Physics 111: Mechanics
Lecture 8

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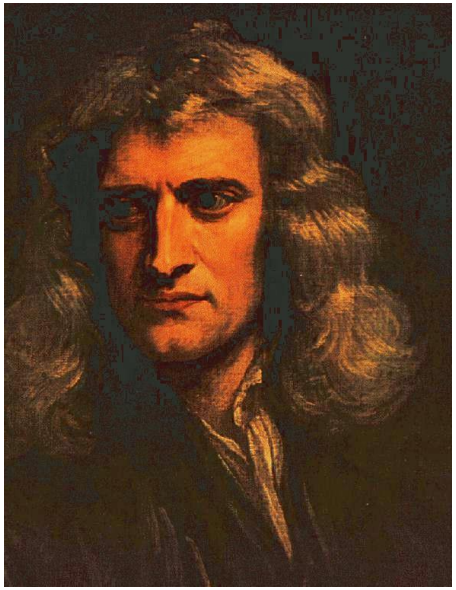
Chapter 8 Momentum, Impulse, and Collisions

- 8.1 Momentum and Impulse
- 8.2 Conservation of Momentum
- 8.3 Momentum Conservation and Collisions
- 8.4 Elastic Collisions
- 8.5 Center of Mass
- 8.6* Rocket Propulsion

*Self study (not required)

Rediscover Newton's 2nd Law



- Newton's 2nd Law:

$$\vec{F}_{net} = \sum \vec{F} = m\vec{a}$$

- Acceleration: $\vec{a} = d\vec{v} / dt$

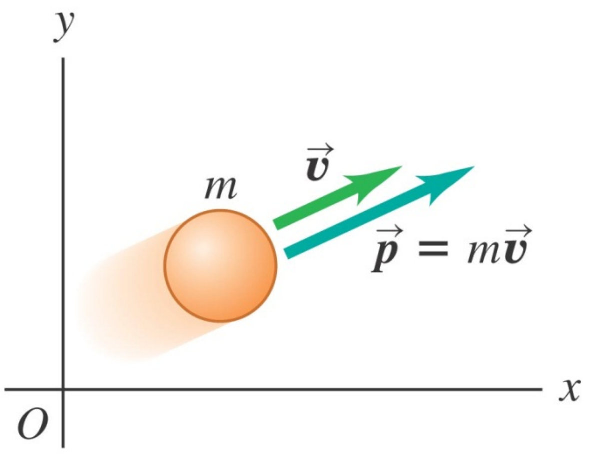
$$\vec{F}_{net} = m\vec{a} = md\vec{v} / dt = d(m\vec{v}) / dt$$

Introducing Momentum

- The **<u>momentum p</u>** of an object of mass m moving with a velocity $\vec{\mathbf{v}}$ is defined to be the product of the mass and velocity:

$$\vec{p} = m\vec{v}$$

  - A new fundamental quantity, like force, energy
  - Momentum depend on an object's mass and velocity
  - The terms momentum and linear momentum will be used interchangeably in the text



Momentum $\vec{p}$ is a vector quantity; a particle's momentum has the same direction as its velocity $\vec{v}$.

Linear Momentum

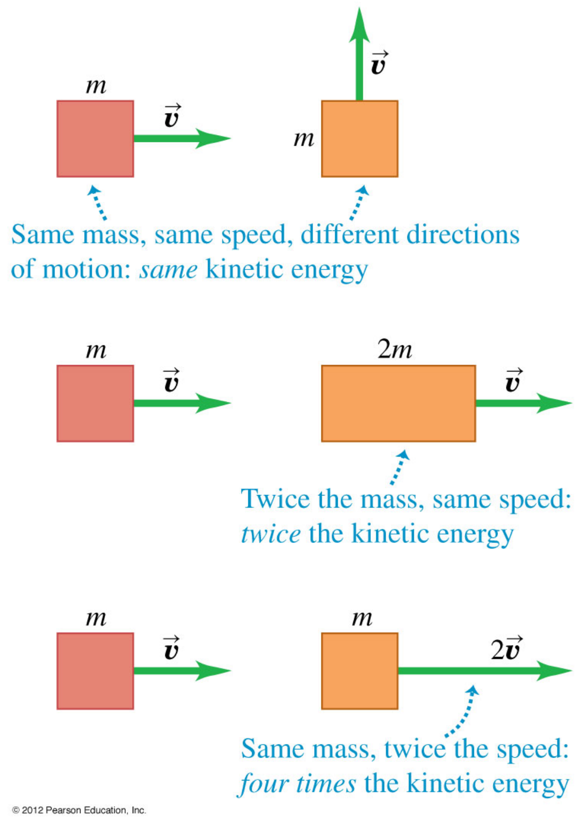
- Linear momentum is a vector quantity

$$\vec{p} = m\vec{v}$$

  - Its direction is the same as the direction of the velocity
  - Momentum can be expressed in its components

$$p_x = mv_x \qquad p_y = mv_y \qquad p_z = mv_z$$

  - The SI units of momentum are kg · m / s



Momentum and Energy

- Two objects with masses $m_1 < m_2$ have equal kinetic energy. How do the magnitudes of their momenta compare?

  (A) $p_1 < p_2$

  (B) $p_1 = p_2$

  (C) $p_1 > p_2$

  (D) Not enough information is given

Newton's Law and Momentum

- Newton's Second Law can be used to relate the momentum of an object to the resultant force acting on it

$$\vec{F}_{net} = m\vec{a} = m d\vec{v} / dt = d(m\vec{v}) / dt = d\vec{p} / dt$$

- The change in an object's momentum divided by the elapsed time equals the net force acting on the object

$$\vec{F}_{net} = d\vec{p} / dt = \text{Change in momentum/time elapsed}$$

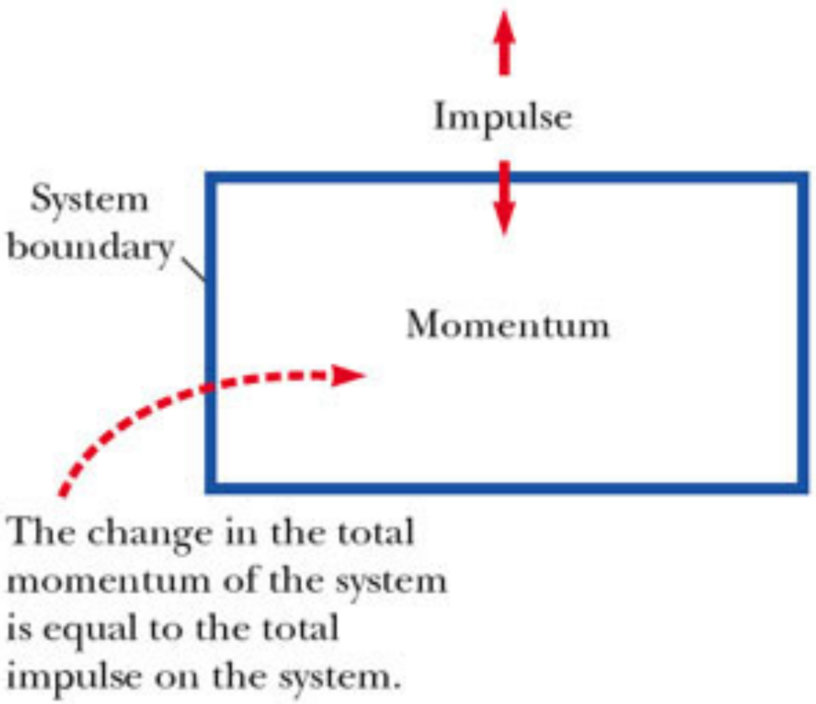
Impulse and Constant Force

- When a single, *constant* force acts on the object, there is an **impulse** delivered to the object
  - is defined as the *impulse*
  - Vector quantity, the direction is the same as the direction of the force

$$\vec{J} = \sum \vec{F}\left(t_2 - t_1\right) = \sum \vec{F}\Delta t$$

Impulse-Momentum Theorem

- The theorem states that the impulse acting on a system is equal to the change in momentum of the system



$$\vec{J} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i$$

$$W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$J_x = p_{fx} - p_{ix} = mv_{fx} - mv_{ix}$$

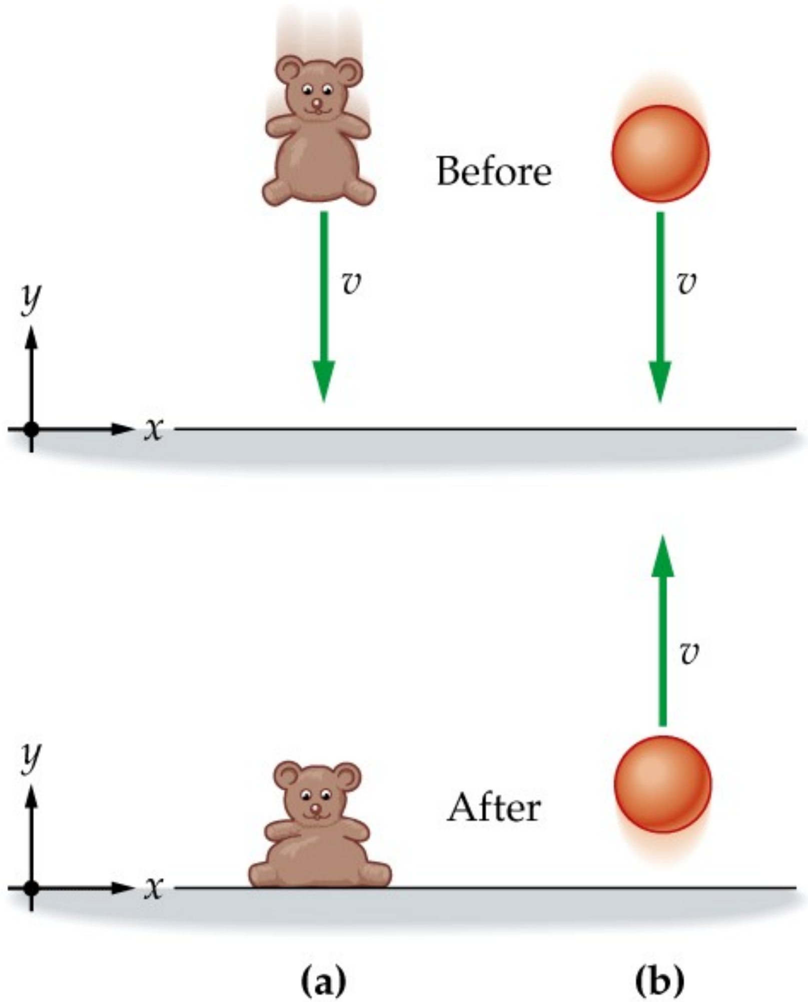
$$J_y = p_{fy} - p_{iy} = mv_{fy} - mv_{iy}$$

Calculating the Change of Momentum

$$\Delta\vec{p} = \vec{p}_2 - \vec{p}_1$$

$$= m\vec{v}_2 - m\vec{v}_1$$

$$= m(\vec{v}_2 - \vec{v}_1)$$



For the teddy bear

$$\Delta p = m\left[0-(-v)\right] = mv$$

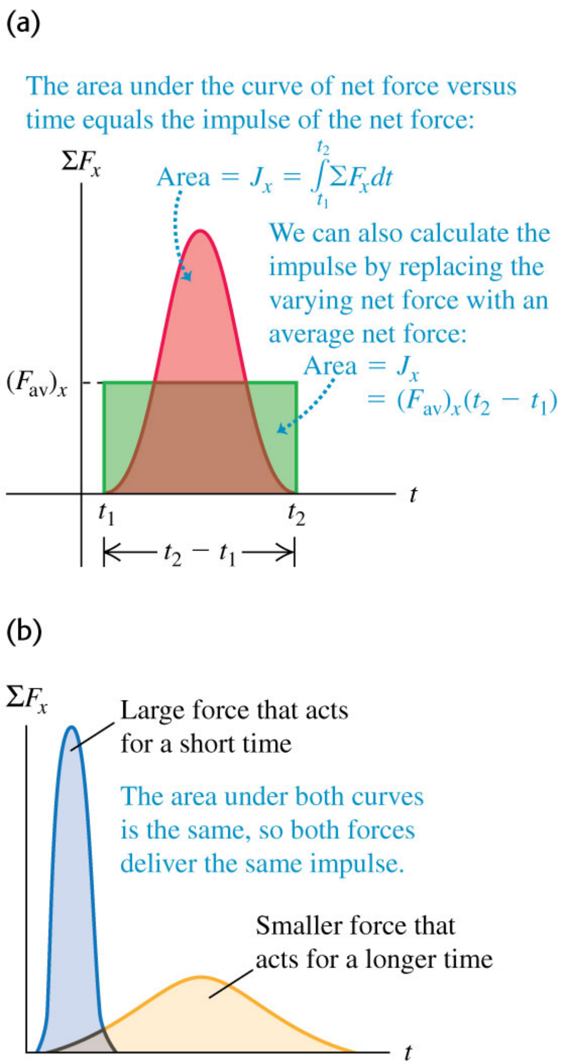
For the bouncing ball

$$\Delta p = m\left[v-(-v)\right] = 2mv$$

Impulse-Momentum Theorem in General

$$\vec{J} = \int_{t_1}^{t_2} \sum \vec{F}\, dt \qquad \vec{J} = \vec{p}_2 - \vec{p}_1$$

- On a graph of ΣF_x versus time, the impulse is equal to the area under the curve, as shown in the figure to the right.
- *Impulse-momentum theorem*: The change in momentum of a particle during a time interval is equal to the impulse of the net force acting on the particle during that interval.



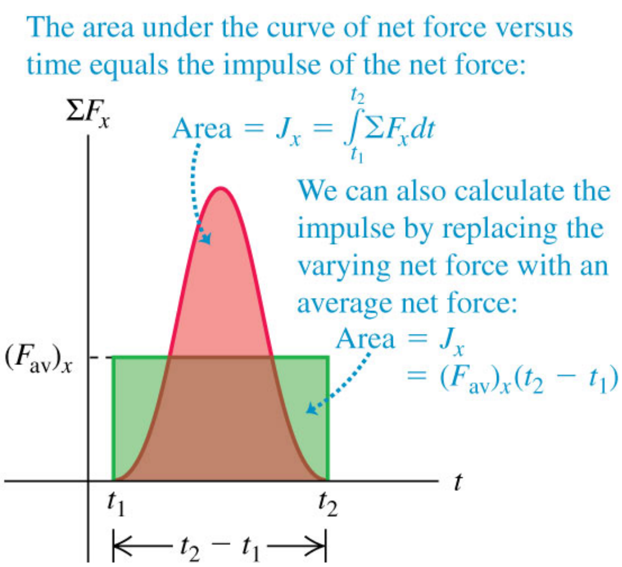
Impulse, Momentum, and Average Force

- Impulse and momentum:

$$\vec{J} = \int_{t_1}^{t_2} \sum \vec{F}\, dt = \vec{p}_2 - \vec{p}_1$$

- Impulse and average force:

$$\vec{J} = \vec{F}_{avg}(t_2 - t_1)$$



- Average force and change of momentum

$$\vec{F}_{avg} = (\vec{p}_2 - \vec{p}_1)/(t_2 - t_1)$$

Airplane hit by a goose

- A 2-kg goose flying at 10 m/s collided head-on with a fast-moving plane at 130 m/s (~291 mph) and smashed on it. The collision occurred in 0.1 s. What is the average force did the plane experience?

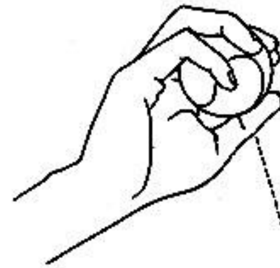
A. 2600 N
B. 5820 N
C. 2400 N
D. 2800 N
E. 5200 N

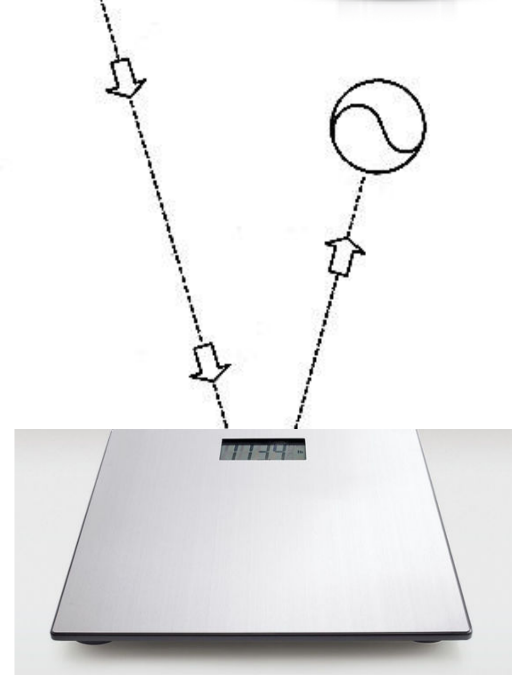
Bouncing Ball

- A ball of mass 0.3 kg and speed 2 m/s hit a weight scale at an angle of 60 degrees w.r.t. the horizontal direction, and then bounces off at the same speed (2 m/s) and same angle as shown in the figure. The weight scale shows an average force of 10 N. How many seconds did the collision take?

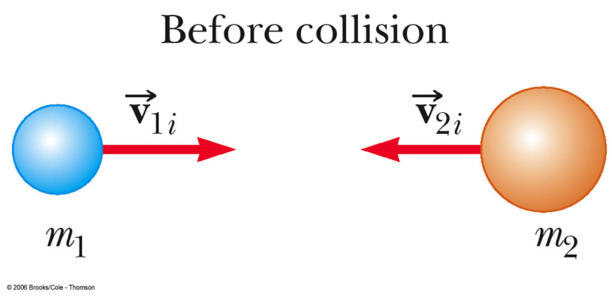
A. 0.5 B. 0.05 C. 0.1

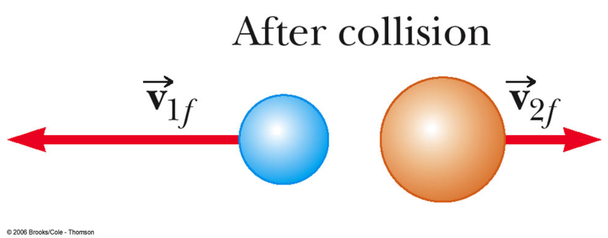
D. 0.12 E. 0.06

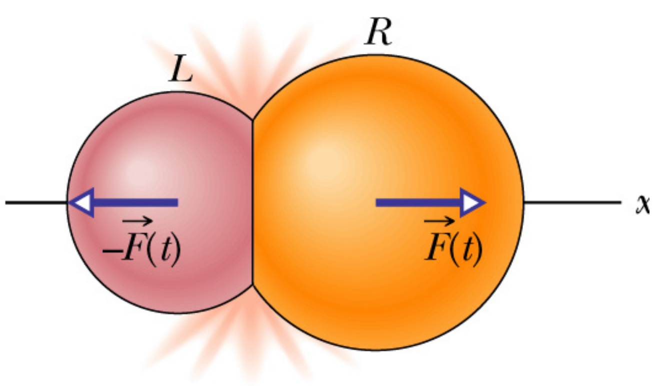




Total Momentum of Colliding Balls







- Start from impulse-momentum theorem

$$\vec{F}_{21}\Delta t = m_1\vec{v}_{1f} - m_1\vec{v}_{1i}$$

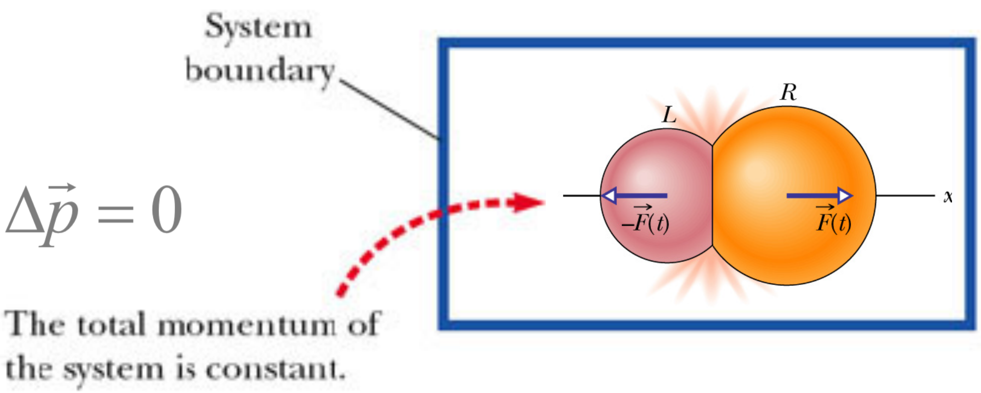
$$\vec{F}_{12}\Delta t = m_2\vec{v}_{2f} - m_2\vec{v}_{2i}$$

- Since $\vec{F}_{21}\Delta t = -\vec{F}_{12}\Delta t$
- Then $m_1\vec{v}_{1f} - m_1\vec{v}_{1i} = -(m_2\vec{v}_{2f} - m_2\vec{v}_{2i})$
- So $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

Conservation of Momentum

- If the vector sum of all external forces on a system is zero, the total momentum of the system remains constant in time

$$\vec{F}_{net}\Delta t = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$



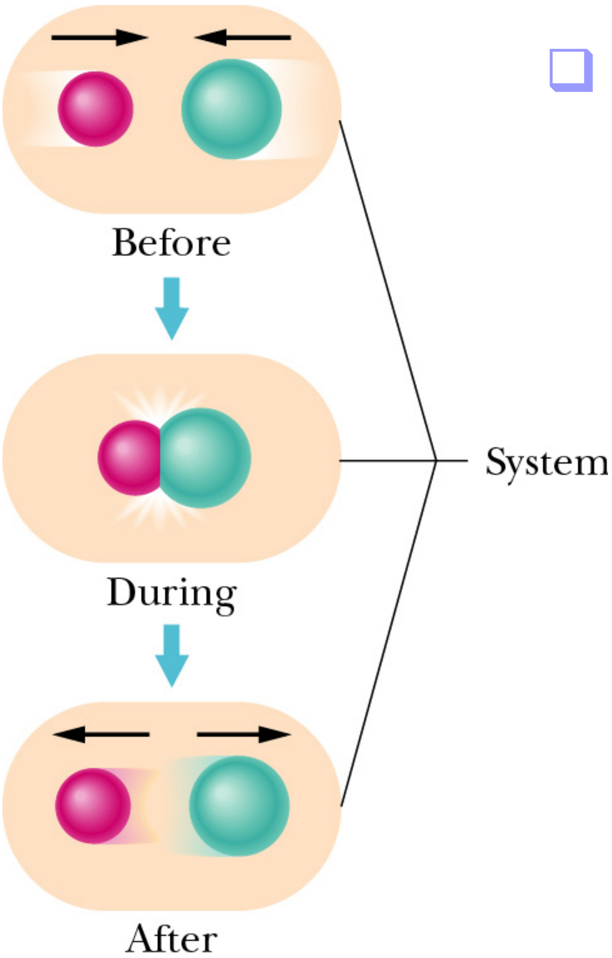
- When $\vec{F}_{net} = 0$ then $\Delta\vec{p} = 0$
- For an isolated system

$$\vec{p}_f = \vec{p}_i$$

- Specifically, the total momentum before the collision will equal the total momentum after the collision. For a two-body system:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Conservation of Momentum



- In an isolated and closed system, the total momentum of the system remains constant in time.
  - Isolated system: no external forces
  - Closed system: no mass enters or leaves
  - The linear momentum of each colliding body may change
  - The total momentum p of the system cannot change.

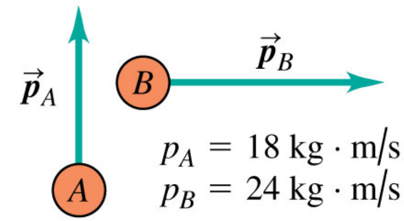
Remember that momentum is a vector!

- When applying conservation of momentum, remember that momentum is a vector quantity!
- Use vector addition to add momenta

$$\vec{P} = \vec{p}_A + \vec{p}_B + \cdots = m_A\vec{v}_A + m_B\vec{v}_B + \cdots$$

- Components of the momenta are also conserved!

$$P_x = p_{Ax} + p_{Bx} + \cdots$$
$$P_y = p_{Ay} + p_{By} + \cdots$$
$$P_z = p_{Az} + p_{Bz} + \cdots$$

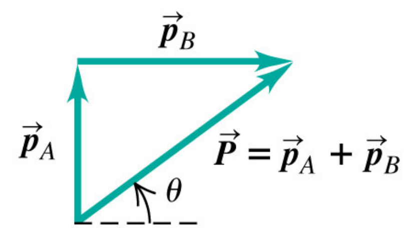


A system of two particles with momenta in different directions

You CANNOT find the magnitude of the total momentum by adding the magnitudes of the individual momenta!

$P = p_A + p_B = 42\ \mathrm{kg\cdot m/s}$ ◀ WRONG

Instead, use vector addition:



◀ RIGHT!

$$P = |\vec{p}_A + \vec{p}_B| = 30\ \mathrm{kg\cdot m/s}\ \text{at}\ \theta = 37^\circ$$

Types of Collisions

- Momentum is approximately conserved in most collisions (why?)
- **Inelastic collisions:** *meatball falls into spaghetti*
  - Kinetic energy is not conserved
  - **Completely inelastic** collisions occur when the objects stick together
- **Elastic collisions:** *billiard ball*
  - both momentum and kinetic energy are conserved
- Most collisions fall between elastic and completely inelastic collisions

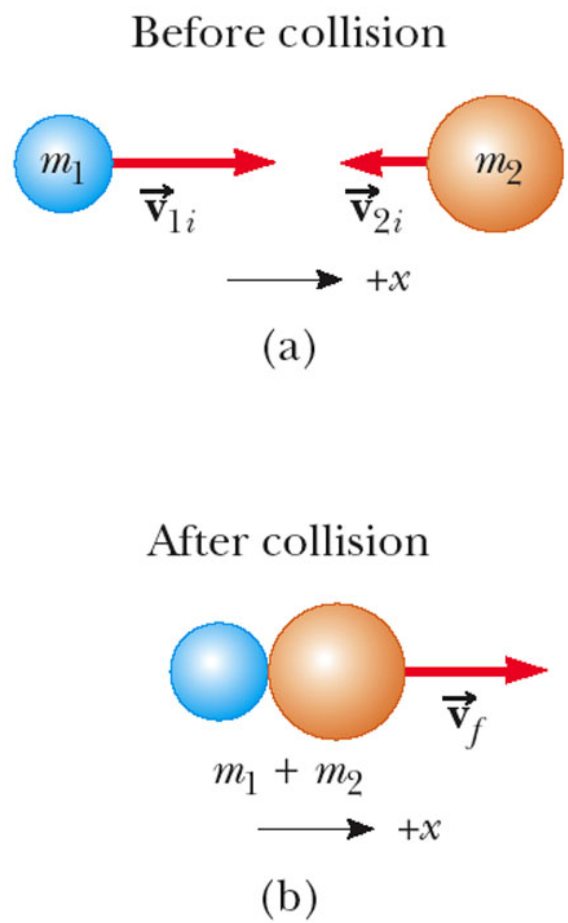
Completely Inelastic Collisions

- When two objects stick together after the collision, they have undergone a completely inelastic collision
- Conservation of momentum

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

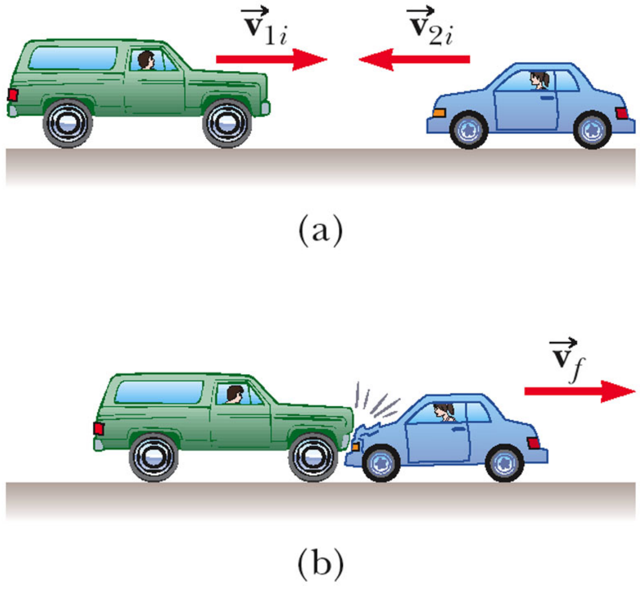
- Kinetic energy is NOT conserved



An SUV Versus a Compact

- An SUV with mass 1.80×10^3 kg is travelling eastbound at +15.0 m/s, while a compact car with mass 9.00×10^2 kg is travelling westbound at -15.0 m/s. The cars collide head-on, becoming entangled.

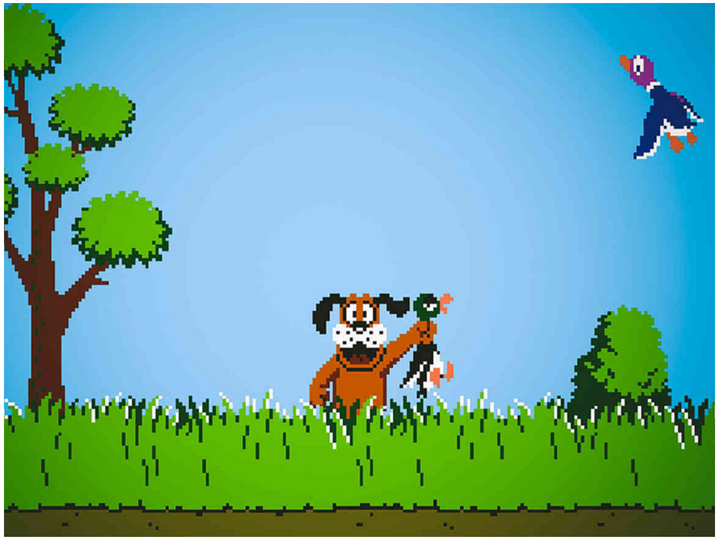
(a) Find the speed of the entangled cars after the collision.
(b) Find the change in the velocity of each car.
(c) Find the change in the kinetic energy of the system consisting of both cars.



Duck Hunt

- A 10-g bullet is fired on a 2-kg duck sitting on a frictionless ice lake. After the bullet hit the duck, they move together at 1 m/s. Find the initial speed of the bullet in m/s.

A. 200
B. 100
C. 50
D. 400
E. 25



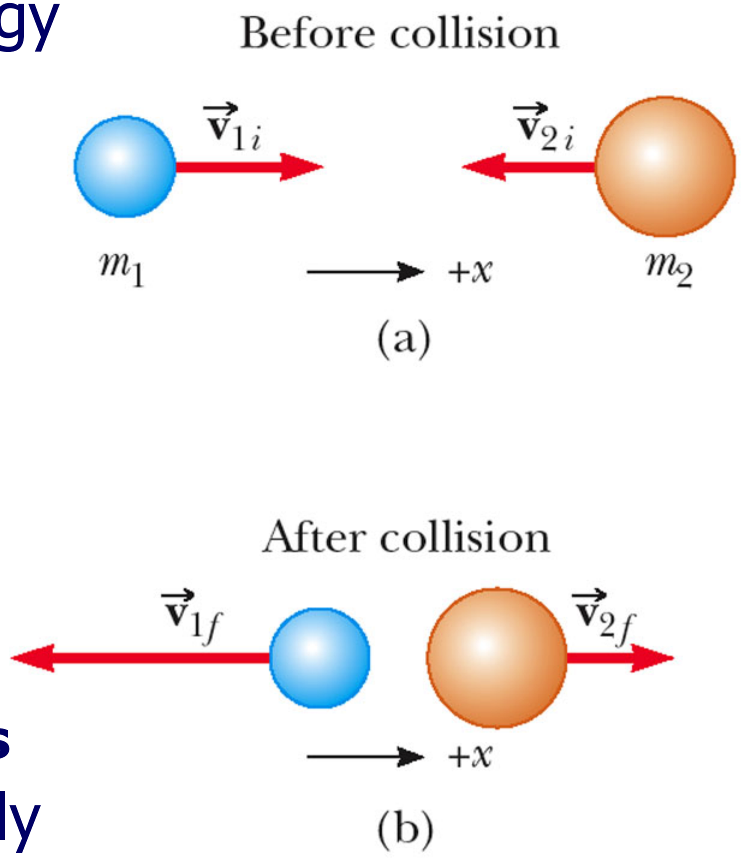
Elastic Collisions

- Both momentum and kinetic energy are conserved

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

- Typically have two unknowns
- Momentum is a vector quantity
  - Direction is important
  - **Be sure to have the correct signs**
- Solve the equations simultaneously



Elastic Collisions

- A simpler equation can be used in place of the kinetic energy equation

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

$$m_1(v_{1i}^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_{2i}^2)$$

$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i})$$

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

$$m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i})$$

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

A special case: One body initially at rest

- $v_{2i}=0$

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

- If $m_2 >> m_1$, $v_{2f} \sim 0$, $v_{1f} \sim -v_{1i}$
- If $m_2 << m_1$, $v_{2f} \sim 2v_{1i}$, $v_{1f} \sim v_{1i}$
- If $m_2 = m_1$, $v_{2f} = v_{1i}$, $v_{1f} = 0$

Elastic Collision

- Glider A of mass 4 kg moves with speed 2 m/s on a horizontal rail without friction. It collides elastically with glider B initially at rest. After the collision, glider A is at rest and glider B moves at 2 m/s. What is the mass of glide B, in kg?

A. 0.5   B. 6   C. 4   D. 8   E. 2

Summary of Types of Collisions

- In an **elastic collision**, both momentum and kinetic energy are conserved

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

- In an **inelastic collision**, momentum is conserved but kinetic energy is not

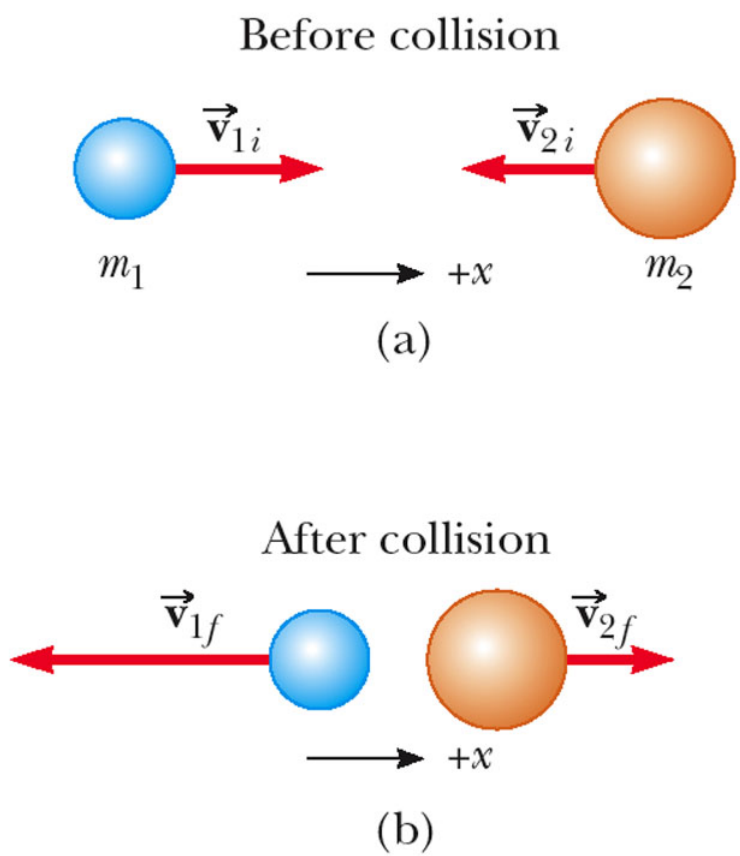
$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

- In a ***completely* inelastic collision**, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

Problem Solving for 1D Collisions, 1

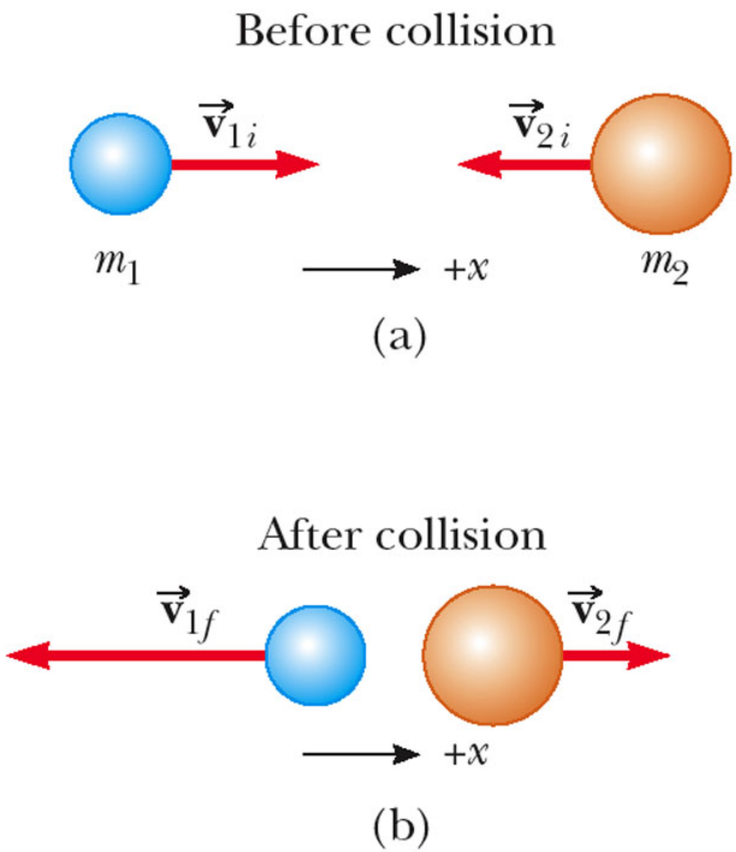
- **Coordinates:** Set up a coordinate axis and define the velocities with respect to this axis
  - It is convenient to make your axis coincide with one of the initial velocities
- **Diagram:** In your sketch, draw all the velocity vectors and label the velocities and the masses



Problem Solving for 1D Collisions, 2

- **Conservation of Momentum:** Write a general expression for the total momentum of the system *before* and *after* the collision
  - Equate the two total momentum expressions
  - Fill in the known values

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

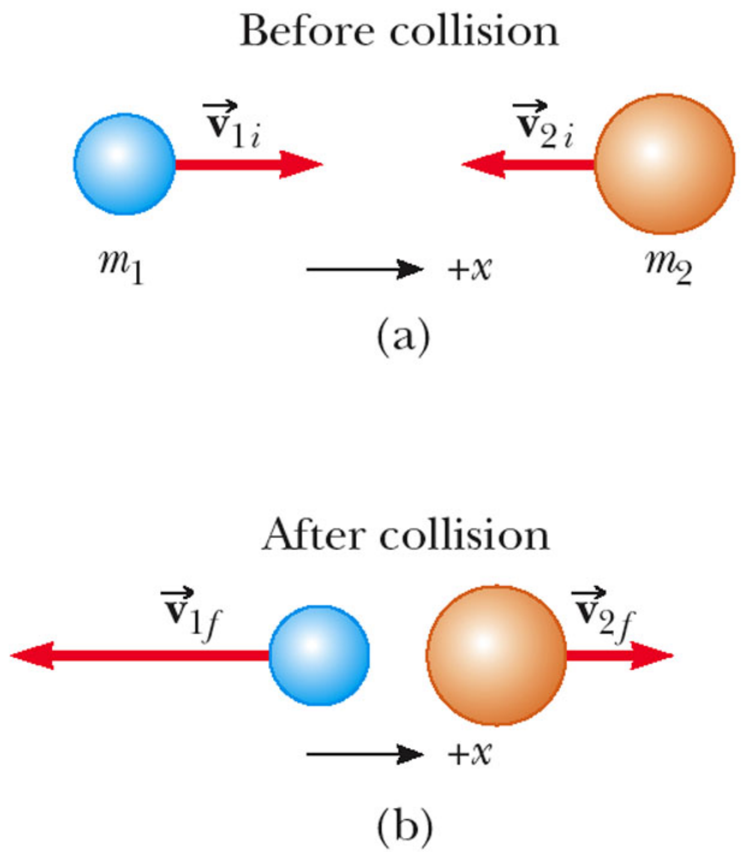


Problem Solving for 1D Collisions, 3

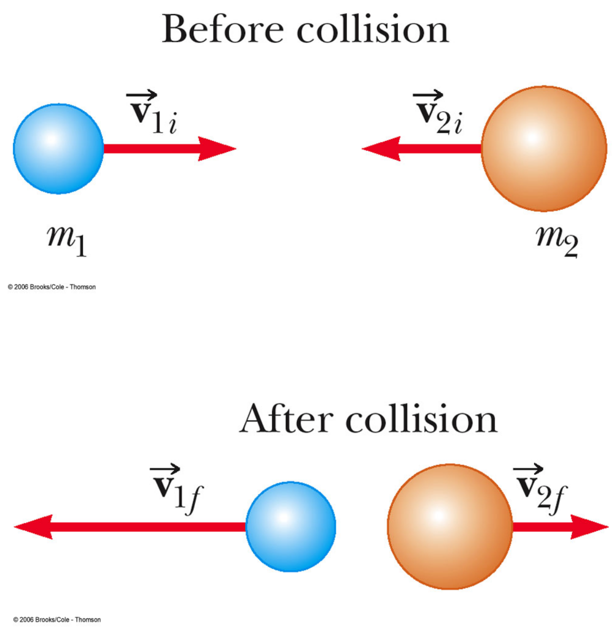
- **Conservation of Energy:** If the collision is elastic, write a second equation for conservation of KE, or the alternative equation
  - This only applies to perfectly elastic collisions

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

- **Solve:** the resulting equations simultaneously



One-Dimension vs Two-Dimension



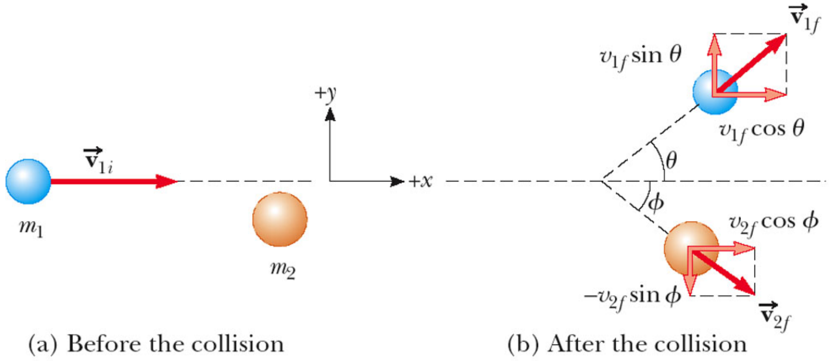
Two-Dimensional Collisions

- For a general collision of two objects in two-dimensional space, the conservation of momentum principle implies that the *total momentum of the system in each direction is conserved*

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$



Two-Dimensional Collisions

- The momentum is conserved in all directions
- Use subscripts for
  - Identifying the object
  - Indicating initial or final values
  - The velocity components

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

- If the collision is elastic, use conservation of kinetic energy as a second equation
  - Remember, the simpler equation can only be used for one-dimensional situations

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

2D Momentum Conservation

A hunter fires a bullet, which weights 0.05 kg moving to the east at 100 m/s at an 1-kg duck, which is flying to the north at 2 m/s. The bullet stays with the duck after it was hit. What is the speed of the duck (and bullet) after the "collision" in m/s?

A. 5.1 B. 4.8 C. 5.4 D. 6.7 E. 7.0

Practice Problems

Impulse and Momentum

Consider two less-than-desirable options. In the first you are driving 30 mph and crash head-on into an identical car also going 30 mph. In the second option you are driving 30 mph and crash head-on into a stationary brick wall. In neither case does your car bounce off the thing it hits, and the collision time is the same in both cases. Which of these two situations would result in the greatest impact force?

- A) hitting the other car
- B) hitting the brick wall
- C) The force would be the same in both cases.
- D) We cannot answer this question without more information.
- E) None of these is true.

Impulse and Momentum

During a collision with a wall, the velocity of a 0.200-kg ball changes from 20.0 m/s toward the wall to 12.0 m/s away from the wall. If the time the ball was in contact with the wall was 60.0 ms, what was the magnitude of the average force applied to the ball?

- A) 40.0 N
- B) 107 N
- C) 16.7 N
- D) 26.7 N
- E) 13.3 N

Momentum

A 480-kg car moving at 14.4 m/s hits from behind a 570-kg car moving at 13.3 m/s in the same direction. If the new speed of the heavier car is 14.0 m/s, what is the speed of the lighter car after the collision, assuming that any unbalanced forces on the system are negligibly small?

- A) 13.6 m/s
- B) 10.5 m/s
- C) 19.9 m/s
- D) 5.24 m/s