + 2a_y(y - y_0)$$

Projectile Motion

- A projectile is any body given an initial velocity that then follows a path determined by the effects of gravity.
  - A projectile moves in a vertical plane that contains the initial velocity vector $\vec{v}_0$.
  - Its trajectory depends only on $\vec{v}_0$ and on the downward acceleration due to gravity.







Projectile Motion: A Simple Case

- 2-D problem and define a coordinate system: x- horizontal, y- vertical (up +)
- Try to pick $x_0 = 0, y_0 = 0$ at $t = 0$
- Horizontal motion + Vertical motion
- Horizontal: $a_x = 0 \,,\ v_{0x} = v_0$
- Vertical: $a_y = -g = -9.8\ m/s^2,\ v_{0y} = 0$
- Equations:

Horizontal	Vertical
$v_x = v_{0x} + a_x t$	$v_y = v_{0y} + a_y t$
$x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2$	$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$
$v_x^2 = {v_{0x}}^2 + 2a_x(x - x_0)$	$v_y^2 = {v_{0y}}^2 + 2a_y(y - y_0)$

Which ball strikes first?

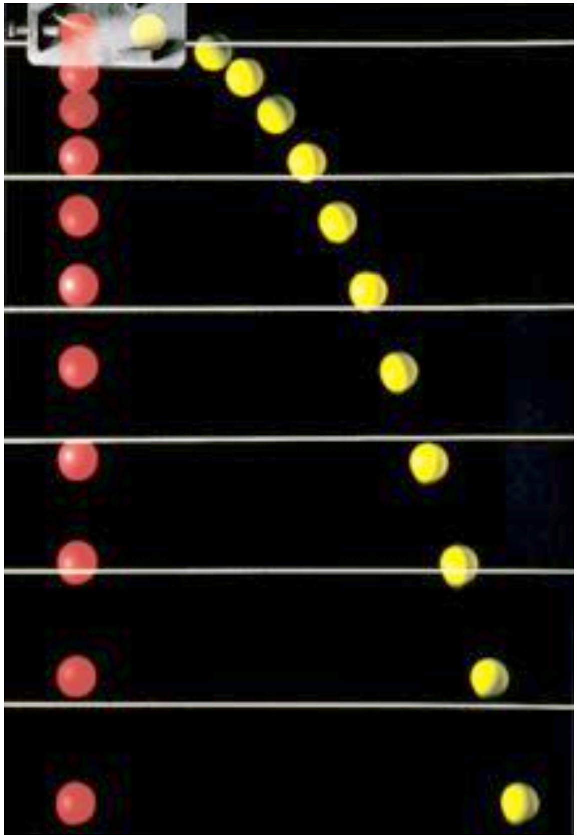
- Consider the motions of two golf balls shown in the right figure, the red one simply released and the yellow one shot horizontally. They have the same initial height and starting time. Which one will strike the ground first?

  (A) the red one

  (B) the yellow one

  (C) at same time

  (D) not enough information is given

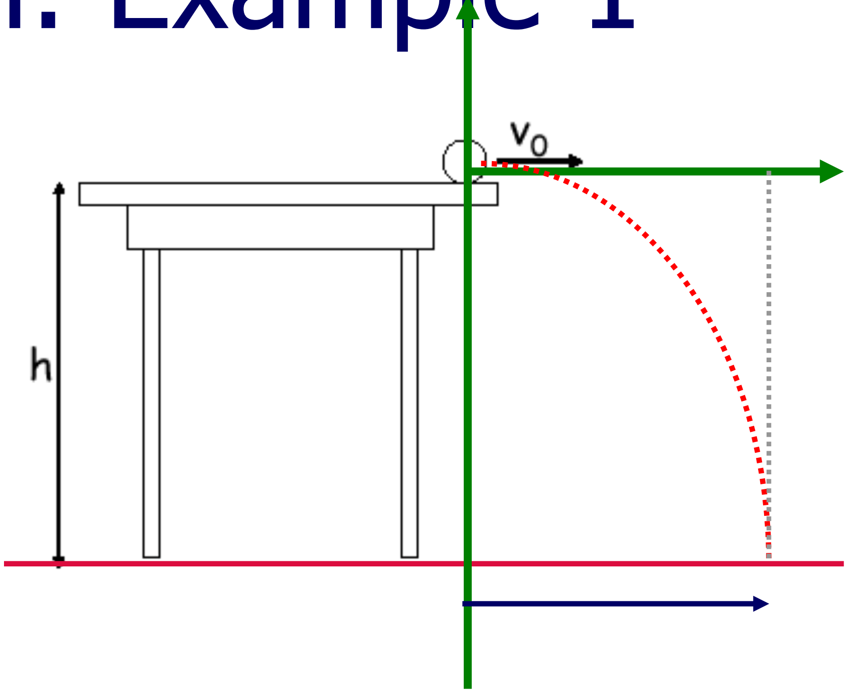


Projectile Motion: Example 1

- A ball rolls off a table of height h. The ball has horizontal velocity v_0 when it leaves the table.
- How far away does it strike the ground?
- How long does it take to reach the ground?

$x_0=?,\ y_0=?,\ v_{0x}=?,\ v_{0y}=?,\ a_x=?,\ a_y=?$

$x=?,\ y=?,\ v_x=?,\ v_y=?$



$x_0 = 0,\ y_0 = 0,\ v_{0x} = v_0,\ v_{0y} = 0,$

$a_x = 0,\ a_y = -g,\ y = -h$

$v_x = v_0 \quad v_y = -gt$

- Vertical Displacement:

$$y - y_0 = -\frac{1}{2}gt^2 = -h$$

- Time needed: $t = \sqrt{2h/g}$
- Horizontal Displacement:

$$x(t) - x_0 = v_0 t = v_0\sqrt{2h/g}$$

Projectile Motion: Generalized

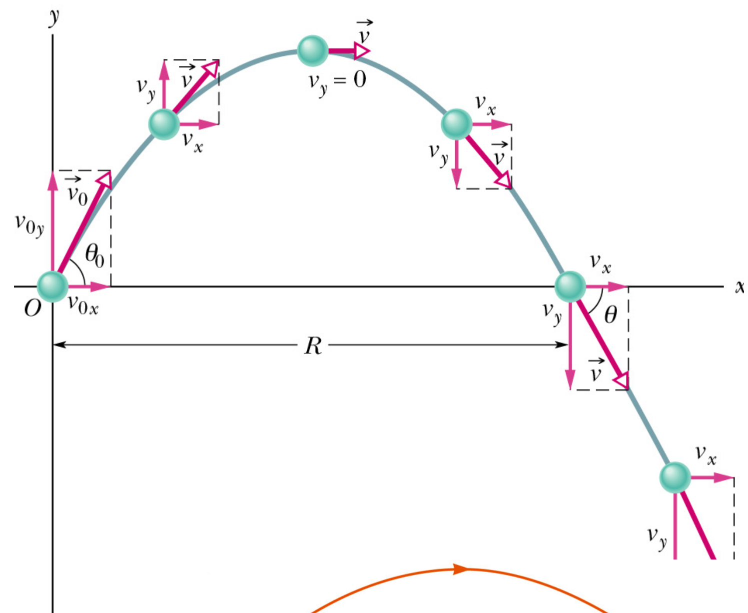
- 2-D problem and define a coordinate system.
- Horizontal: $a_x = 0$ and vertical: $a_y = -g$.
- Try to pick $x_0 = 0$, $y_0 = 0$ at $t = 0$.
- Velocity initial conditions:
  - v_0 can have x, y components.

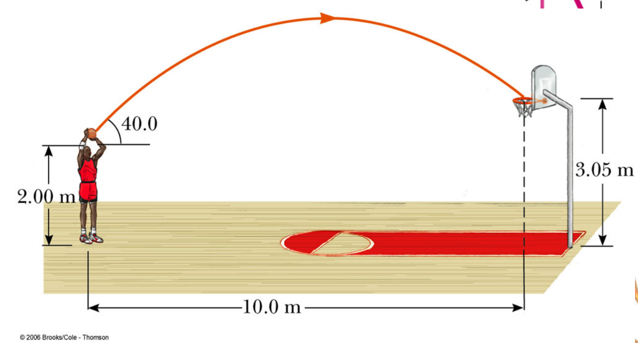
    $$v_{0x} = v_0 \cos\theta_0 \qquad v_{0y} = v_0 \sin\theta_0$$
  - v_x is usually constant.
  - v_y changes continuously.
- Equations:

  Horizontal / Vertical

$$v_x = v_{0x} = v_0 \cos\theta_0 \qquad v_y = v_{0y} - gt = v_0 \sin\theta_0 - gt$$

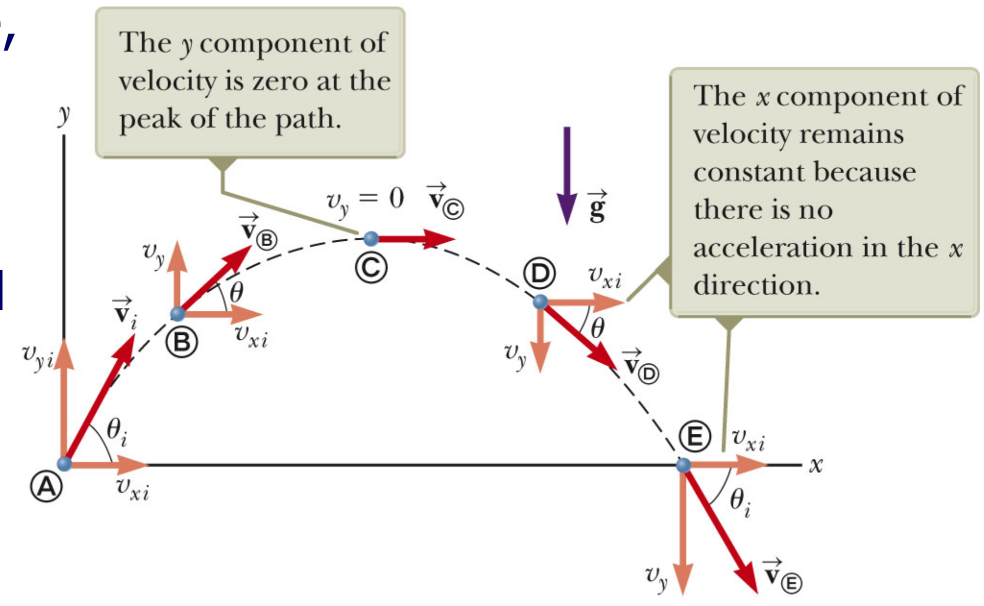
$$x = x_0 + v_{0x}t = (v_0 \cos\theta_0)t \qquad y = y_0 + v_{0y}t - \frac{1}{2}gt^2 = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$$





Projectile Motion

- Provided air resistance is negligible, the horizontal component of the velocity remains constant
- Vertical component of the acceleration is equal to the free fall acceleration $-g$
- Vertical component of the velocity and the displacement in the y-direction are identical to those of a freely falling body
- Projectile motion can be described as a superposition of two independent motion in the x- and y-directions



$$v_x = v_0 \cos\theta_0 \qquad v_y = v_0 \sin\theta_0 - gt$$

$$x = x_0 + v_{0x}t = (v_0 \cos\theta_0)t$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2 = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$$

Mission Impossible 6

- Tom Cruise jumping off rooftop in Mission Impossible 6 (warning: spoil!)
- https://www.youtube.com/watch?v=g_4yYnPEP70

Tom Cruise jumping off rooftop

- In Mission Impossible 6, Tom Cruise jumps off a rooftop and landed on a nearby building that is 5 m lower. The horizontal distance between the two buildings is 10 m. Assume he jumps off the edge nearly horizontally. How fast in m/s should he run in order to make it to the other side (without dying)?

A) 5 m/s B) 10 m/s C) 20 m/s

D) 7 m/s E) 15 m/s

Summary III: Projectile Motion

- Projectile motion is one type of 2-D motion under constant acceleration, where $a_x = 0$, $a_y = -g$.
- The key to analyzing projectile motion is to treat the x- and y-components **separately** and apply 1-D constant acceleration kinematics equations to each direction:

horizontal direction

$$v_x = v_{0x} + a_x t$$

$$x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = {v_{0x}}^2 + 2a_x(x - x_0)$$

Vertical direction

$$v_y = v_{0y} + a_y t$$

$$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$$

$$v_y^2 = {v_{0y}}^2 + 2a_y(y - y_0)$$

$$a_x = 0 \quad a_y = -g$$ (projectile motion, no air resistance)