Physics 111: Mechanics
Lecture 9

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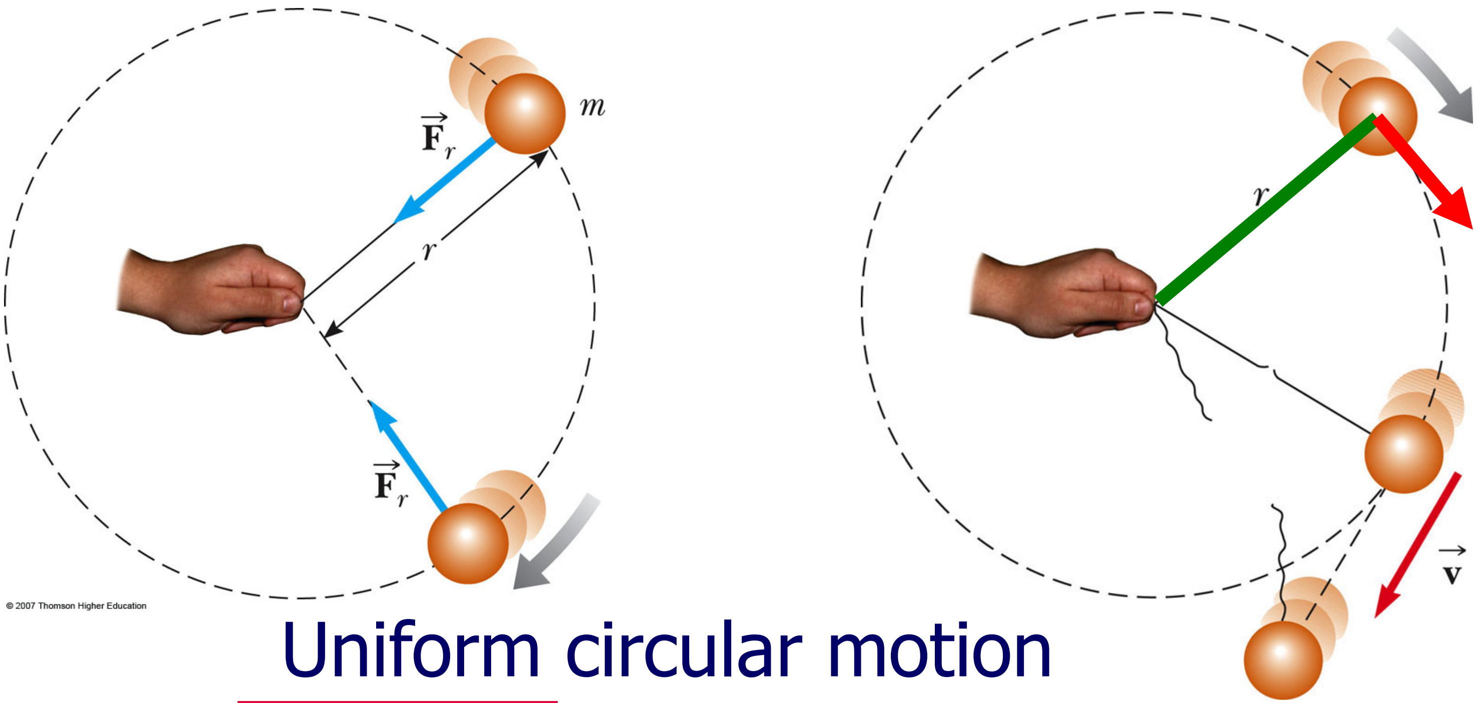
Circular Motion

- 3.4 Motion in a Circle
- 5.4 Dynamics of Circular Motion

If it weren't for the spinning, you would not be thrown up high in the air

If it weren't for the spinning, all the galaxies would collapse into a black hole

Uniform Circular Motion: Definition



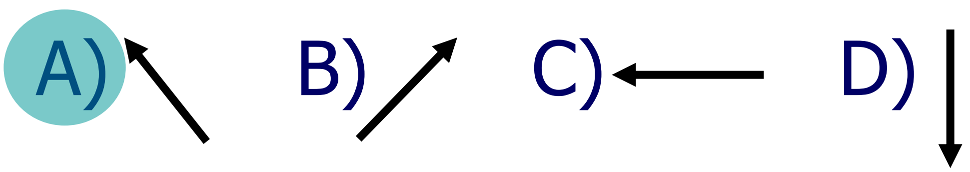
Uniform circular motion

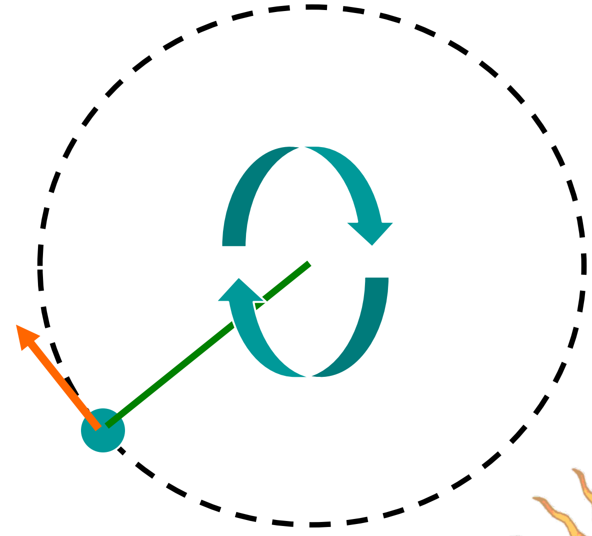
Constant speed, or,
constant magnitude of velocity

Motion along a circle:
Changing direction of velocity

Circular Motion: Velocity

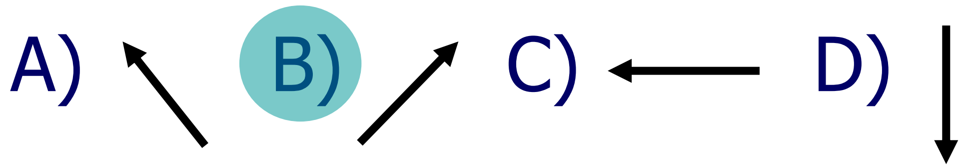
- An object is moving in a clockwise direction around a circle at constant speed. Which vector below represents the direction of the velocity vector when the object is located at that point on the circle?

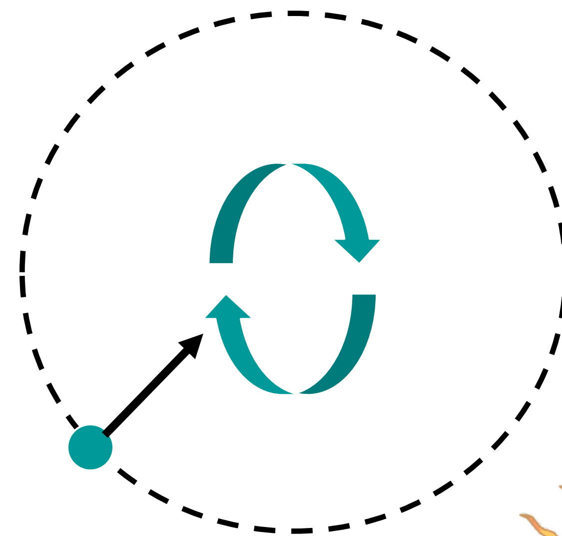




Circular Motion: Acceleration

- An object is moving in a clockwise direction around a circle at constant speed. Which vector below represents the direction of the acceleration vector when the object is located at that point on the circle?





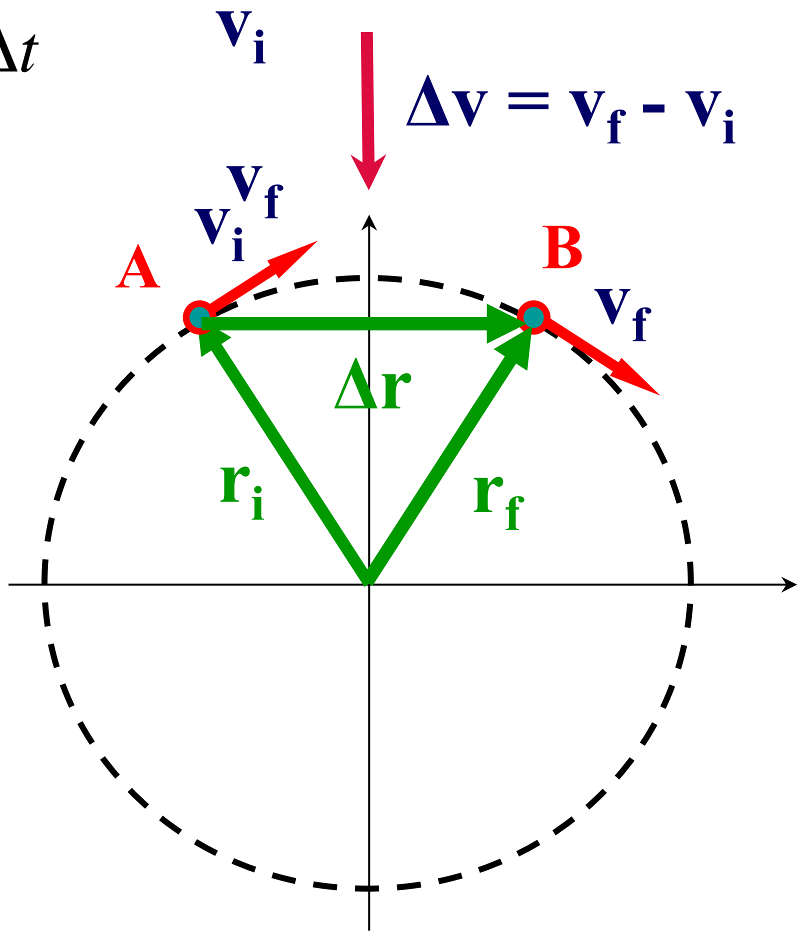
Uniform Circular Motion

- Acceleration: $\vec{a}_{avg} = \Delta\vec{v} / \Delta t$
- Direction: **Centripetal**
- Magnitude:

$$\frac{\Delta v}{v} = \frac{\Delta r}{r} \quad \text{so,} \quad \Delta v = \frac{v\Delta r}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{\Delta r}{\Delta t}\frac{v}{r} = \frac{v^2}{r}$$

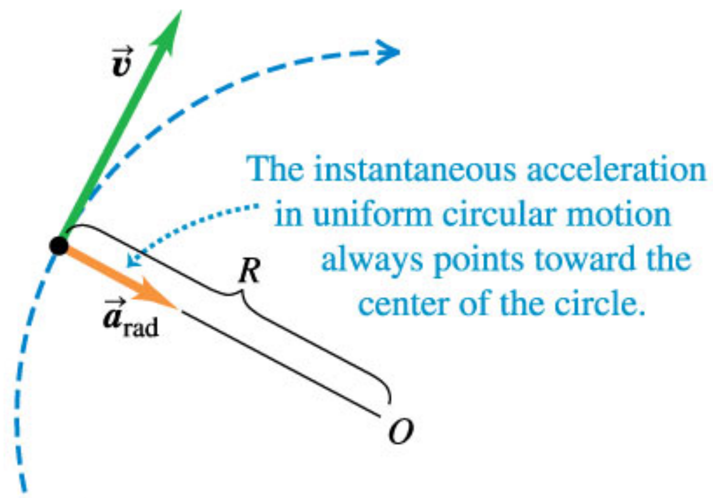
$$a_r = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$



Uniform Circular Motion

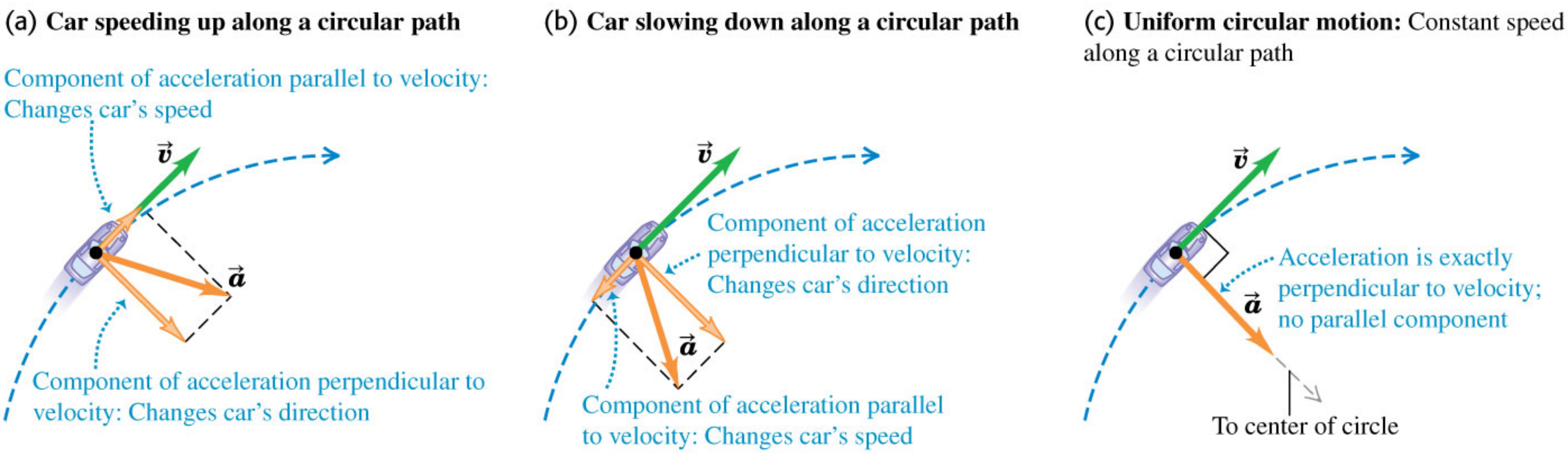
- Velocity:
  - Magnitude: constant v
  - The direction of the velocity is tangent to the circle
- Acceleration:
  - Magnitude: $a_r = v^2 / R$
  - directed toward the center of the circle of motion
- Period:
  - time interval required for one complete revolution of the particle

$$T = 2\pi R / v$$



Non-Uniform Circular Motion

- Uniform circular motion
- Car speeding up along a circular path
- Car slowing down along a circular path



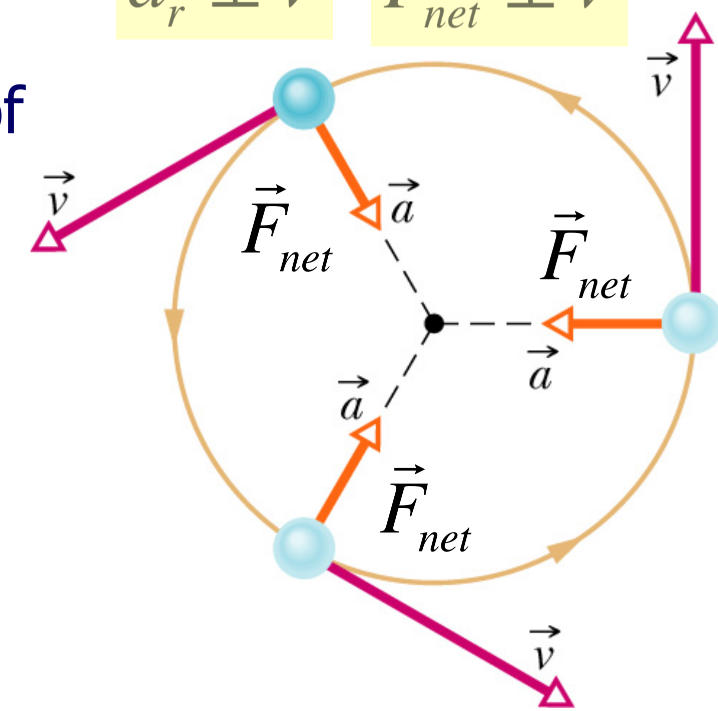
Centripetal Force

- Acceleration: $a_r = \frac{v^2}{R}$
  - Magnitude:
  - Direction: toward the center of the circle of motion
- Force:
  - Start from Newton's 2nd Law
  - Magnitude: $\vec{F}_{net} = m\vec{a}$

$$\vec{F}_{net} = ma_r = \frac{mv^2}{R}$$

  - Direction: toward the center of the circle of motion

$\vec{a}_r \perp \vec{v}$ $\vec{F}_{net} \perp \vec{v}$

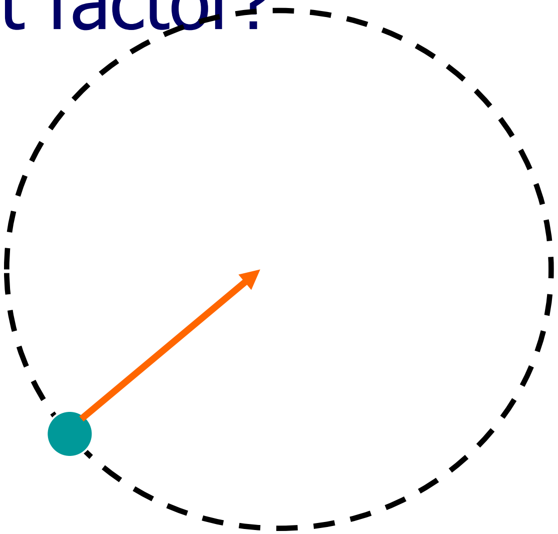


$\vec{a}_r \parallel \vec{F}_{net}$

Centripetal Force

- A motorcycle moves around a curve of radius r with speed v. Later the cycler increases its speed to 4v while traveling along another curve with radius 4r. The centripetal force of the particle has changed by what factor?

A) 8
B) 0.5
C) 2
D) 4
E) unchanged



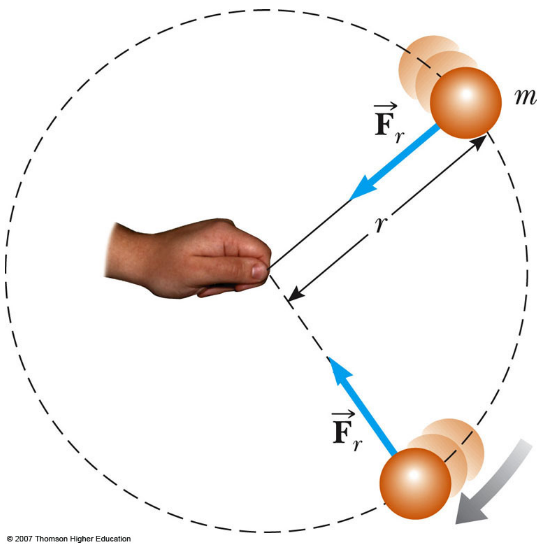
What provide Centripetal Force ?

- Centripetal force is **not** a new kind of force
- Centripetal force stands for any force that keeps an object following a circular path

$$F_c = ma_r = \frac{mv^2}{R}$$

- In our class, centripetal force is likely a combination of
  - Gravitational force **mg**: downward to the ground
  - Normal force **N**: perpendicular to the surface
  - Tension force **T**: along the cord and away from object
  - Static friction force: $f_s^{max} = \mu_s N$

What provide Centripetal Force ?

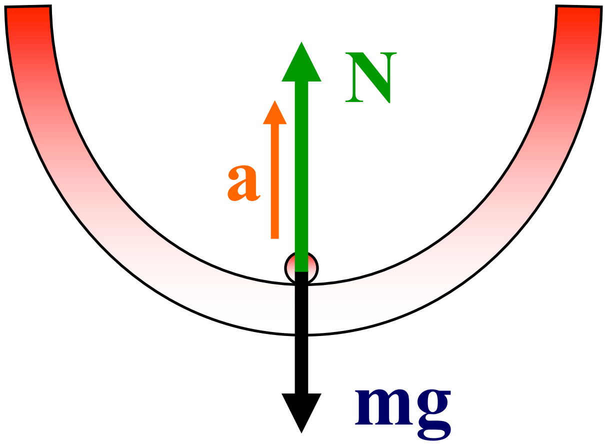


$$F_{net} = T = ma$$

$$T = \frac{mv^2}{r}$$

$$F_{net} = N - mg = ma$$

$$N = mg + m\frac{v^2}{r}$$



Problem Solving Strategy

- **Draw a free body diagram**, showing and labeling all the forces acting on the object(s)
- **Choose a coordinate system** that has one axis perpendicular to the circular path and the other axis tangent to the circular path
- **Find the net force toward the center** of the circular path (this is the force that causes the centripetal acceleration, F_C)
- **Use Newton's second law**
  - The directions will be radial, normal, and tangential
  - The acceleration in the radial direction will be the centripetal acceleration
- **Solve for the unknown(s)**

Level Curves

- The force of static friction directed toward the center of the curve keeps the car moving in a circular path.

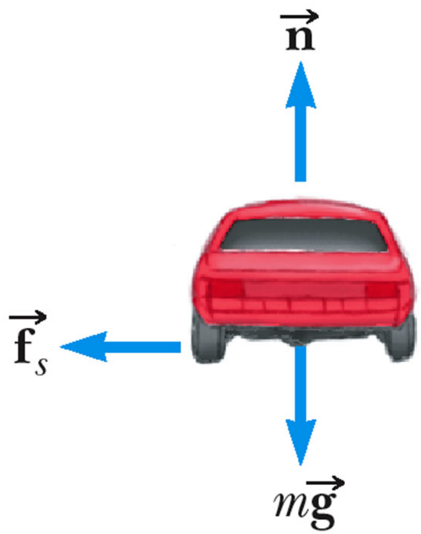
$$f_{s,\max} = \mu_s N = m\frac{v_{\max}^2}{r}$$

$$\sum F_y = N - mg = 0$$

$$N = mg$$

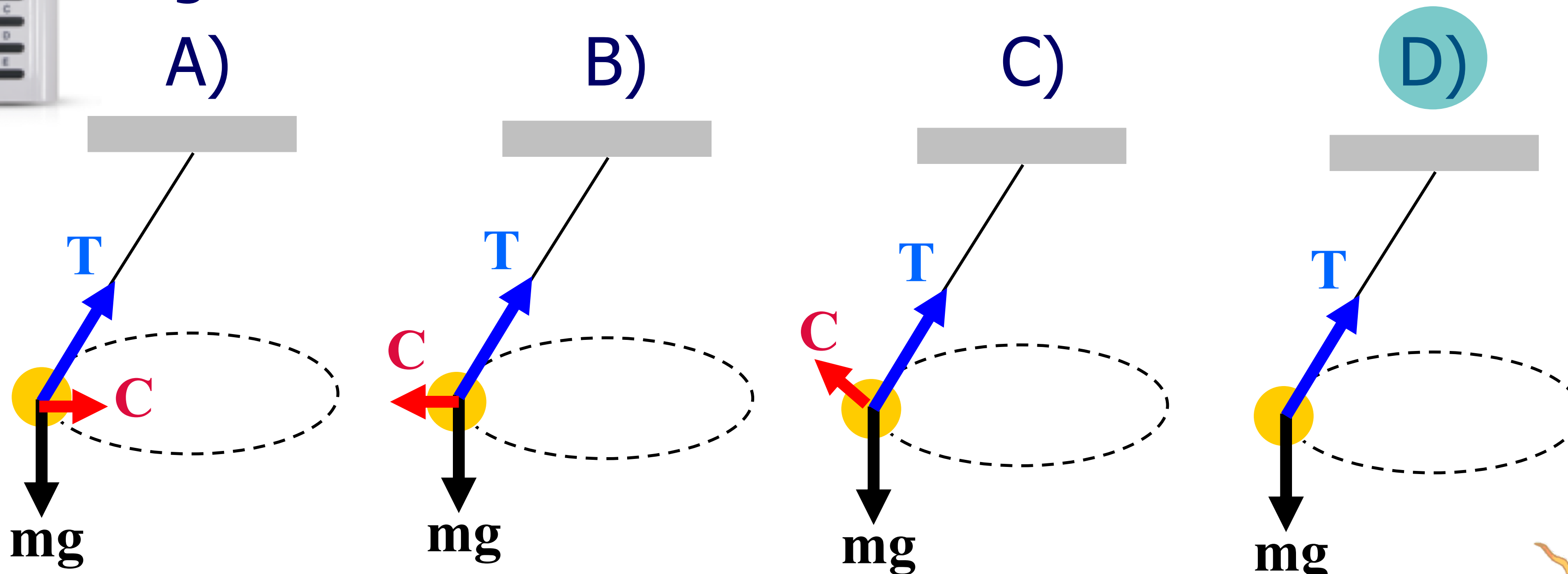
$$v_{\max} = \sqrt{\frac{\mu_s N r}{m}} = \sqrt{\frac{\mu_s m g r}{m}} = \sqrt{\mu_s g r}$$

$$= \sqrt{(0.523)(9.8m/s^2)(35.0m)} = 13.4m/s$$



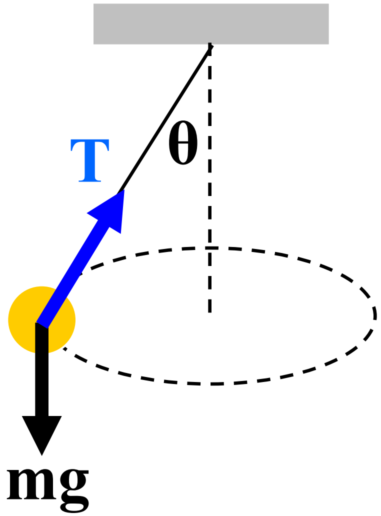
The Conical Pendulum

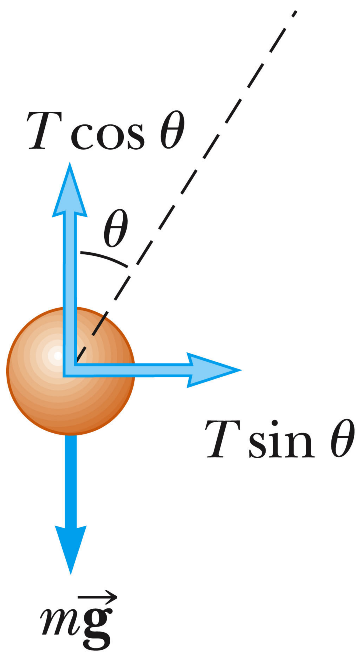
- A small ball is suspended from a string. The ball revolves with constant speed v in a horizontal circle of radius r. Which is the correct free-body diagram for the ball?



The Conical Pendulum

- A small ball of mass m is suspended from a string of length L. The ball revolves with constant speed v in a horizontal circle of radius r. Find an expression for speed v, acceleration a, and period t.





$$m = 5\,kg \quad L = 5\,m \quad r = 2\,m$$

$$\sum F_y = T\cos\theta - mg = 0$$

$$T\cos\theta = mg$$

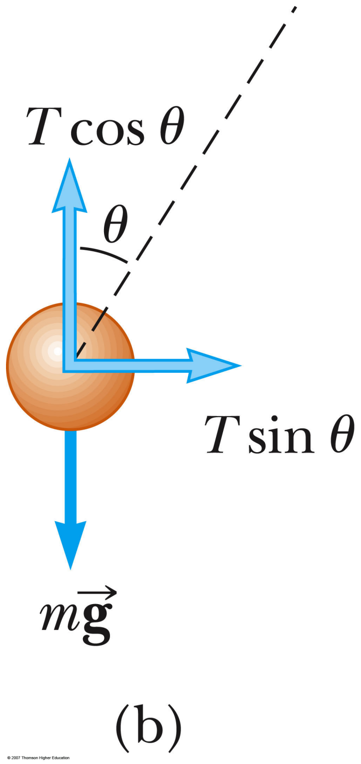
$$\sum F_x = T\sin\theta = ma_c = \frac{mv^2}{r}$$

$$\sin\theta = \frac{r}{L} = 0.4$$

$$\tan\theta = \frac{r}{\sqrt{L^2 - r^2}} = 0.44$$

The Conical Pendulum

- Find v, a, and t



$$m = 5\,kg \quad L = 5\,m \quad r = 2\,m$$

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$$T\cos\theta = mg$$

$$\sum F_x = T\sin\theta = \frac{mv^2}{r}$$

$$\sin\theta = \frac{r}{L} = 0.4$$

$$\tan\theta = \frac{r}{\sqrt{L^2 - r^2}} = 0.44$$

$$T\sin\theta = \frac{mv^2}{r}$$

$$T\cos\theta = mg$$

$$\tan\theta = \frac{v^2}{gr}$$

$$v = \sqrt{rg\tan\theta}$$

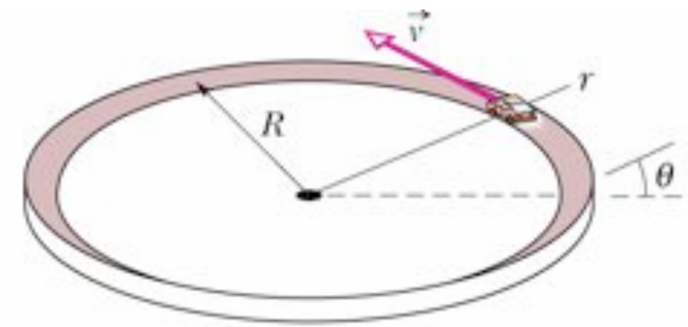
$$v = \sqrt{Lg\sin\theta\tan\theta} = 2.9\ \text{m/s}$$

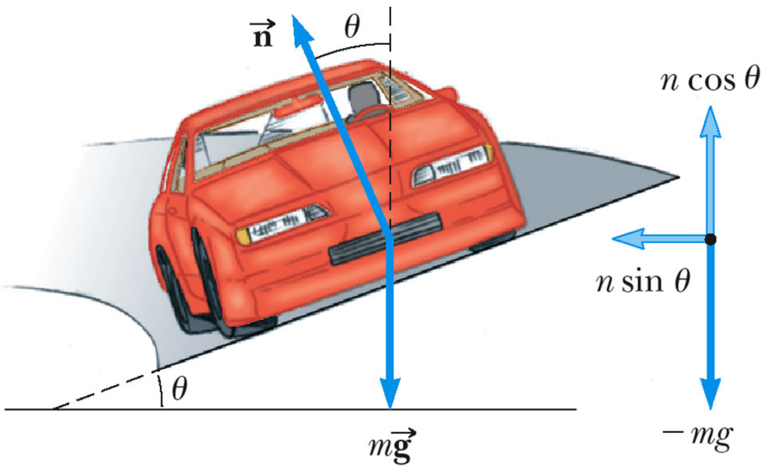
$$a = \frac{v^2}{r} = g\tan\theta = 4.3\ \text{m/s}^2$$

Time to complete one revolution: $T = 2\pi r / v$

Banked Curves

- A car moving at the designated speed can negotiate the curve. Such a ramp is usually banked, which means that the roadway is tilted toward the inside of the curve. Suppose the designated speed for the ramp is 13.4 m/s and the radius of the curve is 35.0 m. At what angle should the curve be banked so that no friction at all is needed?





Banked Curves

$v = 13.4\ \mathrm{m/s} \qquad r = 35.0\ \mathrm{m}$

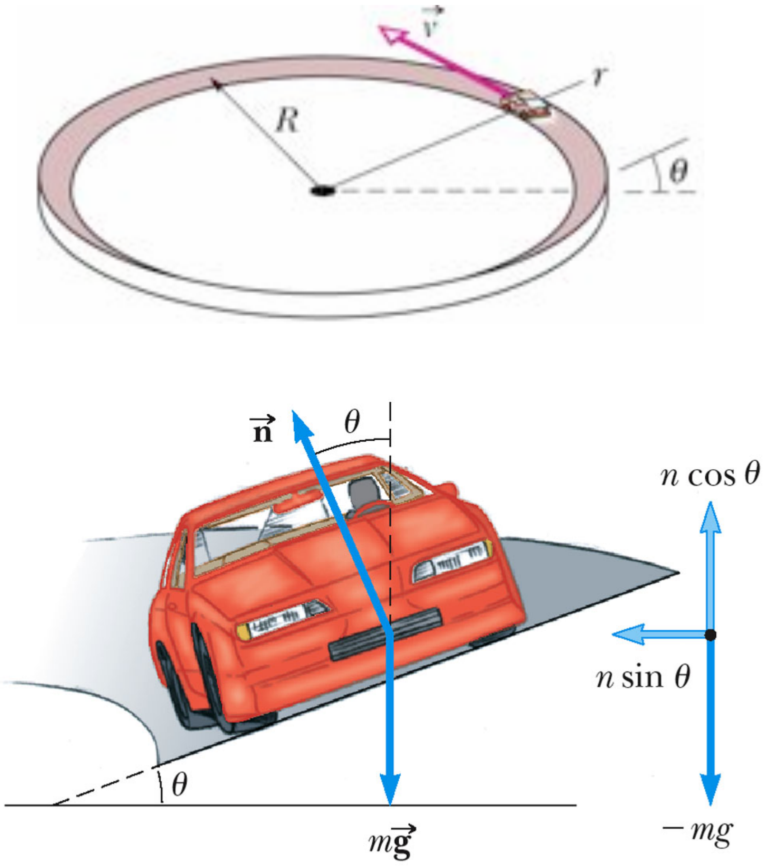
$$\sum F_r = n\sin\theta = ma_c = \frac{mv^2}{r}$$

$$\sum F_y = n\cos\theta - mg = 0$$

$$n\cos\theta = mg$$

$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{13.4\ \mathrm{m/s}}{(35.0\ \mathrm{m})(9.8\ \mathrm{m/s^2})}\right) = 27.6^\circ$$



Practice Problems

Getting off a Parkway ramp

- John is driving his car off the Parkway at 60 mph around a ramp with a curvature of 300 m. The coefficient of friction between the tires and the flat road is 0.20. Will his car slip or not? (1 mile = 1609 m)

A. Yes B. No

Swing a ball

- Maggie is swinging a ball of mass 10.0 kg tied to a thin string in a circle on a frictionless table. The length of the string is 1 m, and the largest force it can sustain is 20 N. What is the maximum speed of the ball it can reach, in m/s, before the string breaks?

A. 14 B. 200 C. 2 D. 1.4