Physics 111: Mechanics
Lecture 12

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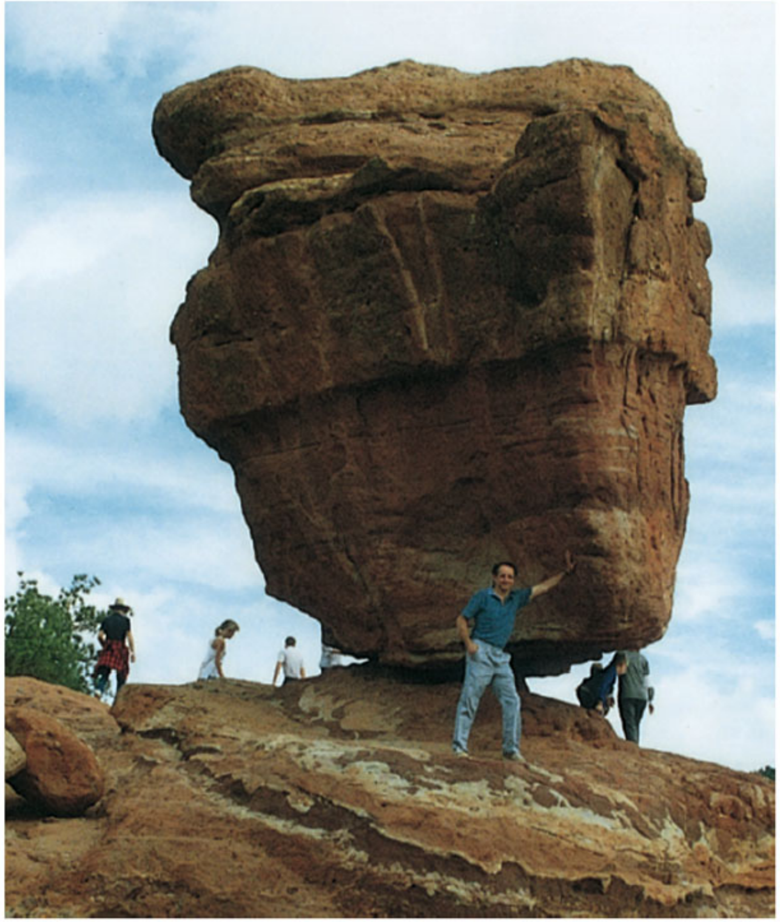
Chapter 11 Equilibrium and Elasticity

- 11.1 Conditions for Equilibrium
- 11.2 Center of Gravity
- 11.3 Solving Rigid-Body Equilibrium Problems
- 11.4* Stress, Strain, and Elastic Moduli
- 11.5* Elasticity and Plasticity



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Static and Dynamic Equilibrium

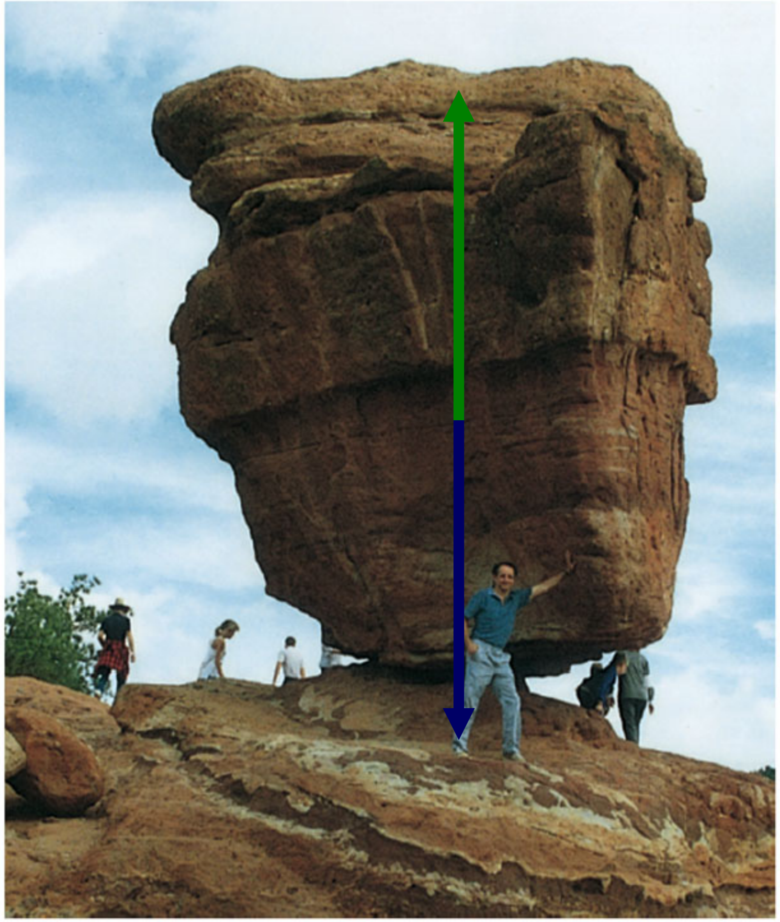
- Equilibrium implies the object is at rest (static) or its center of mass moves with a constant velocity (dynamic)
- This chapter deals only with the special case in which linear and angular velocities are both equal to zero, called "**static equilibrium**" : $v_{CM} = 0$ and $\omega = 0$
- Examples
  - *Book on table*
  - *Puck sliding on ice in a constant velocity*
  - *Ceiling fan – off*
  - *Ceiling fan – on*
  - *Ladder leaning against wall (foot in groove)*

Conditions for Equilibrium

- The first condition of equilibrium is a statement of translational equilibrium
- The net external force on the object must equal zero

$$\vec{F}_{net} = \sum \vec{F}_{ext} = m\vec{a} = 0$$

- It states that the translational acceleration of the object's center of mass must be zero



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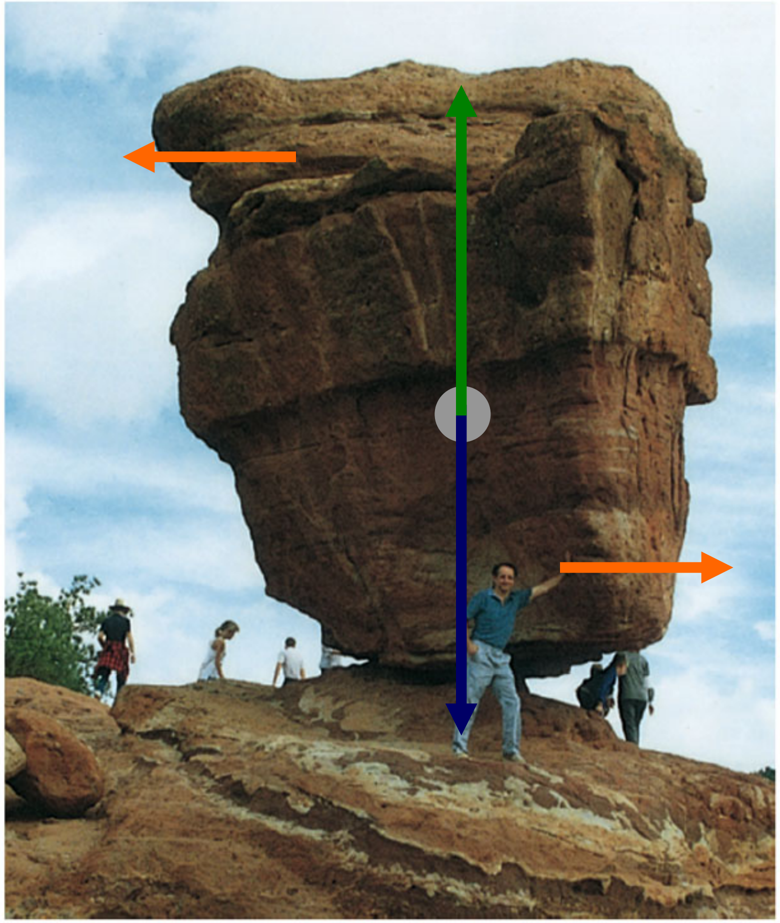
Conditions for Equilibrium

- If the object is modeled as a particle, then this is the only condition that must be satisfied

$$\vec{F}_{net} = \sum \vec{F}_{ext} = 0$$

- For an extended object to be in equilibrium, a second condition must be satisfied
- This second condition involves the rotational motion of the extended object



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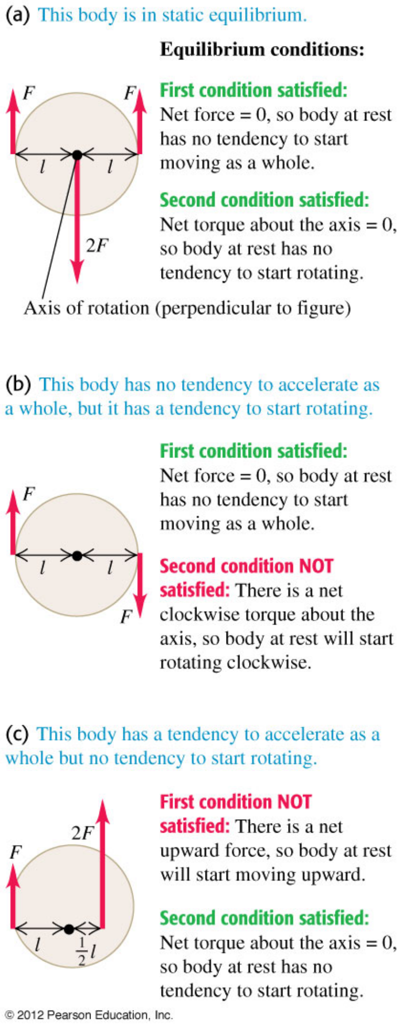
Conditions for Equilibrium

- The **second condition** of equilibrium is a statement of rotational equilibrium
- The net external torque on the object must equal zero

$$\vec{\tau}_{net} = \sum \vec{\tau}_{ext} = I\vec{\alpha} = 0$$

- It states the angular acceleration of the object to be zero
- This must be true for **any** axis of rotation



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Conditions for Equilibrium

- The net force equals zero $\sum \vec{\mathbf{F}} = 0$
  - If the object is modeled as a particle, then this is the only condition that must be satisfied
- The net torque equals zero $\sum \vec{\tau} = 0$
  - This is needed if the object cannot be modeled as a particle
- These conditions describe the rigid objects in equilibrium analysis model

Static Equilibrium

- Consider a light rod subject to the two forces of equal magnitude as shown in figure. Which one of the following is correct:

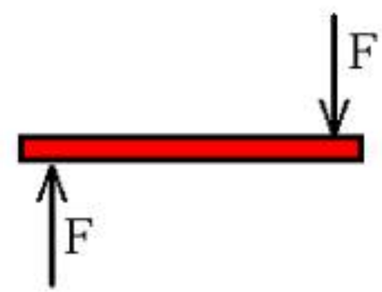
(A) The object is in force equilibrium but not torque equilibrium.

(B) The object is in torque equilibrium but not force equilibrium

(C) The object is in both force equilibrium and torque equilibrium

(D) The object is in neither force equilibrium nor torque equilibrium

(E) The object is in force equilibrium. Need more conditions to determine whether or not in torque equilibrium.



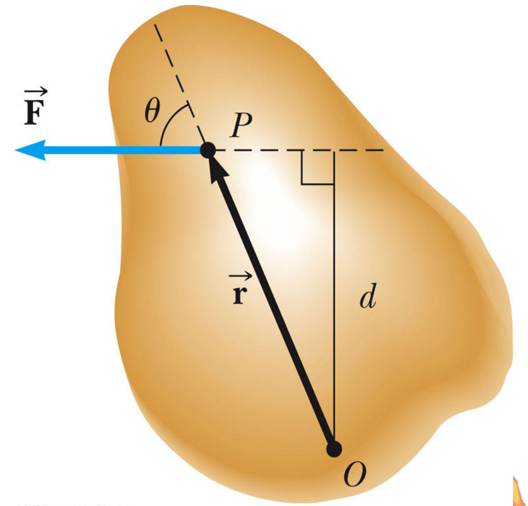
Equilibrium Equations

- Equation 1: $\vec{F}_{net} = \sum \vec{F}_{ext} = 0:\ F_{net,x} = 0\ \ F_{net,y} = 0$ ~~$F_{net,z} = 0$~~
- Equation 2: $\vec{\tau}_{net} = \sum \vec{\tau}_{ext} = 0:$ ~~$\tau_{net,x} = 0$~~ ~~$\tau_{net,y} = 0$~~ $\tau_{net,z} = 0$
- We will restrict the applications to situations in which all the forces lie in the xy plane
- There are three resulting equations

$$F_{net,x} = \sum F_{ext,x} = 0$$

$$F_{net,y} = \sum F_{ext,y} = 0$$

$$\tau_{net,z} = \sum \tau_{ext,z} = 0$$



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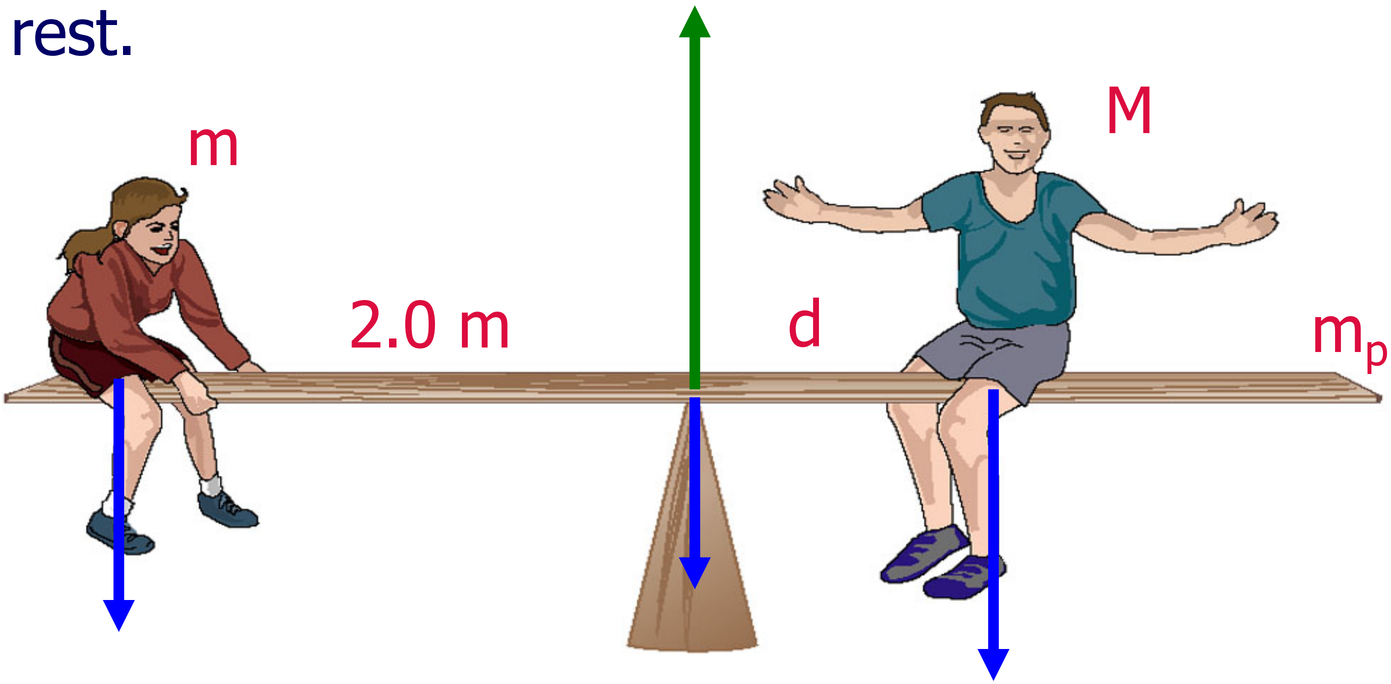
- A seesaw consisting of a uniform board of mass m_{pl} and length L supports at rest a father and daughter with masses M and m, respectively. The support is under the center of gravity of the board, the father is a distance d from the center, and the daughter is a distance 2.00 m from the center.
- A) Find the magnitude of the upward force **n** exerted by the support on the board.
- B) Find where the father should sit to balance the system at rest.



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A) Find the magnitude of the upward force **n** exerted by the support on the board.

B) Find where the father should sit to balance the system at rest.

$$F_{net,y} = n - mg - Mg - m_{pl}g = 0$$

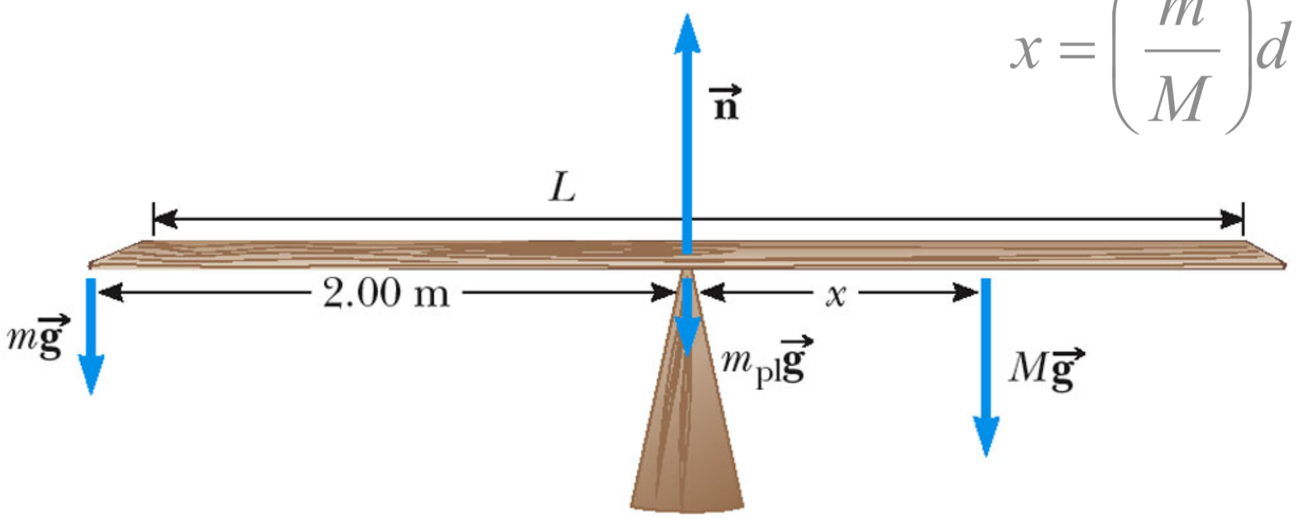
$$n = mg + Mg + m_{pl}g$$

$$\tau_{net,z} = \tau_d + \tau_f + \tau_{pl} + \tau_n$$

$$= mgd - Mgx + 0 + 0 = 0$$

$$mgd = Mgx$$

$$x = \left(\frac{m}{M}\right)d = \frac{2m}{M} < 2.00\text{ m}$$



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$$F_{net,x} = \sum F_{ext,x} = 0$$

$$F_{net,y} = \sum F_{ext,y} = 0$$

$$\tau_{net,z} = \sum \tau_{ext,z} = 0$$

Axis of Rotation

- The net torque is about an axis through any point in the xy plane
- For static equilibrium, does it matter which axis you choose for calculating torques?
- NO. The choice of an axis is arbitrary
- If an object is in equilibrium and the net torque is zero about one axis, then the net torque must be zero about any other axis
- We should be smart to choose a rotation axis to simplify problems

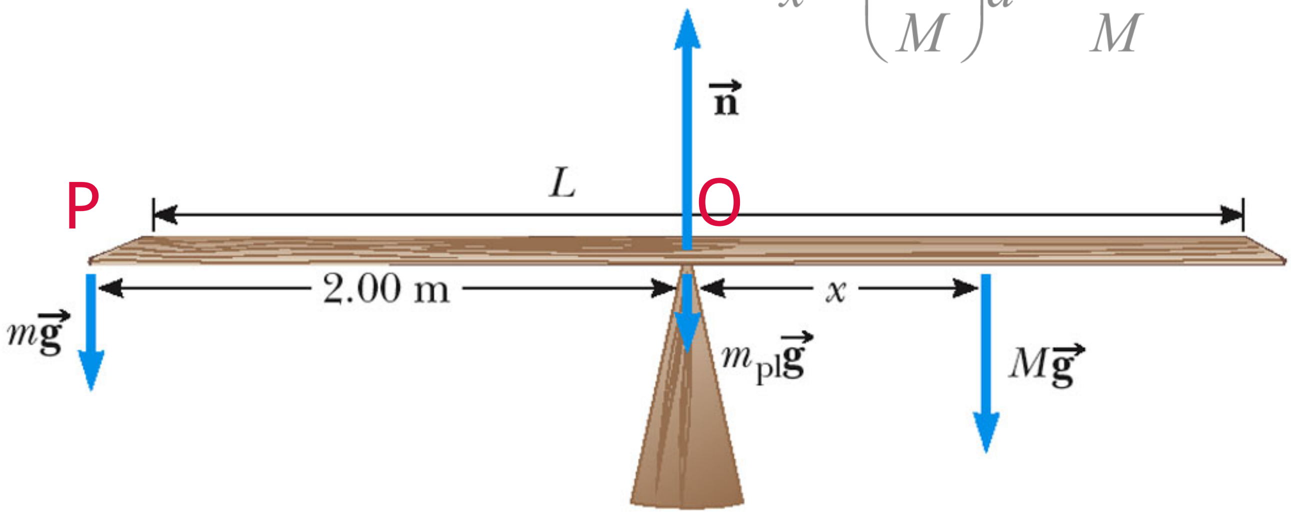
B) Find where the father should sit to balance the system at rest.

Rotation axis O

$$\tau_{net,z} = \tau_d + \tau_f + \tau_{pl} + \tau_n$$
$$= mgd - Mgx + 0 + 0 = 0$$
$$mgd = Mgx$$
$$x = \left(\frac{m}{M}\right)d = \frac{2m}{M}$$

Rotation axis P

$$\tau_{net,z} = \tau_d + \tau_f + \tau_{pl} + \tau_n$$
$$= 0 - Mg(d+x) - m_{pl}gd + nd = 0$$
$$-Mgd - Mgx - m_{pl}gd + (Mg + mg + m_{pl}g)d = 0$$
$$mgd = Mgx$$
$$x = \left(\frac{m}{M}\right)d = \frac{2m}{M}$$



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$$F_{net,x} = \sum F_{ext,x} = 0$$
$$F_{net,y} = \sum F_{ext,y} = 0$$
$$\tau_{net,z} = \sum \tau_{ext,z} = 0$$

Center of Gravity

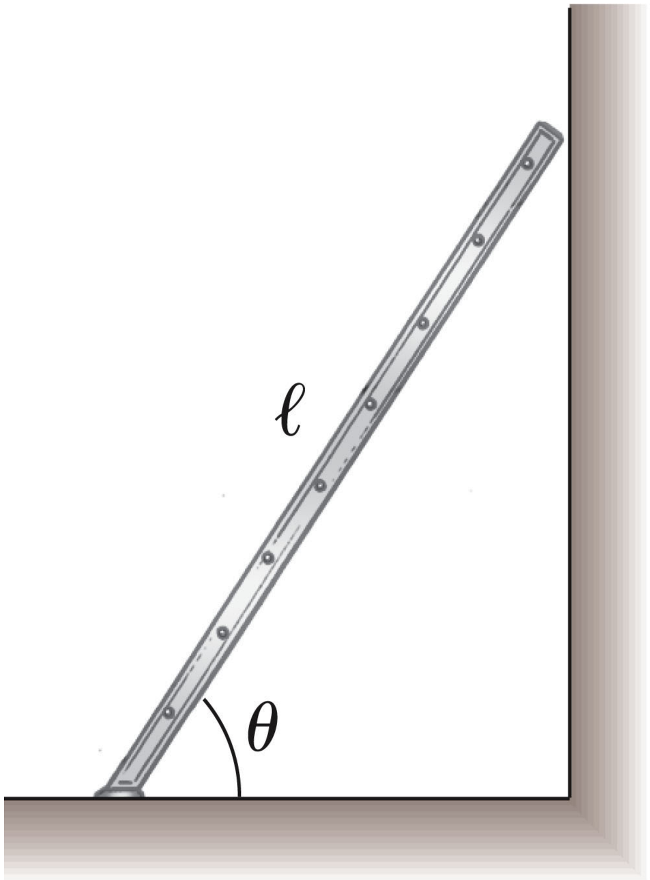
- The torque due to the gravitational force on an object of mass M is the force Mg acting at the center of gravity of the object
- The center of gravity of the object coincides with its center of mass (if the variation in gravitation acceleration over the vertical dimension of the body can be neglected)
- If the object is homogeneous and symmetrical, the center of gravity is at its geometric center

Use an extension ladder safely

A video tutorial: https://www.youtube.com/watch?v=GKNG_Ymf_dk

A Classic Example: Ladder

- A uniform ladder of length l rests against a smooth, vertical wall. The mass of the ladder is m, and the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$. The wall is frictionless. Find the minimum angle θ at which the ladder does not slip.



(a)

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Problem-Solving Strategy 1

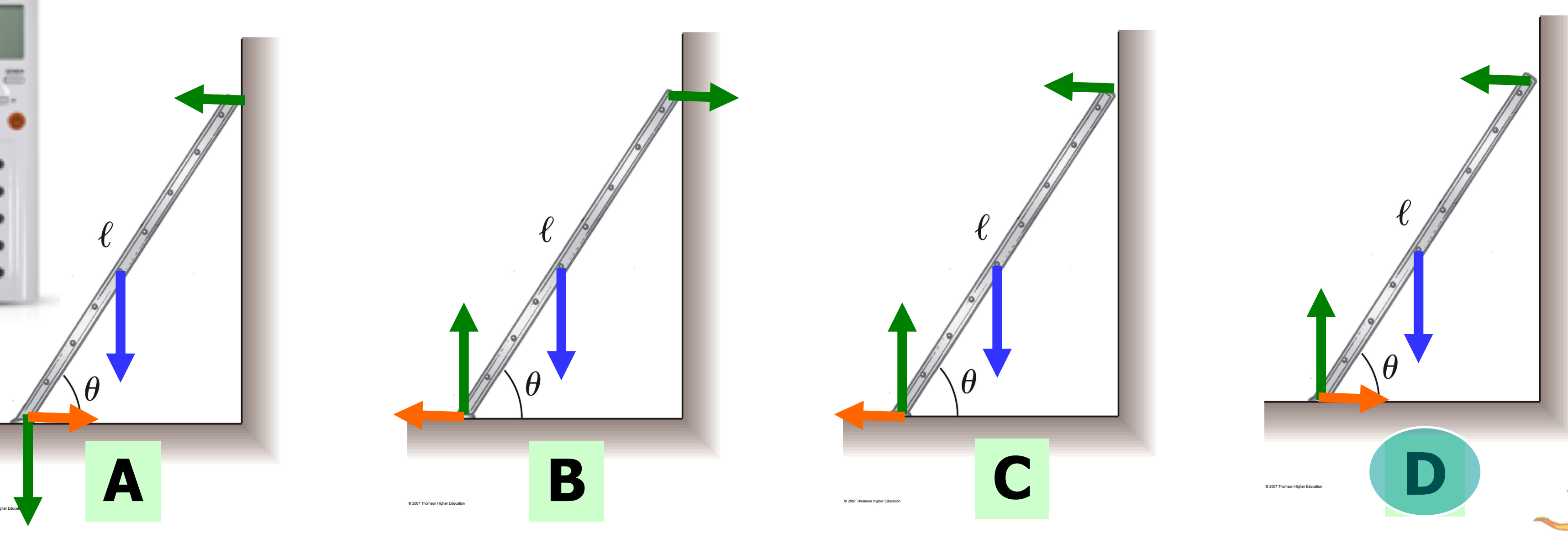
- Draw sketch, decide what is in or out the system
- Draw a free body diagram
- Show and label all external forces acting on the object
- Indicate the locations of all the forces
- Establish a convenient coordinate system
- Find the components of the forces along the two axes
- Apply the first condition for equilibrium
- Be careful of signs

$$F_{net,x} = \sum F_{ext,x} = 0$$

$$F_{net,y} = \sum F_{ext,y} = 0$$

Which free-body diagram is correct?

- A uniform ladder of length l rests against a smooth, vertical wall. The mass of the ladder is m, and the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$. gravity: blue, friction: orange, normal: green



- A uniform ladder of length l rests against a smooth, vertical wall. The mass of the ladder is m, and the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$. Find the minimum angle θ at which the ladder does not slip.

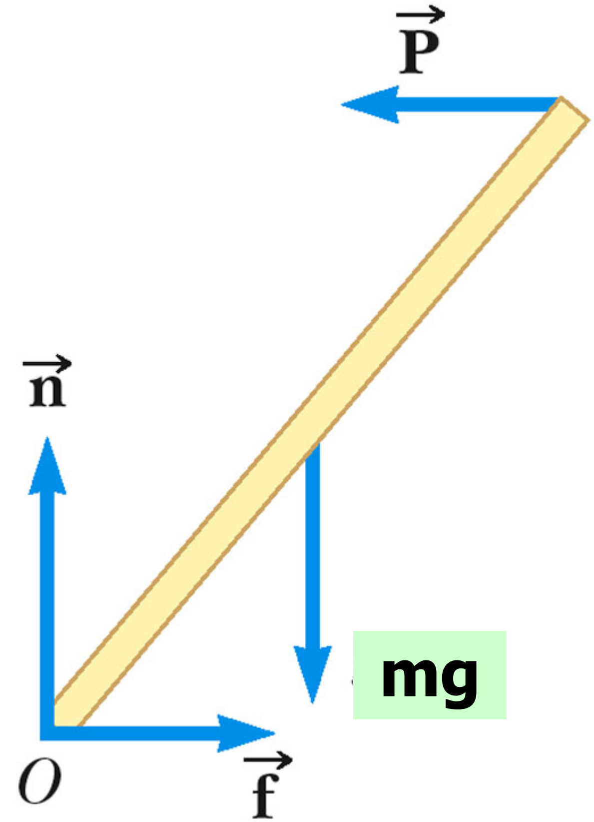
$$\sum F_x = f_x - P = 0$$

$$\sum F_y = n - mg = 0$$

$$P = f_x$$

$$n = mg$$

$$P = f_{x,\max} = \mu_s n = \mu_s mg$$



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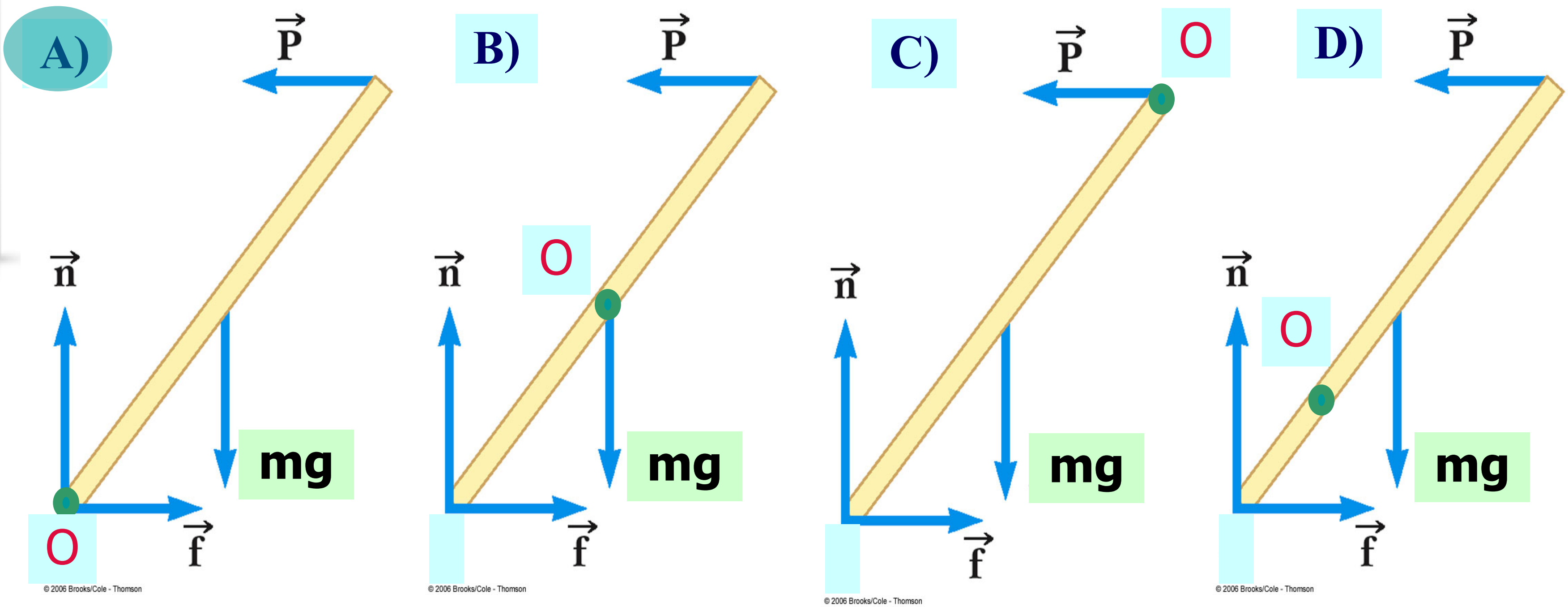
Problem-Solving Strategy 2

- Choose a convenient axis for calculating the net torque on the object
  - Remember the choice of the axis is arbitrary
- Choose an origin that simplifies the calculations as much as possible
  - A force that acts along a line passing through the origin produces a zero torque
- Be careful of sign with respect to rotational axis
  - positive if force tends to rotate object in CCW
  - negative if force tends to rotate object in CW
  - zero if force is on the rotational axis
- Apply the second condition for equilibrium $\tau_{net,z} = \sum \tau_{ext,z} = 0$

Choose an origin O that simplifies the calculations as much as possible?



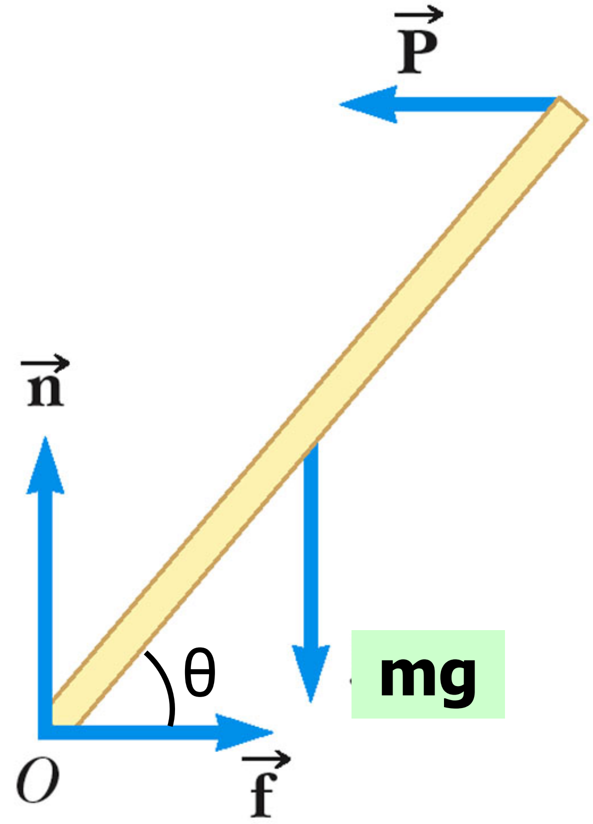
- A uniform ladder of length *l* rests against a smooth, vertical wall. The mass of the ladder is m, and the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$. Find the minimum angle θ at which the ladder does not slip.

$$\sum \tau_O = \tau_n + \tau_f + \tau_g + \tau_P$$

$$= 0 + 0 + Pl\sin\theta_{\min} - mg\frac{l}{2}\cos\theta_{\min} = 0$$

$$\frac{\sin\theta_{\min}}{\cos\theta_{\min}} = \tan\theta_{\min} = \frac{mg}{2P} = \frac{mg}{2\mu_s mg} = \frac{1}{2\mu_s}$$

$$\theta_{\min} = \tan^{-1}\left(\frac{1}{2\mu_s}\right) = \tan^{-1}\left[\frac{1}{2(0.4)}\right] = 51^\circ$$



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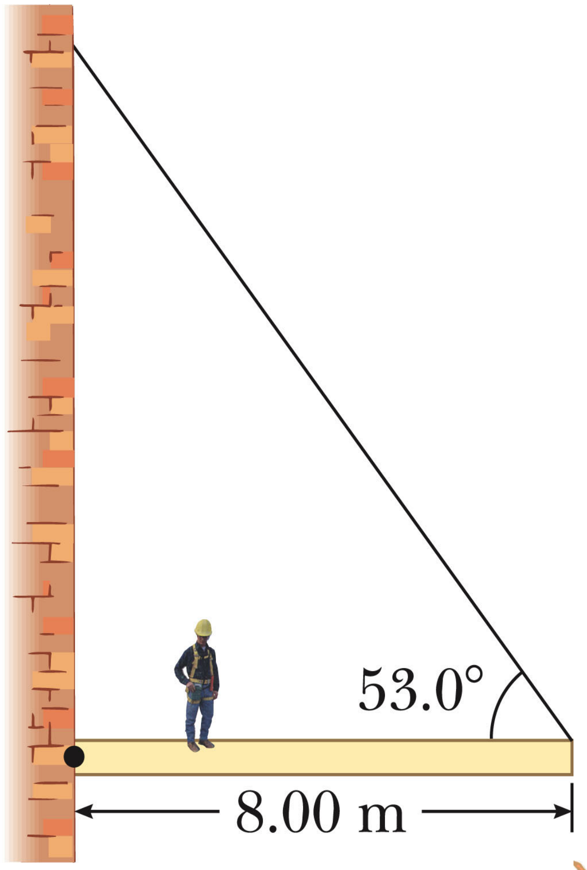
Problem-Solving Strategy 3

- The two conditions of equilibrium will give a system of equations
- Solve the equations simultaneously
- Make sure your results are consistent with your free body diagram
- If the solution gives a negative for a force, it is in the opposite direction to what you drew in the free body diagram
- Check your results to confirm

$$F_{net,x} = \sum F_{ext,x} = 0$$
$$F_{net,y} = \sum F_{ext,y} = 0$$
$$\tau_{net,z} = \sum \tau_{ext,z} = 0$$

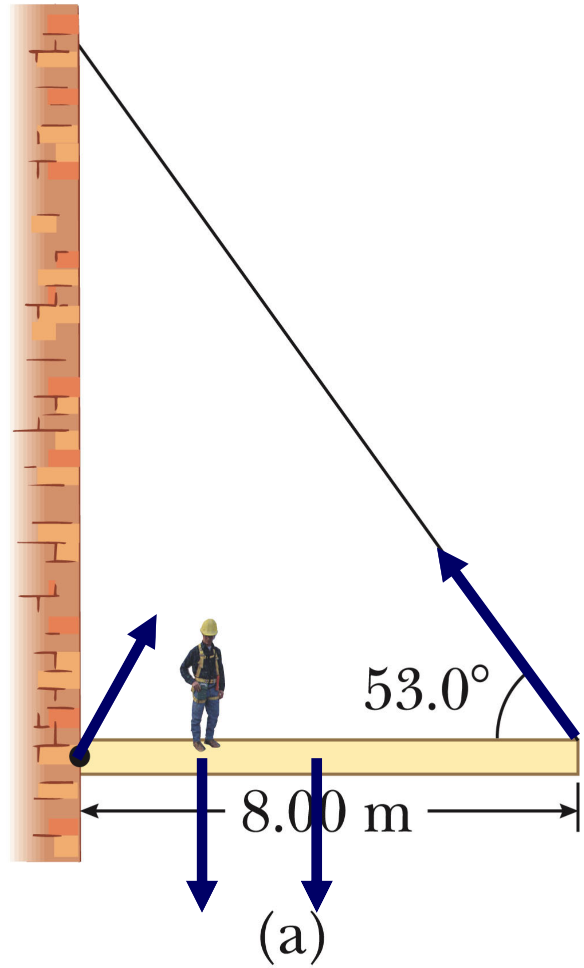
Another Example: Horizontal Beam

- A uniform horizontal beam with a length of l = 8.00 m and a weight of W_b = 200 N is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle of $\phi = 53^\circ$ with the beam. A person of weight W_p = 600 N stands a distance d = 2.00 m from the wall. Find the tension in the cable as well as the magnitude and direction of the force exerted by the wall on the beam.



Understand the problem

- The beam is uniform
  - So the center of gravity is at the geometric center of the beam
- How many forces are there acting on the beam?
- What are their locations and directions?
- Draw a free body diagram



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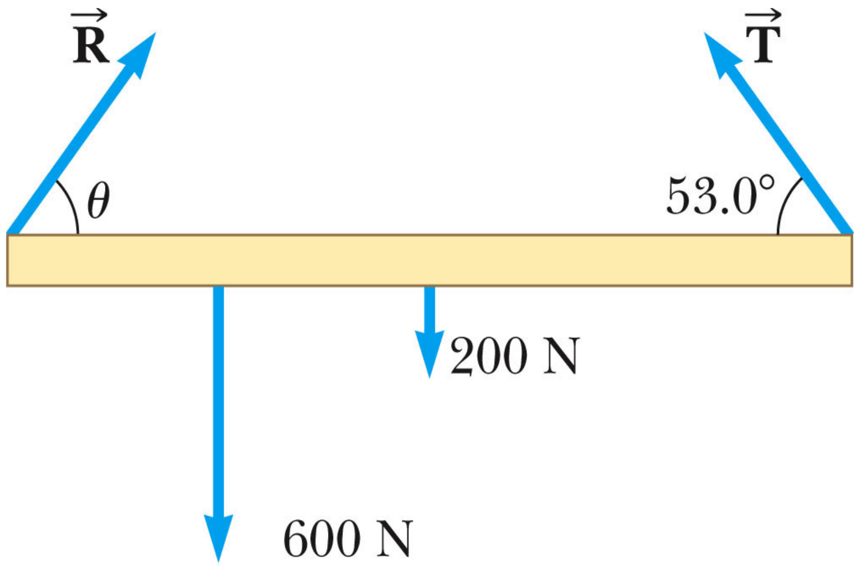
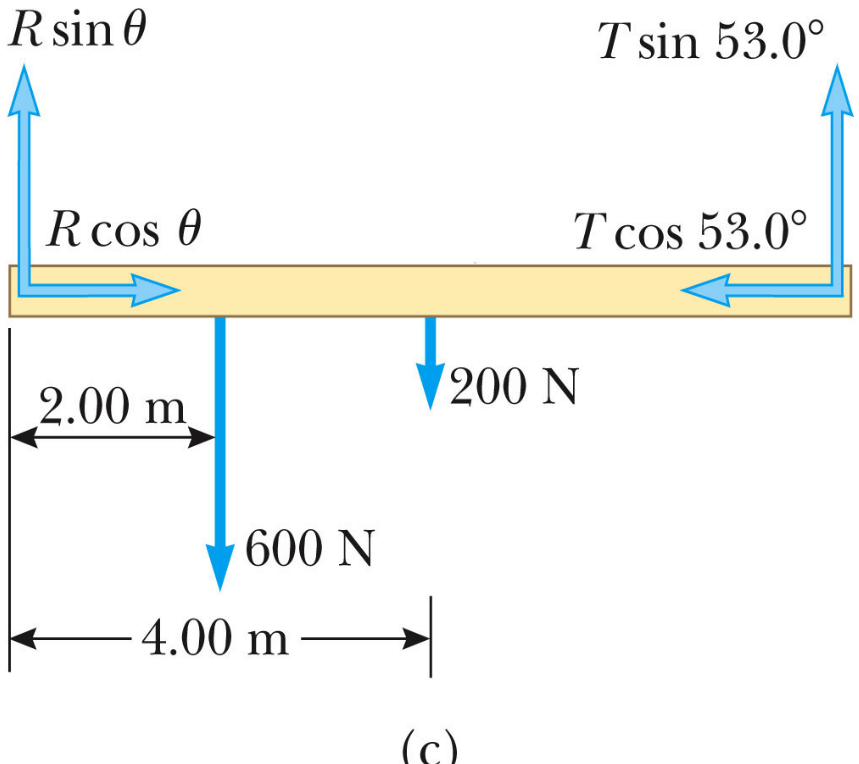
Free Body Diagram

- Analyze
  - Use the pivot in the problem (at the wall) as the pivot
    - This will generally be the easiest
  - Note there are three unknowns (T, R, θ)



Free Body Diagram (cont'd)

- The forces can be broken into components in the free body diagram
- Apply the two conditions of equilibrium to obtain three equations
- Solve for the unknowns



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Horizontal Beam: Solution

$$\sum \tau_z = (T\sin\phi)(l) - W_p d - W_b(\frac{l}{2}) = 0$$

$$T = \frac{W_p d + W_b(\frac{l}{2})}{l\sin\phi} = \frac{(600N)(2m) + (200N)(4m)}{(8m)\sin 53^\circ} = 313N$$

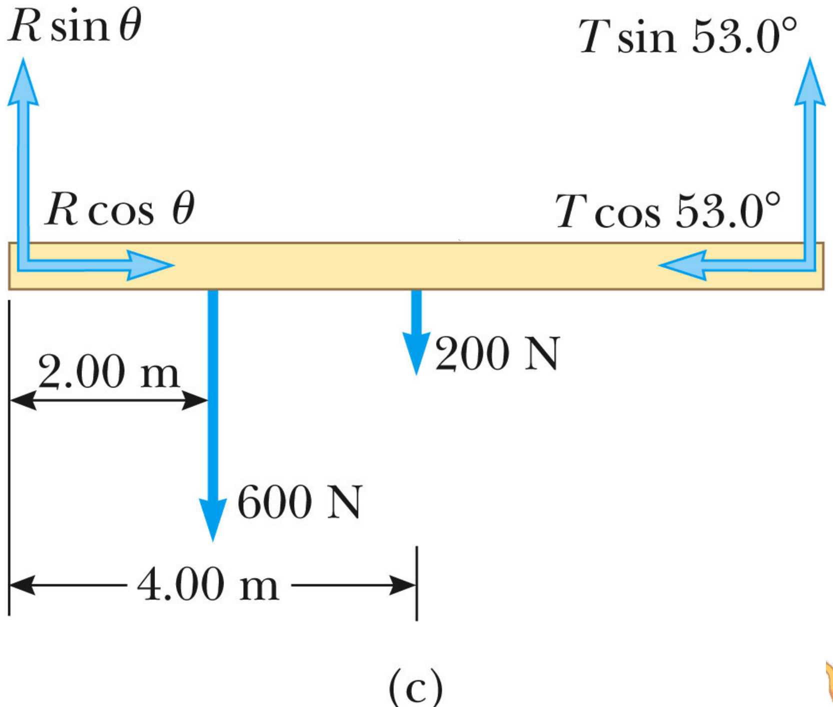
$$\sum F_x = R\cos\theta - T\cos\phi = 0$$

$$\sum F_y = R\sin\theta + T\sin\phi - W_p - W_b = 0$$

$$\frac{R\sin\theta}{R\cos\theta} = \tan\theta = \frac{W_p + W_b - T\sin\phi}{T\sin\phi}$$

$$\theta = \tan^{-1}\left(\frac{W_p + W_b - T\sin\phi}{T\sin\phi}\right) = 71.7^\circ$$

$$R = \frac{T\cos\phi}{\cos\theta} = \frac{(313N)\cos 53^\circ}{\cos 71.7^\circ} = 581N$$



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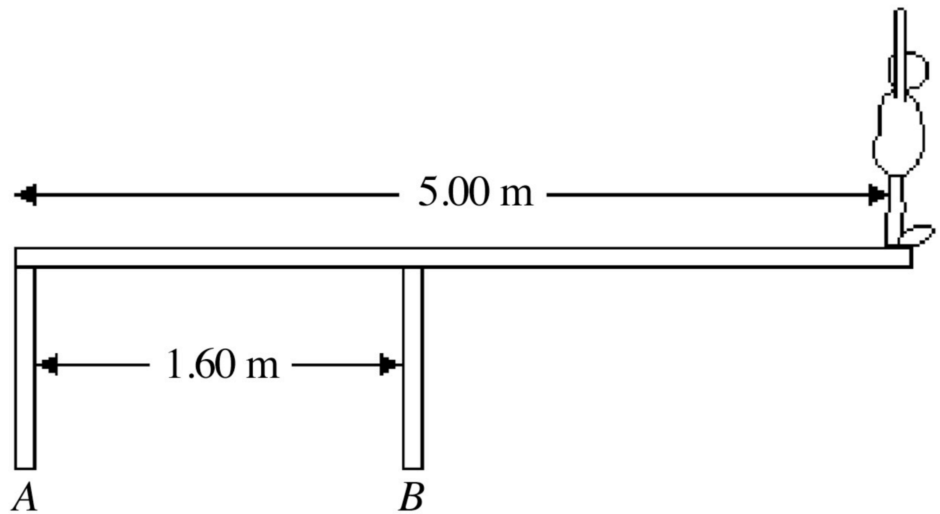
Practice Problems

An 82.0 kg-diver stands at the edge of a light 5.00-m diving board, which is supported by two narrow pillars 1.60 m apart, as shown in the figure. Find the magnitude and direction of the force exerted on the diving board

(a) by pillar *A*.

(b) by pillar *B*.



First, what is the direction of the forces?

a) A upward, B downward
b) A downward, B downward
c) A downward, B upward
d) A upward, B upward

Answer:

Pillar A 1.71 kN downwards
Pillar B 2.51 kN upwards

In the figure, the horizontal lower arm has a mass of 2.8 kg and its center of gravity is 12 cm from the elbow joint pivot. How much force F_M must the vertical extensor muscle in the upper arm, located 2.5 cm away from the elbow joint, exert on the lower arm to hold a 7.5 kg shot put?

A) 100 N
B) 500 N
C) 750 N
D) 1000 N
E) 1500 N