Physics 111: Mechanics
Lecture 7

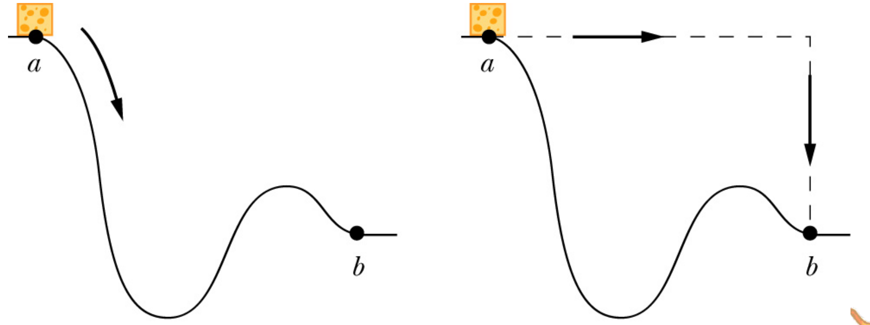
Bin Chen

NJIT Physics Department

Chapter 7 Potential Energy and Energy Conservation

- 7.1 Gravitational Potential Energy
- 7.2 Elastic (Spring) Potential Energy
- 7.3 Conservative and Non-conservative Forces
- 7.4* Force and Potential Energy
- 7.5* Energy Diagrams

*Self study (not required)

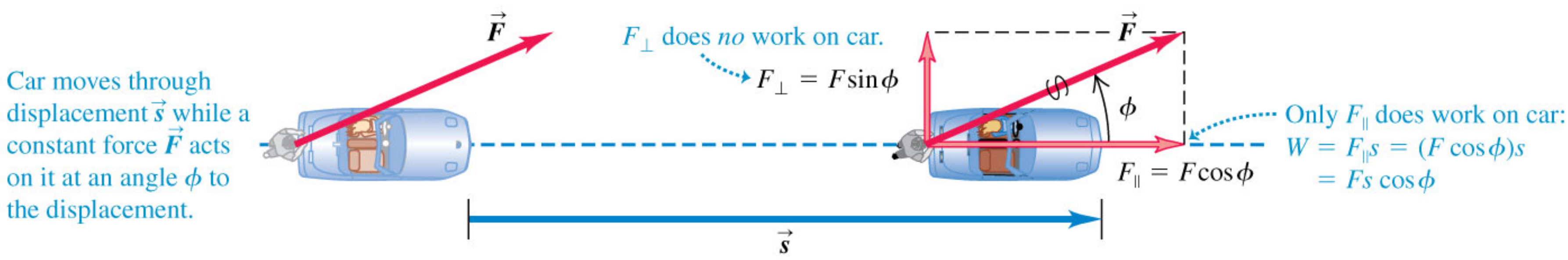


Definition of Work W

- The work, *W*, done by a constant force on an object is defined as the product of the component of the force along the direction of displacement and the magnitude of the displacement

$$W = \vec{F} \cdot \vec{s}$$
$$= Fs\cos\theta$$

- F is the magnitude of the force
- s is the magnitude of the object's displacement
- q is the angle between F and s

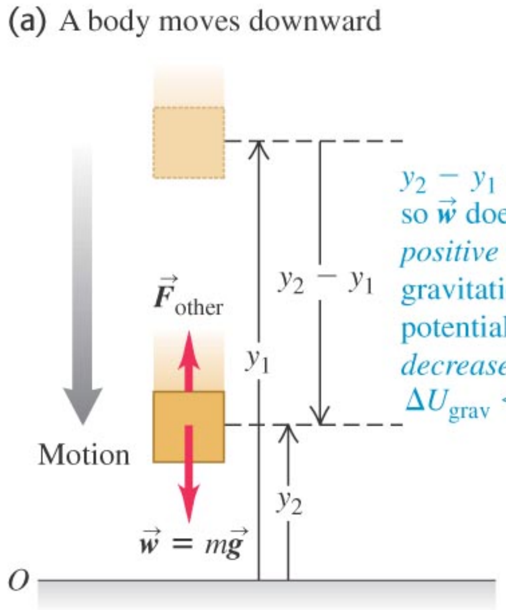


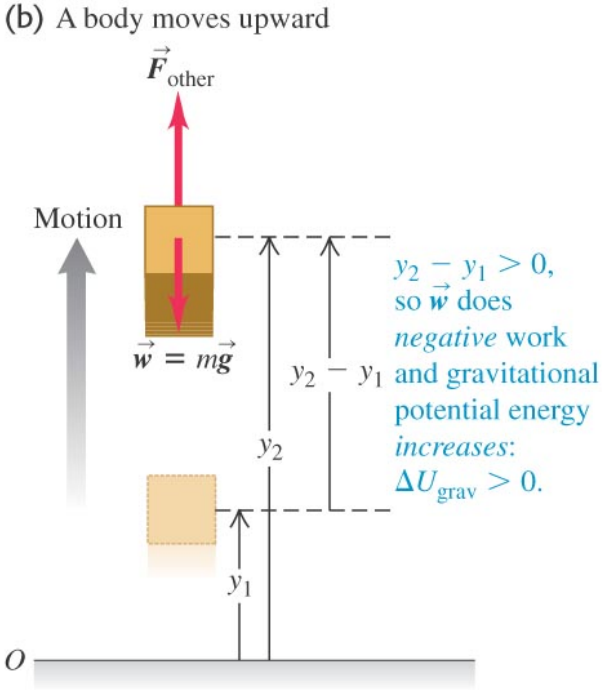
Work done by Gravity and Gravitational Potential Energy

$$W_{\text{grav}} = Fs = w(y_1 - y_2) = mgy_1 - mgy_2$$

$$U_{grav} \equiv mgy$$

$$W_{\text{grav}} = U_{\text{grav},1} - U_{\text{grav},2} = -\left(U_{\text{grav},2} - U_{\text{grav},1}\right) = -\Delta U_{\text{grav}}$$



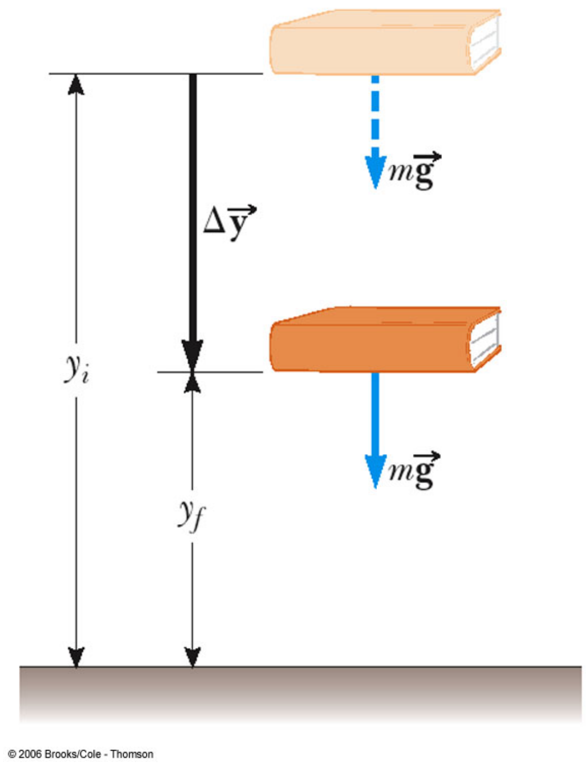


Potential Energy

- Potential energy is associated with the position of the object
- Gravitational Potential Energy is the energy associated with the relative position of an object in space near the Earth's surface
- Shared by both the object and Earth
- The gravitational potential energy

$$U_{grav} \equiv mgy$$

  - m is the mass of an object
  - g is the acceleration of gravity
  - y is the vertical position of the mass relative the surface of the Earth
  - SI unit: joule (J)



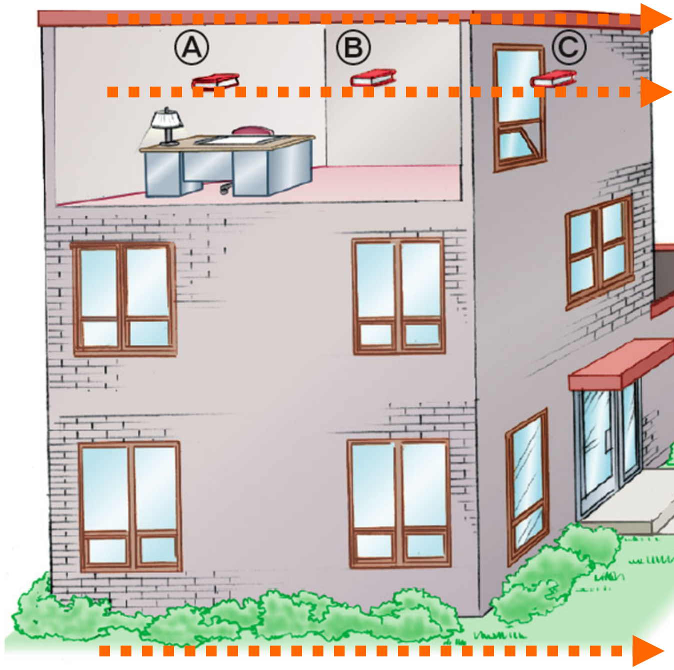
Reference Level

- A location where the gravitational potential energy is zero must be chosen for each problem
  - The choice is arbitrary since the change in the potential energy is the important quantity
  - Choose a convenient location for the zero reference height
    - often the Earth's surface
    - may be some other point suggested by the problem
  - Once the position is chosen, it must remain fixed for the entire problem

Reference Levels

- The gravitational potential energy of an object

(a) is always positive
(b) is always negative
(c) never equals to zero
(d) can be negative or positive



Work-Kinetic Energy Theorem

- When work is done by a net force on an object and the only change in the object is its speed, the work done is equal to the change in the object's kinetic energy
  - Speed will increase if work is positive
  - Speed will decrease if work is negative

$$W_{\text{tot}} = K_2 - K_1 = \Delta K$$

$$W_{\text{tot}} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

Extended Work-Energy Theorem with Gravitational Potential Energy

- The work-kinetic energy theorem can be extended to include gravitational potential energy:

$$W_{\text{tot}} = K_2 - K_1 = \Delta K$$

$$W_{\text{grav}} = U_{\text{grav},1} - U_{\text{grav},2} = -\left(U_{\text{grav},2} - U_{\text{grav},1}\right) = -\Delta U_{\text{grav}}$$

- **If we only have gravitational force and all work done by all rest forces are zero**, then

$$W_{tot} = W_{grav}$$

$$K_1 + U_{\text{grav},1} = K_2 + U_{\text{grav},2} \quad \text{(If only gravity does work)}$$

Conservation of Mechanical Energy

- We denote the total mechanical energy by

$$E = K + U_{grav}$$

- Since $$K_1 + U_{\text{grav},1} = K_2 + U_{\text{grav},2}$$
- So

$$E = K + U_{grav} = \text{constant}$$

- The total mechanical energy is conserved and remains the same at all times

$$\tfrac{1}{2} m\upsilon_1^{\ 2} + mgy_1 = \tfrac{1}{2} m\upsilon_2^{\ 2} + mgy_2$$

(If only gravity does work)

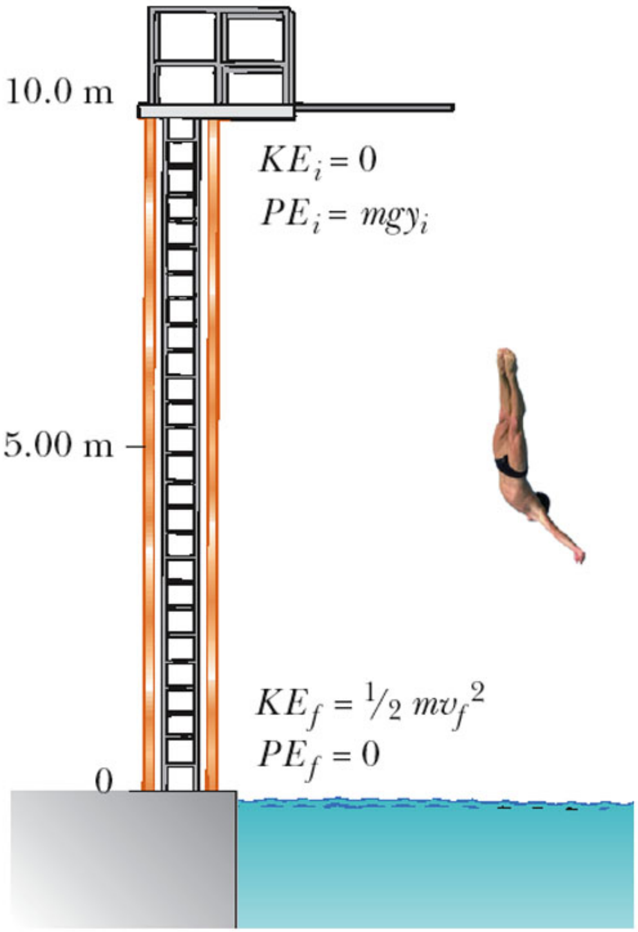
Problem-Solving Strategy

- Define the system
- Select the location of zero gravitational potential energy
  - Do *not* change this location while solving the problem
- Identify two points the object of interest moves between
  - One point should be where information is given
  - The other point should be where you want to find out something

Platform Diver

- A diver of mass m drops from a board 10.0 m above the water's surface. Neglect air resistance.

- Find his speed as he hits the water



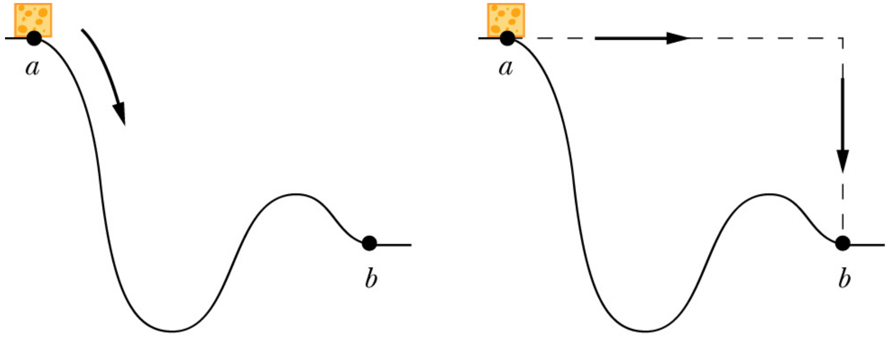
Platform Diver

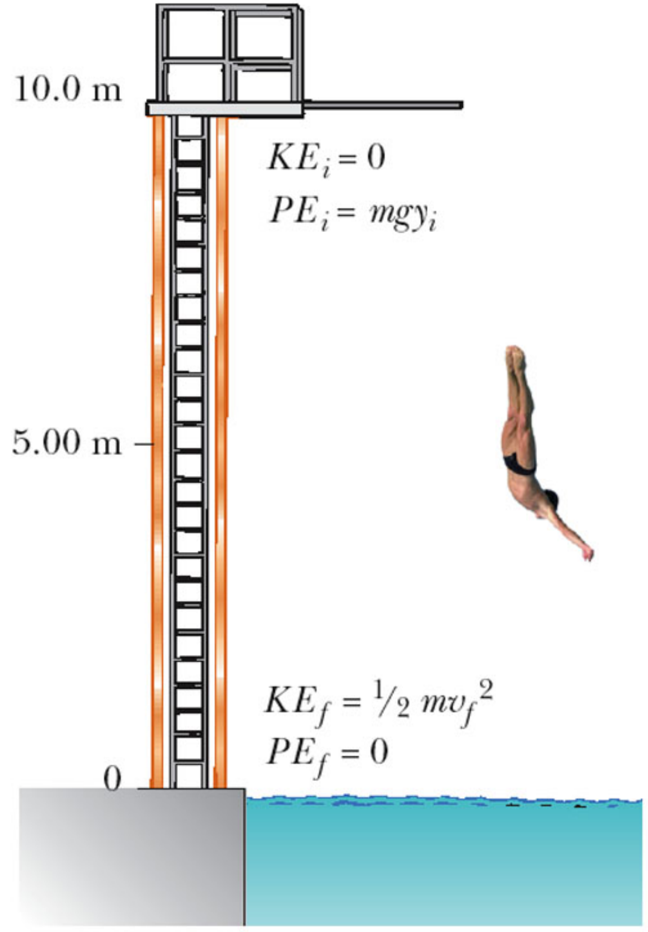
- Find his speed as he hits the water

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

$$0 + mgy_i = \frac{1}{2}mv_f^2 + 0$$

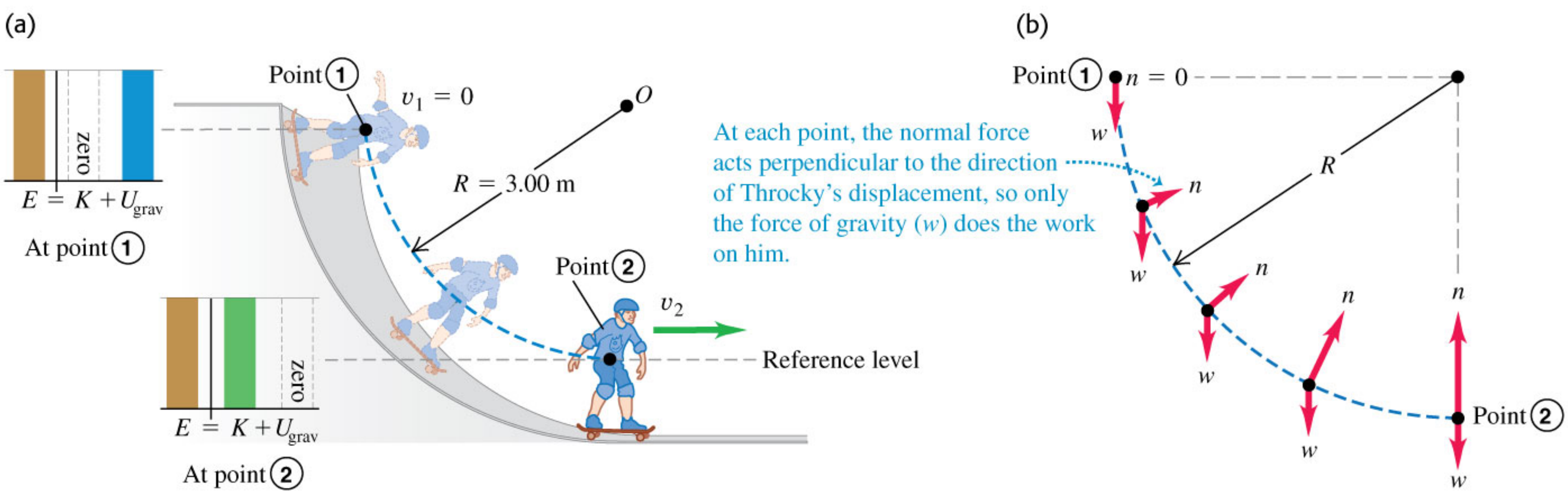
$$v_f = \sqrt{2gy_i} = 14m/s$$





Skateboarding

- A boy skateboards from rest down a curved frictionless ramp. He moves through a quarter-circle with radius R=3m. The boy and his skateboard have a total mass of 25 kg.
- Find his speed at the bottom of the ramp



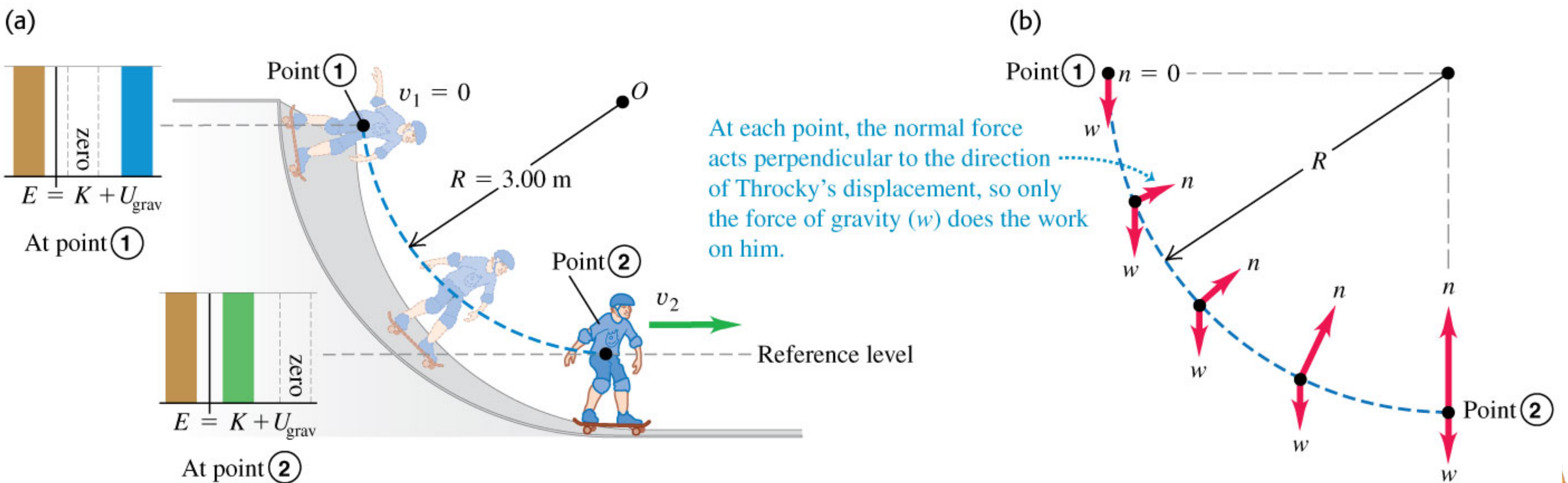
Skateboarding

- Find his speed at the bottom of the ramp

$$K_1 = 0 \qquad U_{grav,1} = mgR \qquad \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2$$

$$K_2 = \tfrac{1}{2}mv_2^2 \qquad U_{grav,2} = 0$$

$$0 + mgR = \frac{1}{2}mv_2^2 + 0 \qquad v_2 = \sqrt{2gR} = 7.67 m/s$$

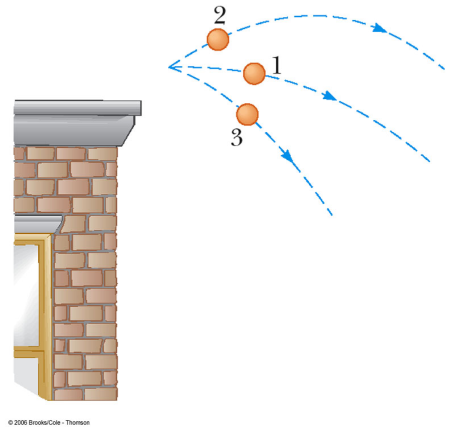


Prof. Walter Lewin's Pendulum

- https://www.youtube.com/watch?v=xXXF2C-vrQE

Conservation of Mechanical Energy

- Three identical balls are thrown from the top of a building, all with the same initial speed. The first ball is thrown horizontally, the second at some angle above the horizontal, and the third at some angle below the horizontal. Neglecting air resistance, rank the speeds of the balls as they reach the ground, from fastest to slowest.



(a) 1, 2, 3
(b) 2, 1, 3
(c) 3, 1 ,2
(d) 3, 2, 1
(e) all three ball strike the ground at the same speed

When Forces other than Gravity Do Work

- The work-kinetic energy theorem can be extended to include potential energy: $W_{\text{tot}} = K_2 - K_1 = \Delta K$

$$W_{tot} = W_{grav} + W_{other} = K_2 - K_1$$

- Since $$W_{\text{grav}} = U_{\text{grav},1} - U_{\text{grav},2} = -\left(U_{\text{grav},2} - U_{\text{grav},1}\right) = -\Delta U_{\text{grav}}$$
- Then

$$W_{other} + U_{grav,1} - U_{grav,2} = K_2 - K_1$$

- If forces other than gravity do work

$$K_1 + U_{grav,1} + W_{other} = K_2 + U_{grav,2}$$

$$\frac{1}{2}mv_1^2 + mgy_1 + W_{other} = \frac{1}{2}mv_2^2 + mgy_2$$

Lifting bucket from well

- A person lifted a 10.0-kg bucket from the bottom of a 10-m deep well. The bucket started from rest and had a 1.00 m/s speed as it reached the top. How much work, in J, is done by this person? ($g=9.8\ m/s^2$. Neglect friction.)

A. 975

B. 985

C. 980

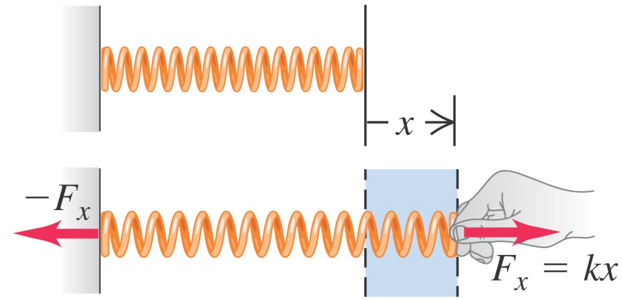
D. -985

E. -975

Spring Force: An Elastic Force

- Hooke's Law gives the force

$$F_x = kx$$

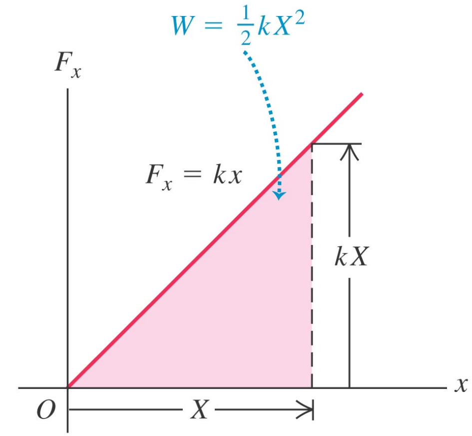


Where F_x is the force **exerted on the spring** in the same direction of x

- Work done on the spring from x_1 to x_2

$$W = \int_{x_1}^{x_2} F_x dx = \int_{x_1}^{x_2} kx dx = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$

The area under the graph represents the work done on the spring as the spring is stretched from $x = 0$ to a maximum value X:



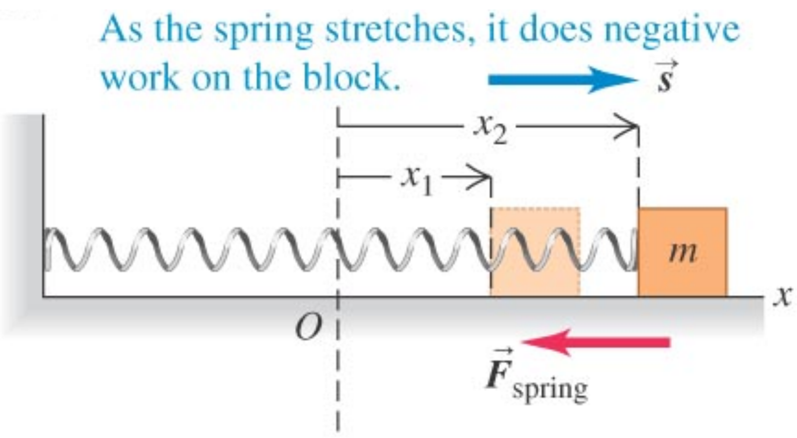
- The force **exerted by the spring** is

$$F_s = -F_x = -kx$$

Potential Energy in a Spring

- Work done ***by*** the spring

$$W_s = \int_{x_1}^{x_2} F_s dx = \int_{x_1}^{x_2} -kx dx = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$



- Elastic Potential Energy:

$$U_{el} = \frac{1}{2}kx^2$$

  - SI unit: Joule (J)
  - related to the work required to compress a spring from its equilibrium position to some final, arbitrary, position

Extended Work-Energy Theorem with Elastic Potential Energy

- The work-kinetic energy theorem can be extended to include elastic potential energy:

$$W_{\text{tot}} = K_2 - K_1 = \Delta K$$

$$W_{\text{el}} = \tfrac{1}{2}kx_1^{\ 2} - \tfrac{1}{2}kx_2^{\ 2} = U_{\text{el},1} - U_{\text{el},2} = -\ \Delta U_{\text{el}}$$

- **If we only have spring force and all work done by all rest forces are zero**, then $W_{tot} = W_{el}$

$$K_1 + U_{\text{el},1} = K_2 + U_{\text{el},2}$$

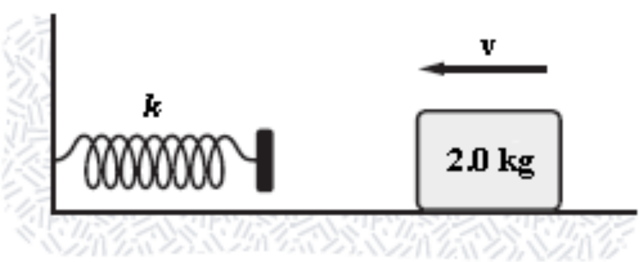
$$\tfrac{1}{2}mv_1^{\ 2} + \tfrac{1}{2}kx_1^{\ 2} = \tfrac{1}{2}mv_2^{\ 2} + \tfrac{1}{2}kx_2^{\ 2}$$

(If only the elastic force does work)

Hitting a Spring

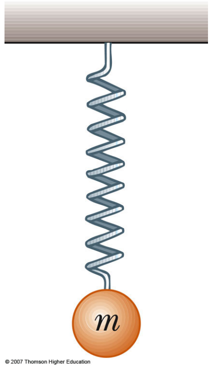
- A 2 kg block slides with no friction and with an initial speed of 4 m/s. It hits a spring with spring constant k=1400 N/m. The block compresses the spring in a straight line for a distance 0.1 m. What is the block's speed, in m/s, at that point?

   A. 1.0
   B. 2.0
   C. 3.0
   D. 4.0
   E. 5.0



Extended Work-Energy Theorem with **BOTH** Gravitational and Elastic Potential Energy

- The work-kinetic energy theorem can be extended to include both types of potential energy:



$$W_{\text{tot}} = K_2 - K_1 = \Delta K$$

$$W_{\text{grav}} = U_{\text{grav},1} - U_{\text{grav},2} = -\left(U_{\text{grav},2} - U_{\text{grav},1}\right) = -\,\Delta U_{\text{grav}}$$

$$W_{\text{el}} = \tfrac{1}{2}kx_1^{\,2} - \tfrac{1}{2}kx_2^{\,2} = U_{\text{el},1} - U_{\text{el},2} = -\,\Delta U_{\text{el}}$$

- If the system only involves gravitational force and spring force or all work done by all rest forces are zero, then

$$W_{tot} = W_{grav} + W_{el}$$

$$K_1 + U_{grav,1} + U_{el,1} = K_2 + U_{grav,2} + U_{el,2}$$

$$\frac{1}{2}mv_1^2 + mgy_1 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}kx_2^2$$

Mechanical Energy Conservation with **BOTH** Gravitational and Elastic Potential Energy

- We denote the total mechanical energy

$$E = K + U = K + U_{grav} + U_{el}$$

- Since $E_2 = E_1$
- The total mechanical energy is conserved

$$\frac{1}{2}mv_1^2 + mgy_1 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}kx_2^2$$

If other forces do work

- If work done by other forces that **cannot** be described in terms of potential energy

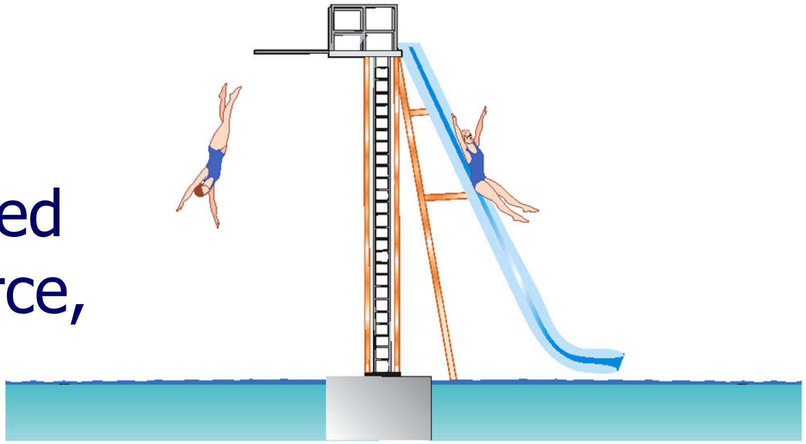
$$W_{net} = W_{other} + W_{grav} + W_{el}$$

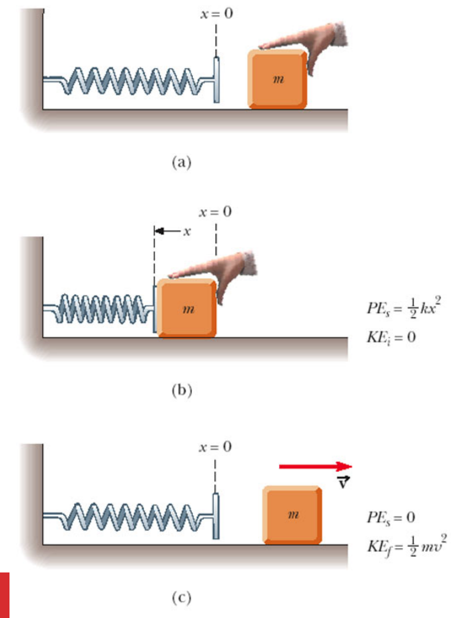
- General case $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

- If W_{other} is positive, E = K + U increases
- If W_{other} is negative, E = K + U decreases
- If W_{other} is zero, E = K + U keeps constant

Types of Forces

- Conservative forces
  - Work and energy associated with the force can be recovered
  - Examples: Gravity, Spring Force, EM forces
- Nonconservative forces
  - The forces are generally dissipative and work done against it cannot easily be recovered
  - Examples: Kinetic friction, air drag forces, normal forces, tension forces, applied forces …





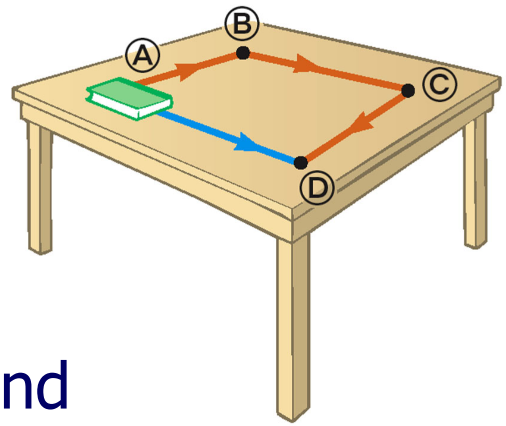
Conservative Forces

- A force is conservative if the work it does on an object moving between two points is independent of the path the objects take between the points
  - The work depends only upon the initial and final positions of the object
  - Any conservative force can have a potential energy function associated with it
  - Work done by gravity
  - Work done by spring force

Nonconservative Forces

- A force is nonconservative if the work it does on an object depends on the path taken by the object between its final and starting points.
  - The work depends upon the movement path
  - For a non-conservative force, potential energy can NOT be defined
  - Work done by a nonconservative force

$$W_{nc} = \sum \vec{F} \cdot \vec{d} = -f_k d + \sum W_{otherforces}$$



  - It is generally dissipative. The dispersal of energy takes the form of heat or sound

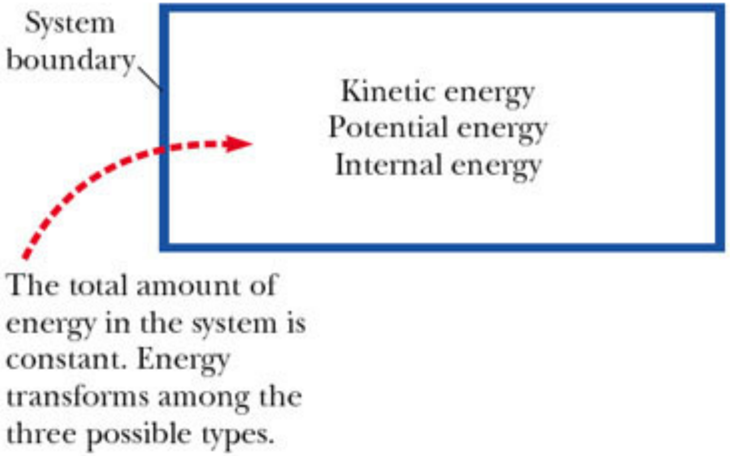
Conservation of Energy in General

- Any work done by conservative forces can be accounted for by changes in potential energy

$$W_c = U_1 - U_2 = -(U_2 - U_1) = -\Delta U$$

$$W_{nc} = \Delta K + \Delta U$$

$$-W_{nc} = \Delta U_{\text{int}}$$



- Law of conservation of energy

$$\Delta K + \Delta U + \Delta U_{\text{int}} = 0$$

- Energy is never created or destroyed. It only changes form.

Problem-Solving Strategy

- Define the system to see if it includes non-conservative forces (especially friction, drag force …)
- Without non-conservative forces

$$\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2$$

- With non-conservative forces $W_{nc} = (KE_f + PE_f) - (KE_i + PE_i)$

$$-fd + \sum W_{otherforces} = (\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2) - (\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2)$$

- Select the location of zero potential energy
  - Do *not* change this location while solving the problem
- Identify two points the object of interest moves between
  - One point should be where information is given
  - The other point should be where you want to find out something

Energy is conserved

- Energy cannot be created nor destroyed
- It can be transferred from one object to another or change in form
- If the total amount of energy in a system changes, it can only be due to the fact that energy has crossed the boundary of the system by some method of energy transfer