Physics 111: Mechanics
Lecture 6

Bin Chen

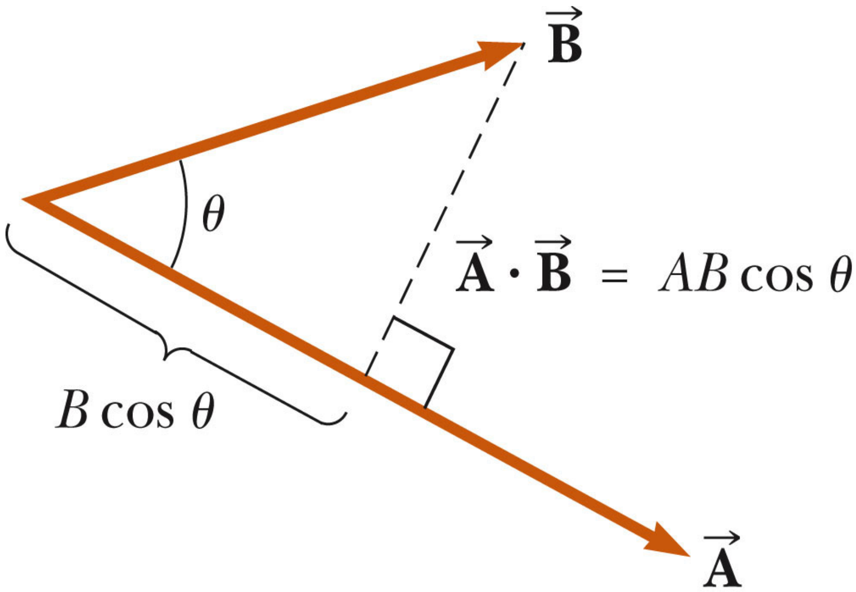
NJIT Physics Department

Chapter 6 Work and Kinetic Energy

- 1.10 Scalar Product of Vectors
- 6.1 Work
- 6.2 Kinetic Energy and the Work-Energy Theorem
- 6.3 Work and Energy with Varying Forces
- 6.4 Power

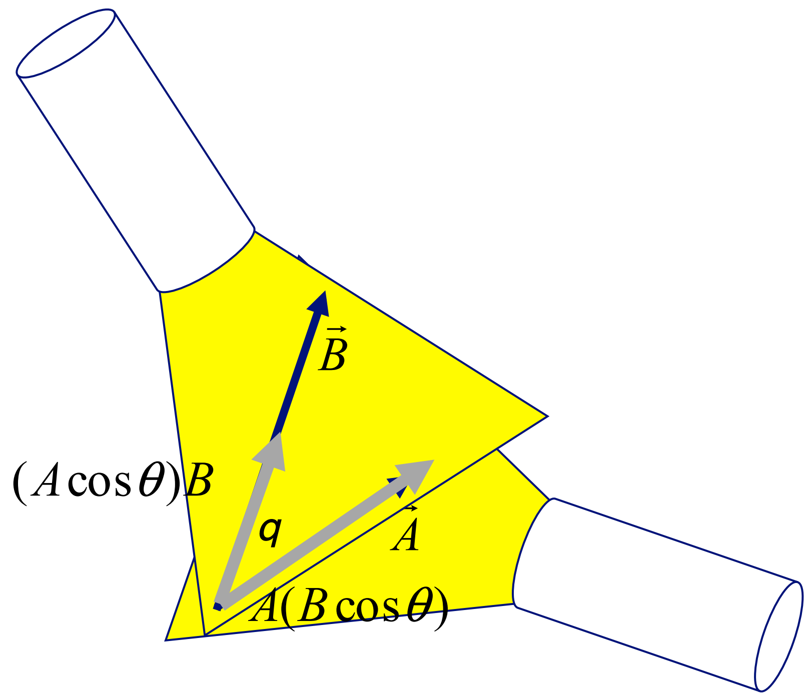
Scalar Product of Two Vectors

- The scalar product of two vectors is written as $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$
  - It is also called the dot product
- $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \equiv A\ B\ cos\ \theta$
  - *q* is the angle *between* *A* and *B*



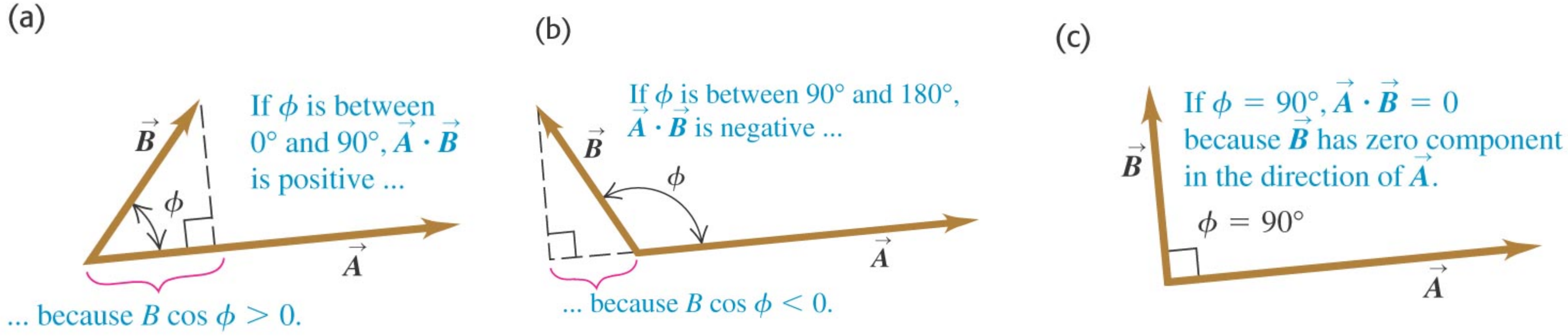
Scalar Product: A Graphic Representation

- The scalar, or "dot" product says something about how parallel two vectors are.
- The scalar product of two vectors can be thought of as the projection of one onto the direction of the other.
  - [?] is the angle between the vectors
  - Scalar product of any perpendicular vectors = zero
  - Scalar product is maximum for parallel vectors



Scalar Product is a Scalar

- Not a vector
- May be positive, negative, or zero



Scalar Product in Components

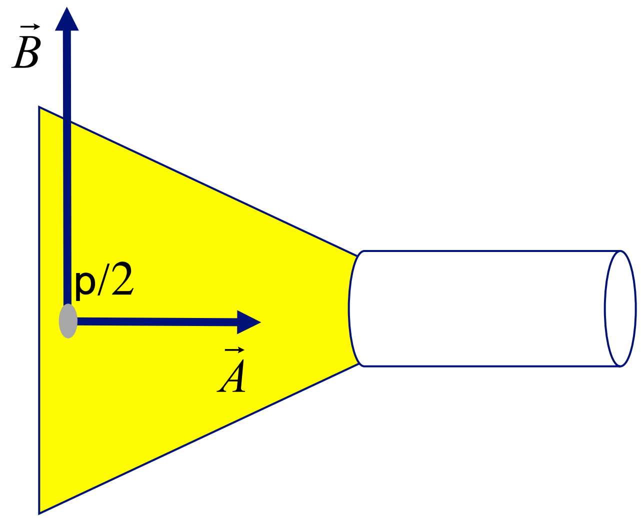
- The scalar product of two vectors can be thought of as the projection of one onto the direction of the other.
- What is the scalar products of unit vectors?
- Components of a vector can be regarded as the scalar product of the vector and the unit vectors

$$\hat{i}\cdot\hat{j}=0;\ \hat{i}\cdot\hat{k}=0;\ \hat{j}\cdot\hat{k}=0$$

$$\hat{i}\cdot\hat{i}=1;\ \hat{j}\cdot\hat{j}=1;\ \hat{k}\cdot\hat{k}=1$$

$$\vec{A}\cdot\vec{B}=AB\cos\theta$$

$$\vec{A}\cdot\hat{i}=A\cos\theta=A_x$$



Calculating Scalar Product Using Components

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

- Start with $$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$ $$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$
- Then $$\begin{aligned}\vec{A}\cdot\vec{B} &= (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) \\ &= A_x\hat{i}\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) + A_y\hat{j}\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) + A_z\hat{k}\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})\end{aligned}$$
- But $$\hat{i}\cdot\hat{j} = 0;\ \hat{i}\cdot\hat{k} = 0;\ \hat{j}\cdot\hat{k} = 0$$ $$\hat{i}\cdot\hat{i} = 1;\ \hat{j}\cdot\hat{j} = 1;\ \hat{k}\cdot\hat{k} = 1$$
- So $$\begin{aligned}\vec{A}\cdot\vec{B} &= A_x\hat{i}\cdot B_x\hat{i} + A_y\hat{j}\cdot B_y\hat{j} + A_z\hat{k}\cdot B_z\hat{k} \\ &= A_xB_x + A_yB_y + A_zB_z\end{aligned}$$

Scalar Product: An Example

- The vectors $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = -\hat{i} + 2\hat{j}$
- Determine the scalar product $\vec{A} \cdot \vec{B} = ?$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y = 2 \cdot (-1) + 3 \cdot 2 = -2 + 6 = 4$$

- Find the angle θ between these two vectors

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{2^2 + 3^2} = \sqrt{13} \qquad B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$$

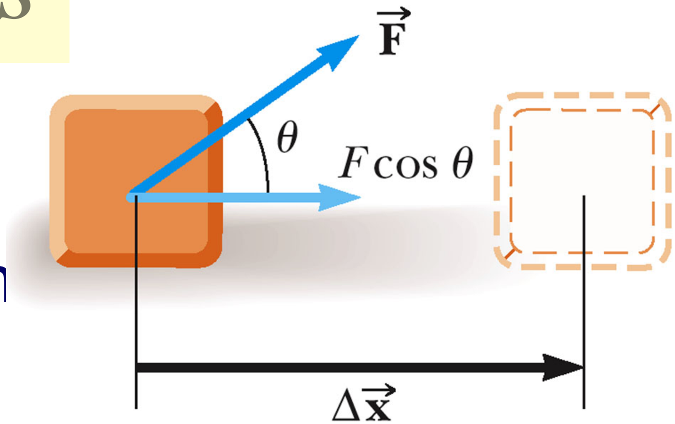
$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1} \frac{4}{\sqrt{65}} = 60.3^\circ$$

Definition of Work W

- The work, *W*, done by a constant force on an object is defined as the scalar (dot) product of the component of the force along the direction of displacement and the magnitude of the displacement

$$W \equiv (F\cos\phi)s = \vec{F}\cdot\vec{s}$$



  - F is the magnitude of the force
  - Δx is the the object's displacemen
  - q is the angle between F and Δx

Unit of Work

- This gives no information about
  - the time it took for the displacement to occur
  - the velocity or acceleration of the object
- Work is a scalar quantity $W \equiv (F\cos\phi)s = \vec{F}\cdot\vec{s}$
- Can be calculated using components

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z \qquad \vec{F}\cdot\vec{s} = F_xx + F_yy + F_zz$$

- SI Unit
  - N • m = J
  - J = (kg • m / s^2) • m

Scalar Product and Work

- Steve apply a force $\vec{F} = 3\hat{i} + 3\hat{j}$ N on a car and makes it to move a displacement of $\vec{s} = -3\hat{i} + 5\hat{j}$ m. How much work (in J) does Steve do in this case?

A) $9\hat{i}$

B) $-9\hat{i}$

C) 6

D) 9

E) 15

$$W \equiv (F\cos\phi)s = \vec{F} \cdot \vec{s}$$

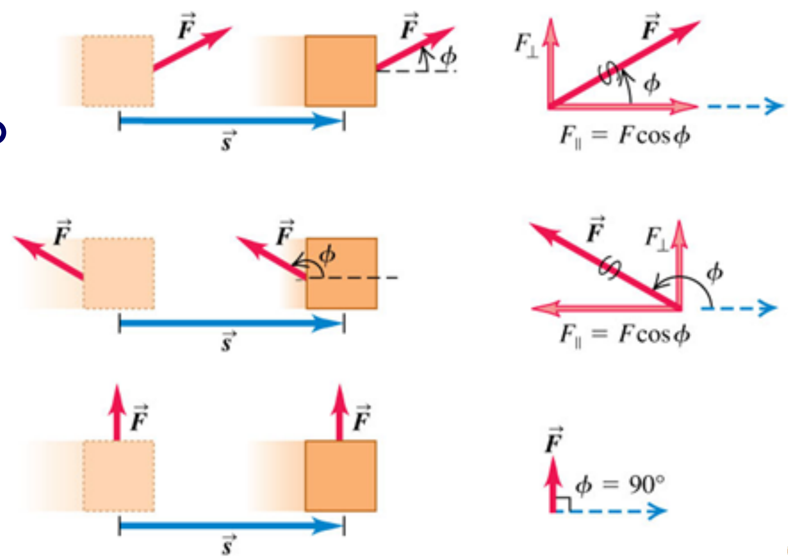
$$W = \vec{F} \cdot \vec{s} = F_x x + F_y y = 3 \cdot (\text{-}3) + 3 \cdot 5 = \text{-}9 + 15 = 6$$

Work: Positive or Negative?

- Work can be positive, negative, or zero. The sign of the work depends on the direction of the force relative to the displacement

$$W \equiv (F\cos\phi)s = \vec{F}\cdot\vec{s}$$

- Work positive: if 0°< ? < 90°
- Work negative: if 90°< ? <180°
- Work zero: W = 0 if ? = 90°
- Work maximum if ? = 0°
- Work minimum if ? = 180°



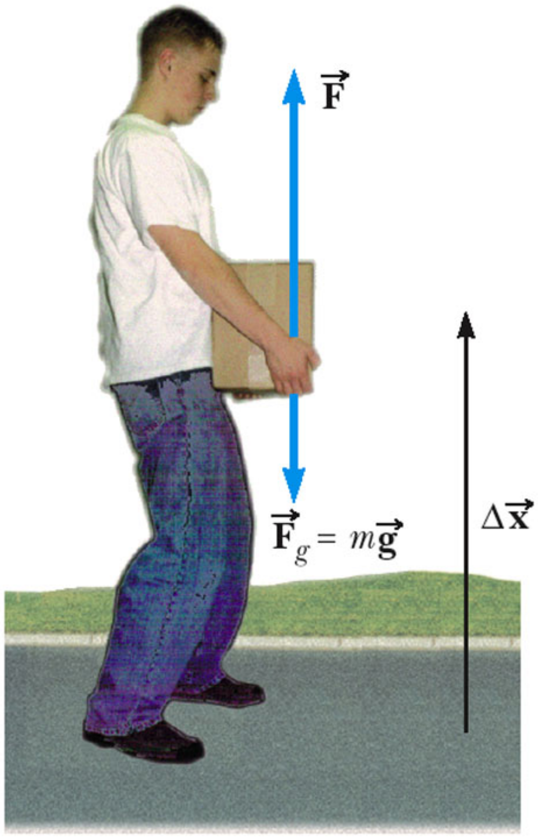
Example: When Work is Zero

- A man carries a bucket of water horizontally at constant velocity.
- The force does no work on the bucket
- Displacement is horizontal
- Force is vertical
- cos 90° = 0

$$W \equiv (F\cos\phi)s = \vec{F}\cdot\vec{s}$$

Example: Work Can Be Positive or Negative

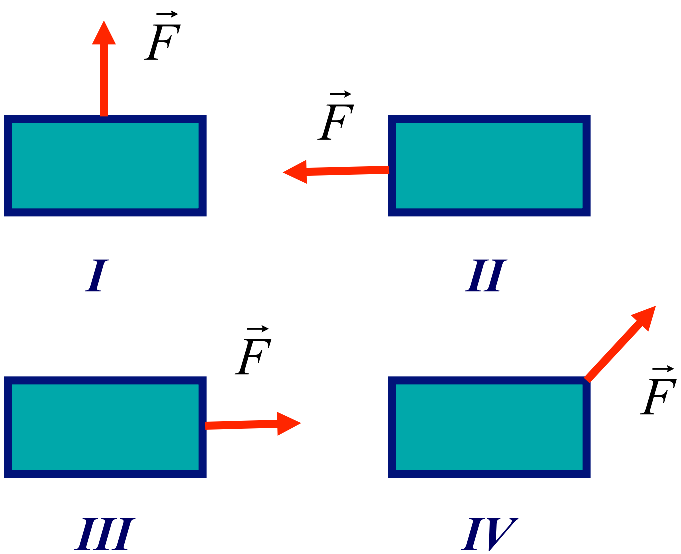
- Is the work positive or negative when lifting the box?
- ✓ Positive
- Is the work positive or negative when lowering the box?
- ✓ Negative



Work Done by a Constant Force

1. The right figure shows four situations in which a force is applied to an object. In all four cases, the force has the same magnitude, and the displacement of the object is to the right and of the same magnitude. Rank the situations in order of the work done by the force on the object, from most positive to most negative.

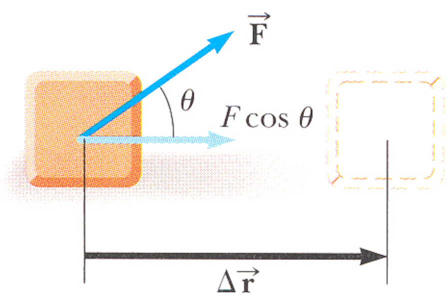
A. I, IV, III, II
B. II, I, IV, III
C. III, II, IV, I
D. I, IV, II, III
E. III, IV, I, II

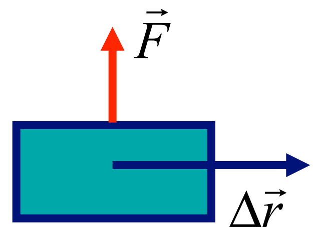


Work Done by a Constant Force

- The work W done a system by an agent exerting a constant force on the system is the product of the magnitude F of the force, the magnitude Δr of the displacement of the point of application of the force, and $\cos\theta$, where θ is the angle between the force and displacement vectors:

$$W \equiv \vec{F} \cdot \Delta\vec{r} = F\Delta r \cos\theta$$



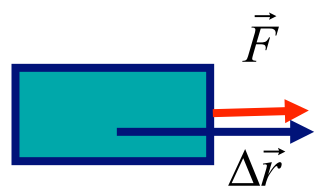


I

$$W_I = 0$$

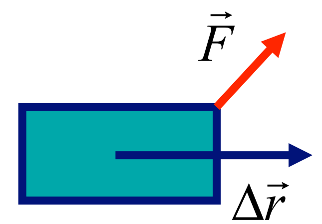
II

$$W_{II} = -F\Delta r$$



III

$$W_{III} = F\Delta r$$



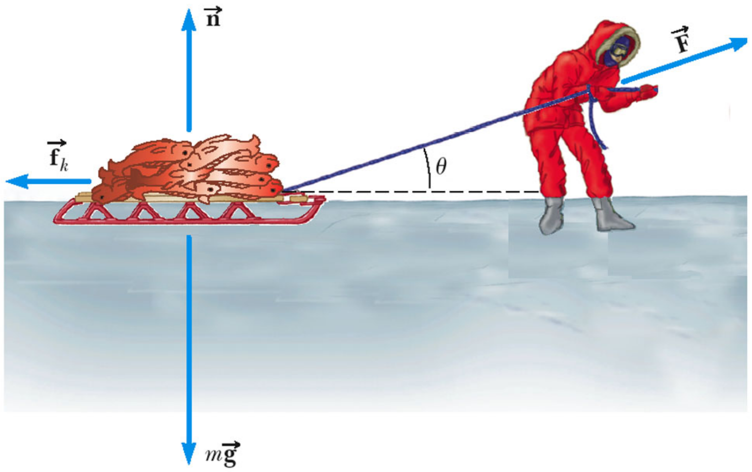
IV

$$W_{IV} = F\Delta r\cos\theta$$

Work and Force

- An Eskimo returning pulls a sled as shown. The total mass of the sled is 50.0 kg, and he exerts a force of 120 N on the sled by pulling on the rope. How much work does he do on the sled if θ = 30° and he pulls the sled 5.0 m ?

$$W = (F\cos\phi)s$$
$$= (1.20\times10^2 N)(\cos 30^\circ)(5.0m)$$
$$= 5.2\times10^2 J$$



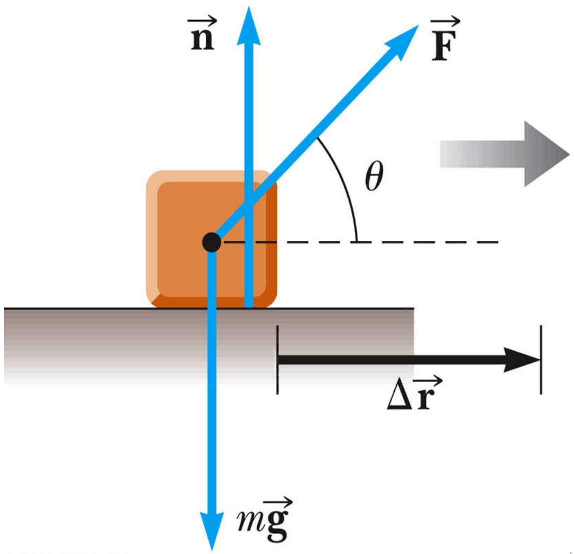
Work Done by Multiple Forces

- If more than one force acts on an object, then the total work is equal to the algebraic sum of the work done by the individual forces

$$W_{net} = \sum W_{\text{by individual forces}}$$

  - Remember work is a scalar, so this is the algebraic sum

$$W_{net} = W_g + W_N + W_F = (F\cos\theta)\Delta r$$

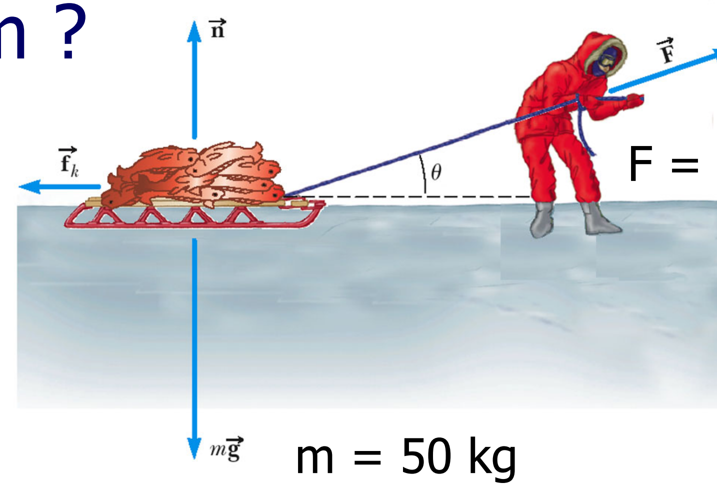


Work and Multiple Forces

- Suppose μ_k = 0.200, How much work done on the sled by friction, and the net work if θ = 30° and he pulls the sled 5.0 m ?

$$F_{net,y} = N - mg + F\sin\theta = 0$$
$$N = mg - F\sin\theta$$

$$W_{fric} = (f_k \cos 180^\circ)\Delta r = -f_k \Delta r$$
$$= -\mu_k N \Delta r = -\mu_k (mg - F\sin\theta)\Delta r$$
$$= -(0.200)(50.0kg \cdot 9.8m/s^2$$
$$-1.2\times 10^2 N \sin 30^\circ)(5.0m)$$
$$= -4.3\times 10^2 J$$

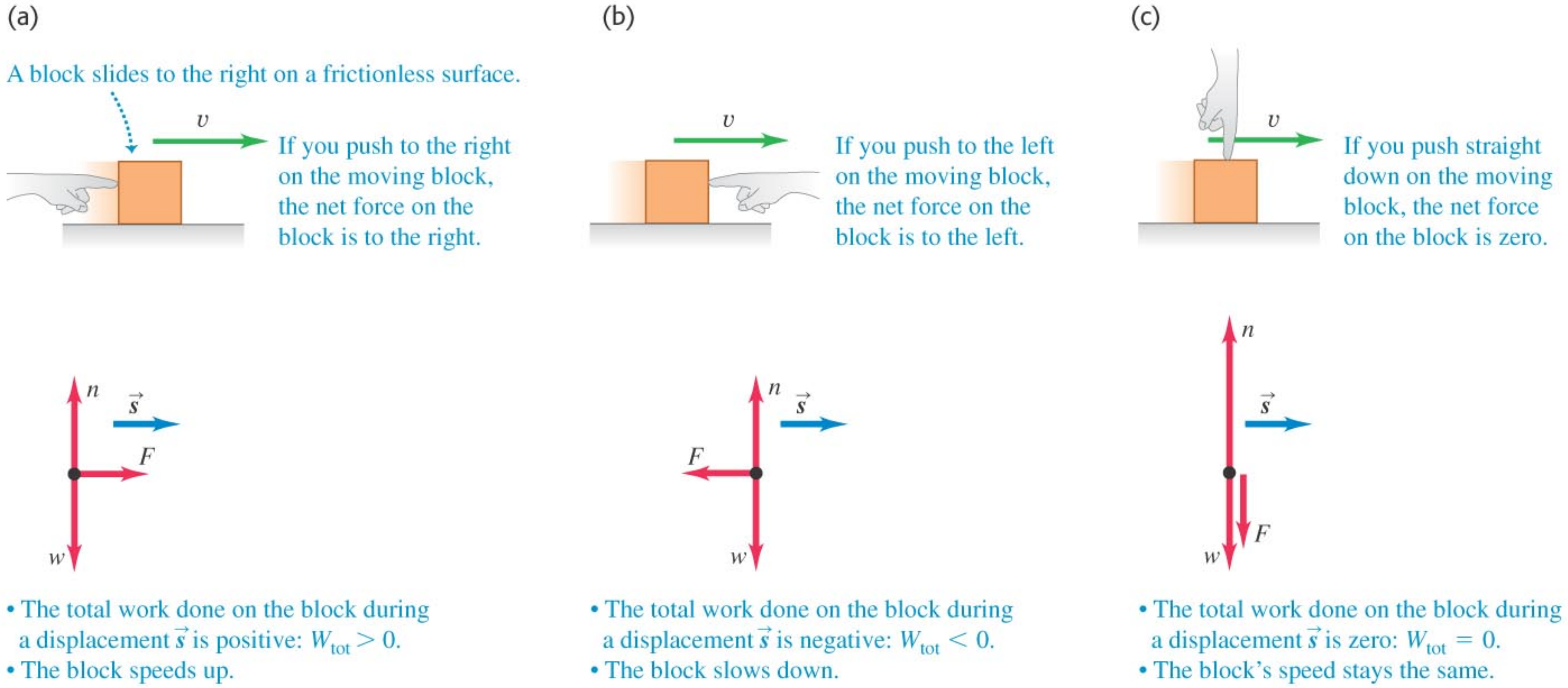


F = 120 N

m = 50 kg

$$W_{net} = W_F + W_{fric} + W_N + W_g$$
$$= 5.2\times 10^2 J - 4.3\times 10^2 J + 0 + 0$$
$$= 90.0J$$

Work and Motion



Special case: Constant Acceleration

Remember result eliminating t:

$$v^2 - v_0^2 = 2a(x - x_0)$$

Multiply by ½ m:

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = ma(x - x_0)$$

$$= ma\Delta x$$

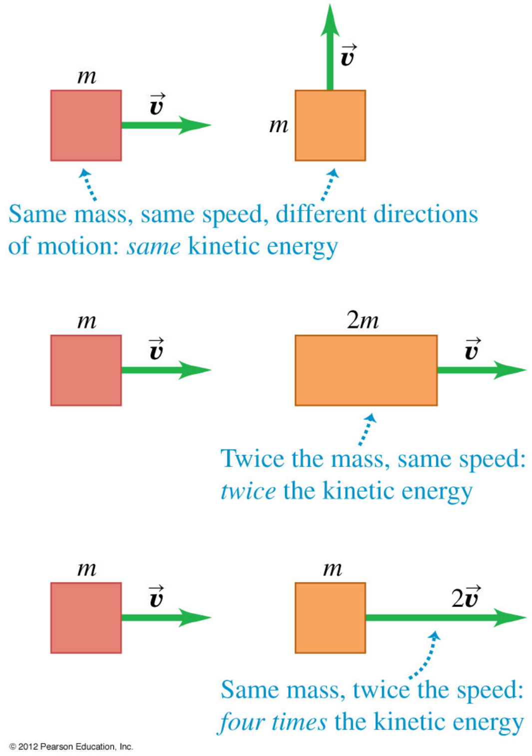
But $F=ma!$

$$\Delta(\frac{1}{2}mv^2) = F\Delta x$$

Kinetic Energy

- For an object m moving with a speed of v

$$K = \frac{1}{2} m v^2$$



- Kinetic Energy is energy associated with the state of motion of an object
- SI unit: joule (J)

$$1 \text{ joule} = 1 \text{ J} = 1 \text{ kg m}^2/\text{s}^2$$

Work-Energy Theorem

- When work is done by a net force on an object and the only change in the object is its speed, the work done is equal to the change in the object's kinetic energy

$$W_{\text{tot}} = K_2 - K_1 = \Delta K$$

  - Speed will increase if work is positive
  - Speed will decrease if work is negative

Work with Varying Forces

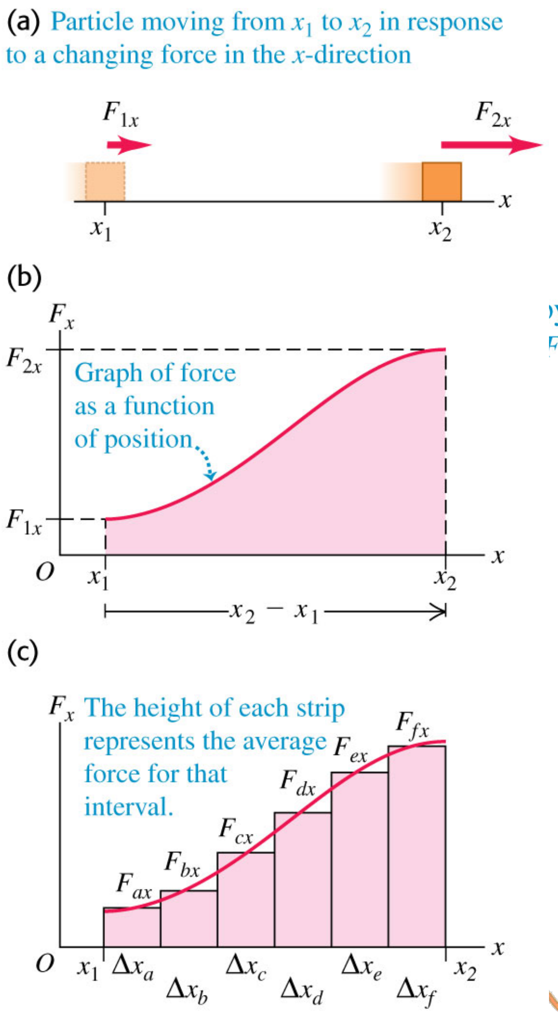
- On a graph of force as a function of position, **the total work done by the force** is represented by **the area under the curve** between the initial and the final position
- Note there could be negative work!
- Straight-line motion

$$W = F_{ax}\Delta x_a + F_{bx}\Delta x_b + \ldots\ldots$$

$$W = \int_{x_1}^{x_2} F_x dx$$

- Motion along a curve

$$W = \int_{P_1}^{P_2} F\cos\phi dl = \int_{P_1}^{P_2} F_{\|} dl = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{l}$$



Work-Energy with Varying Forces

- Work-energy theorem W_{tol} = [?]K holds for varying forces as well as for constant ones

$$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\frac{dx}{dt} = v_x \frac{dv_x}{dx}$$

$$W_{tot} = \int_{x_1}^{x_2} F_x dx = \int_{x_1}^{x_2} m a_x dx = \int_{x_1}^{x_2} m v_x \frac{dv_x}{dx} dx$$

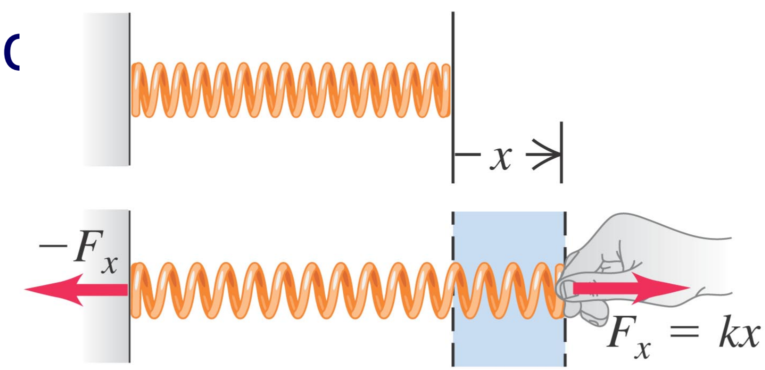
$$W_{tot} = \int_{v_1}^{v_2} m v_x dv_x$$

$$W_{tot} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = \Delta K$$

Spring Force: a Varying Force

- Involves the *spring constant*, k
- Hooke's Law gives the force

  - Where F_x is the force ***exerted on the spring*** in the same direction of x
  - The force ***exerted by the spring*** is $F_s = -F_x = -kx$
  - k depends c

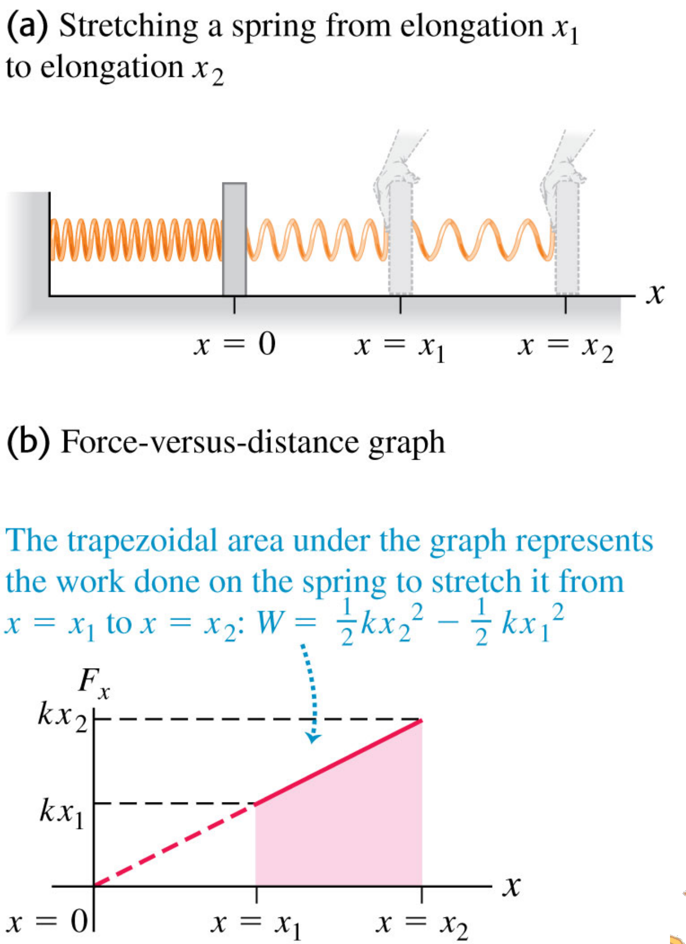


of. Unit: N/m.

Work done on a Spring

- To stretch a spring, we must do work
- We apply equal and opposite forces to the ends of the spring and gradually increase the forces
- The work we must do to stretch the spring from x1 to x2

$$W = \int_{x_1}^{x_2} F_x dx = \int_{x_1}^{x_2} kx dx = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$



Power

- Work does not depend on time interval
- The rate at which energy is transferred is important in the design and use of practical device
- The time rate of energy transfer is called power
- The average power is given by

$$\overline{P} = \frac{W}{\Delta t}$$

  - when the method of energy transfer is work

Instantaneous Power

- Power is the time rate of energy transfer. Power is valid for any means of energy transfer
- Other expression

$$\overline{P} = \frac{W}{\Delta t} = \frac{F\Delta x}{\Delta t} = F\overline{v}$$

- A more general definition of instantaneous power

$$P = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

$$P = \vec{F} \cdot \vec{v} = Fv\cos\theta$$

Units of Power

- The SI unit of power is called the watt
  - 1 watt = 1 joule / second = 1 kg · m^2 / s^3
- A unit of power in the US Customary system is horsepower
  - 1 hp = 550 ft · lb/s = 746 W
- Units of power can also be used to express units of work or energy
  - 1 kWh = (1000 W)(3600 s) = 3.6 x10^6 J