Physics 111: Mechanics
Lecture 13

Bin Chen

NJIT Physics Department

Chapter 12 Fluid Mechanics

- 12.1 Density
- 12.2 Pressure in a Fluid
- 12.3 Buoyancy
- 12.4 Fluid Flow
- 12.5 Bernoulli's Equation
- 12.6* Viscosity and Turbulence

Density

- The **density** of a material is its mass per unit volume:
- SI unit of density is kg/m^3
- Objects made of the same material have the same density even though they may have different masses and different volumes
- The **specific gravity** of a material is its density compared to that of water at 4°C (better named "relative density")

$$\rho = \frac{m}{V}$$

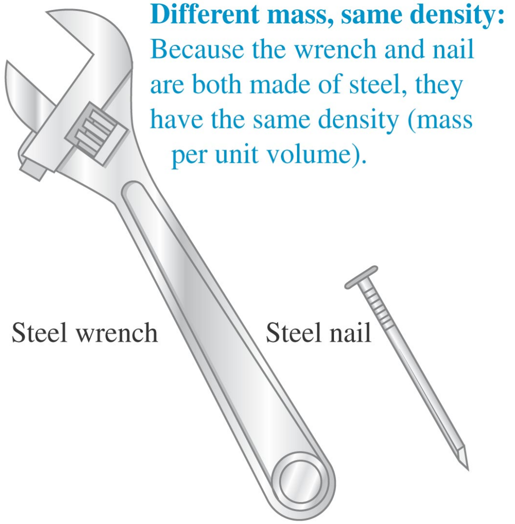


Table 12.1 Densities of Some Common Substances

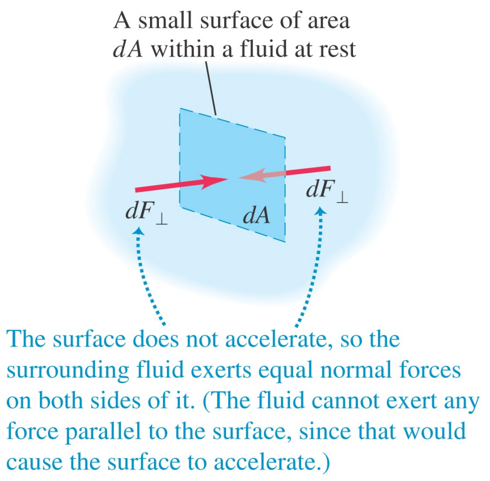
Material	Density (kg/m^3)*	Material	Density (kg/m^3)*
Air (1 atm, 20°C)	1.20	Iron, steel	7.8×10^3
Ethanol	0.81×10^3	Brass	8.6×10^3
Benzene	0.90×10^3	Copper	8.9×10^3
Ice	0.92×10^3	Silver	10.5×10^3
Water	1.00×10^3	Lead	11.3×10^3
Seawater	1.03×10^3	Mercury	13.6×10^3
Blood	1.06×10^3	Gold	19.3×10^3
Glycerine	1.26×10^3	Platinum	21.4×10^3
Concrete	2×10^3	White dwarf star	10^{10}
Aluminum	2.7×10^3	Neutron star	10^{18}

*To obtain the densities in grams per cubic centimeter, simply divide by 10^3.

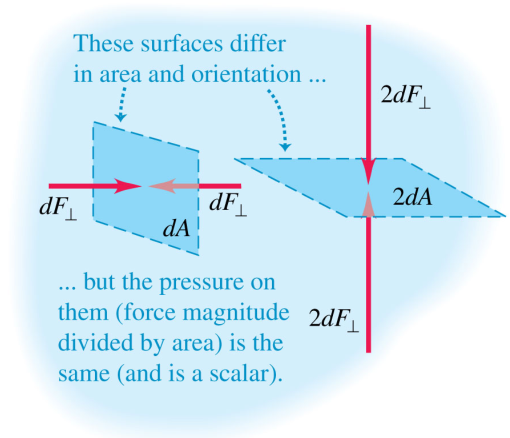
Pressure in a Fluid

- The pressure in a fluid is the normal force per unit area:

$$p = \frac{dF_\perp}{dA}$$



- SI unit of pressure is the pascal
  - 1 pascal = 1 Pa = 1 N/m^2
  - 1 bar = 10^5 Pa
  - 1 millibar = 100 Pa
- Atmospheric pressure is the pressure of the earth's atmosphere
- Normal atmospheric pressure at sea level is 1 atmosphere (atm)
  - 1 atm = 1.013×10^5 Pa = 1.013 bar



Pressure at Depth in a Fluid

- Consider an element of a fluid at rest with area A and thickness dy

$$dw = gdm = g\rho dV = \rho gAdy$$

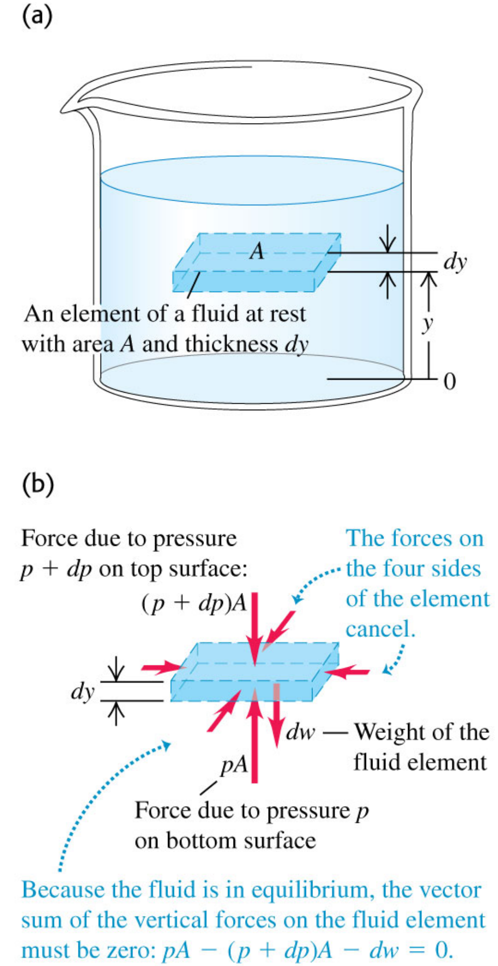
- The fluid element is in equilibrium

$$\sum F_y = 0$$

$$pA - (p + dp)A - \rho gAdy = 0$$

$$\frac{dp}{dy} = -\rho g$$

- When y increases, p decreases. As we move upward in the fluid, pressure decreases



Pressure at Depth in a Fluid

- If p_1 and p_2 are the pressures at elevations y_1 and y_2, respectively

$$\frac{dp}{dy} = -\rho g$$

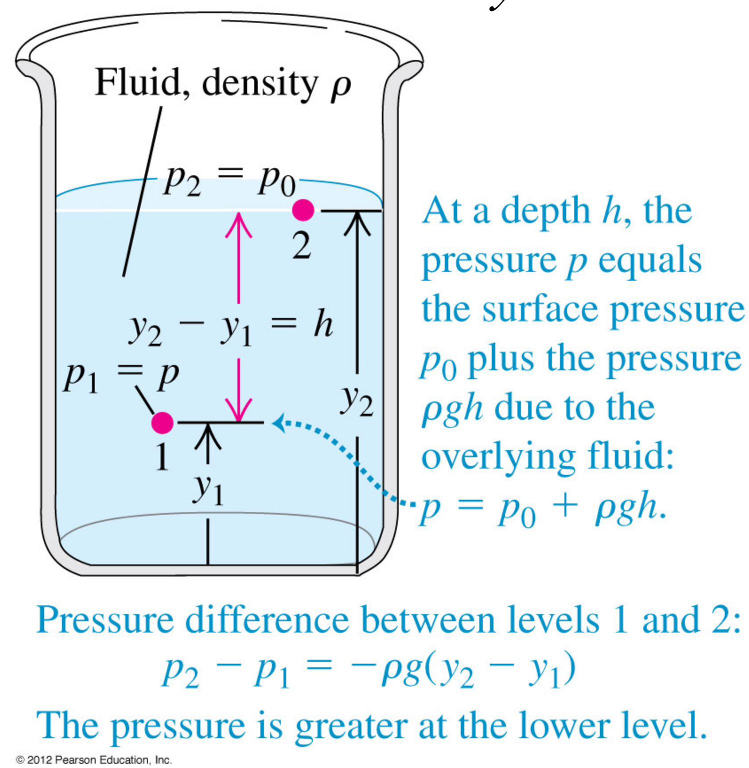
$$p_2 - p_1 = -\rho g(y_2 - y_1)$$

- Take point 2 at the surface of the fluid, where the pressure is p_0

$$p_0 - p_1 = -\rho g(y_2 - y_1) = -\rho g h$$

- The pressure p at a depth h is greater than p_0 *at the surface by an amount* ?gh

$$p = p_0 + \rho g h$$



Pressure at Depth in a Fluid

- Rank the pressures at different levels from big to small

A) D, C, A, B, E

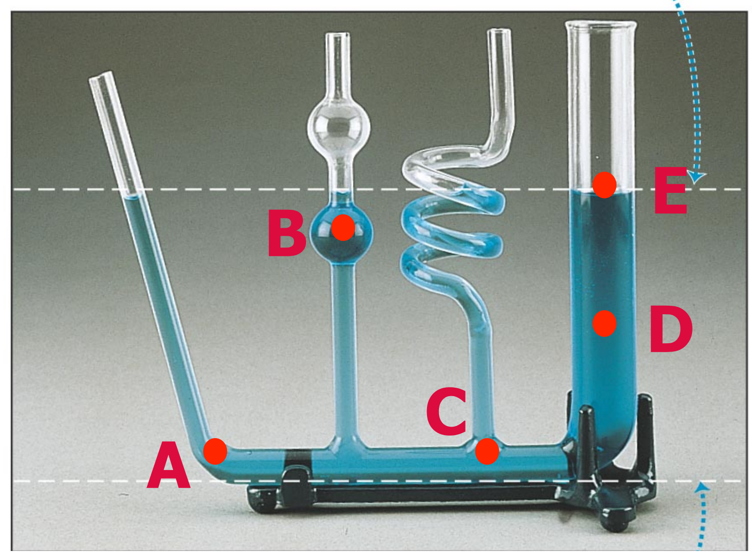
B) E, B, D, C, A

C) A and C tie, D, B, E

D) D, C and A tie, B, E

E) Neel more information

The pressure at the top of each liquid column is atmospheric pressure, p_0.



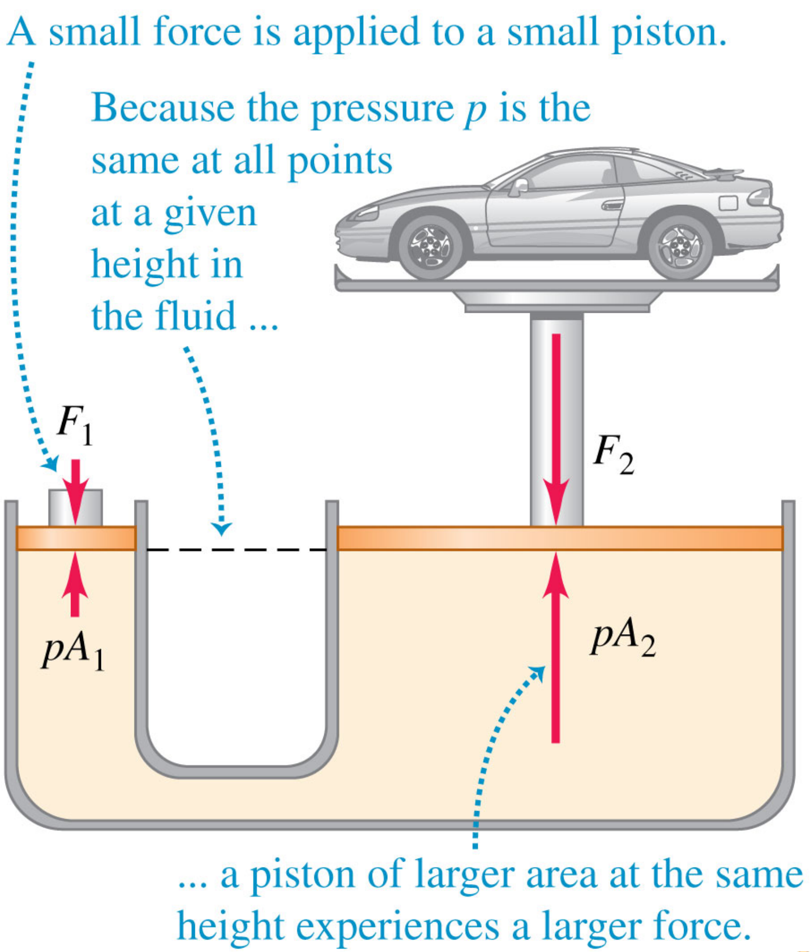
$$p = p_0 + \rho g h$$

Pascal's law

- Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel

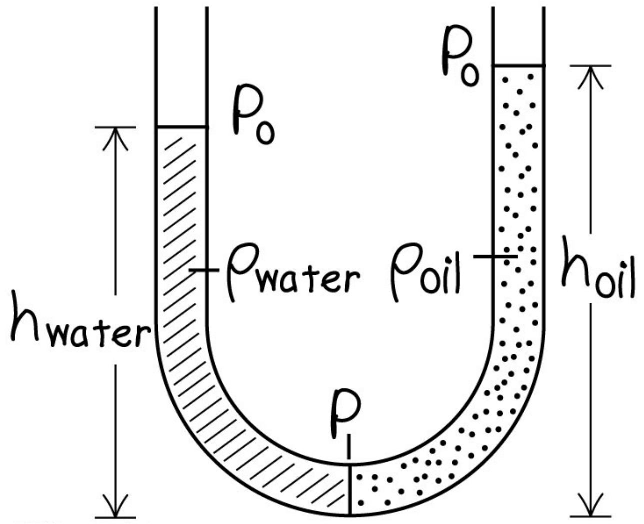
$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2} \qquad F_2 = \frac{A_2}{A_1} F_1$$

- The hydraulic lift is a force-multiplying device with a multiplication factor equal to the ratio of the areas of the two pistons



A tale of two fluids

- A manometer tube is partially filled with water. Oil is poured into the left arm of the tube until the oil-water interface is at the midpoint of the tube as shown. Both arms of the tube are open to the air. Find the relationship between the heights h_{oil} and h_{water}.



$$p = p_0 + \rho_{water} g h_{water}$$

$$p = p_0 + \rho_{oil} g h_{oil}$$

$$h_{oil} = \frac{\rho_{water}}{\rho_{oil}} h_{water}$$

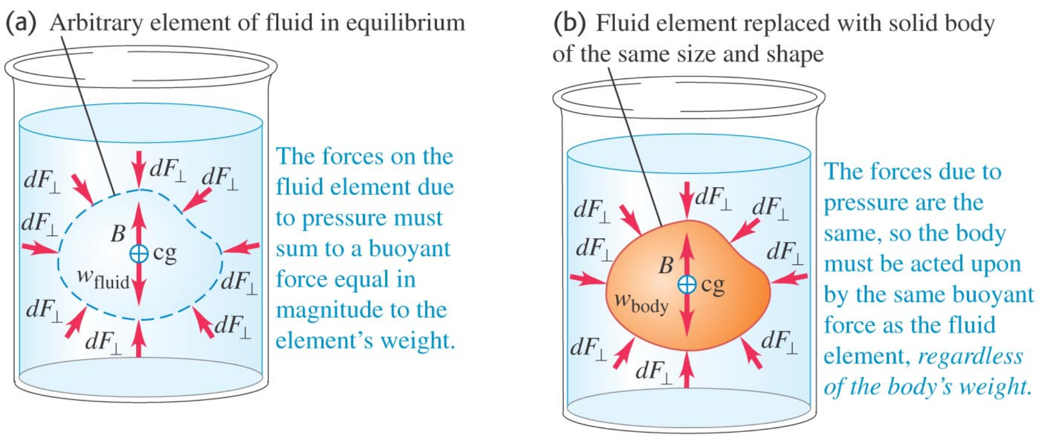
Buoyancy

- **Archimedes' Principle:** When a body is completely or partially immersed in a fluid, the fluid exerts an upward force (the "buoyant force") on the body equal to the weight of the fluid displaced by the body
- Buoyant force B

$$B = \rho_{fluid} V g$$

- Gravity of the body

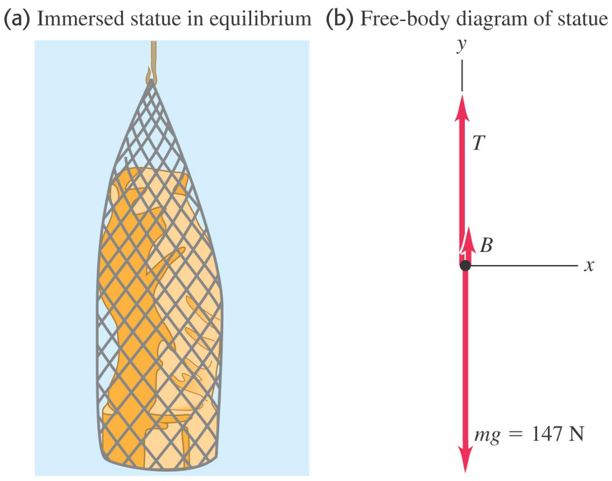
$$w = \rho_{body} V g$$



- When the body is less dense than the fluid, it floats

Buoyancy

- A 15.0-kg solid gold statue is raised from the sea bottom as shown in the figure. What is the tension in the hoisting cable (assumed massless) when the statue is (a) at rest and completely underwater and (b) at rest and completely out of the water?



$$V = \frac{m_{statue}}{\rho_{statue}} = \frac{15.0\text{ kg}}{19.3\times10^3\text{ kg/m}^3} = 7.77\times10^{-4}\text{ m}^3$$

$$B_{sw} = w_{sw} = m_{sw}g = \rho_{sw}Vg = 7.84\text{ N}$$

$$\sum F_y = B_{sw} + T_{sw} + (-m_{statue}g) = 0$$

$$T_{sw} = m_{statue}g - B_{sw} = 139\text{ N}$$

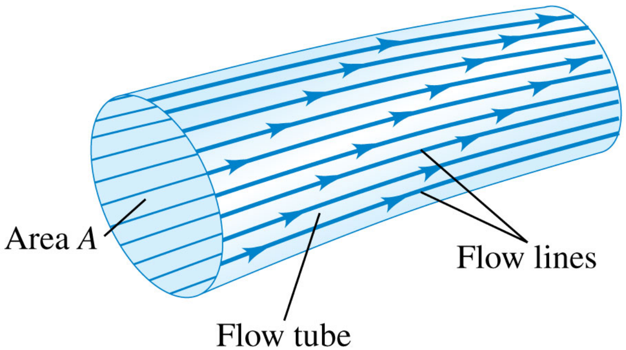
$$B_{air} = m_{air}g = \rho_{air}Vg = 9.1\times10^{-3}\text{ N}$$

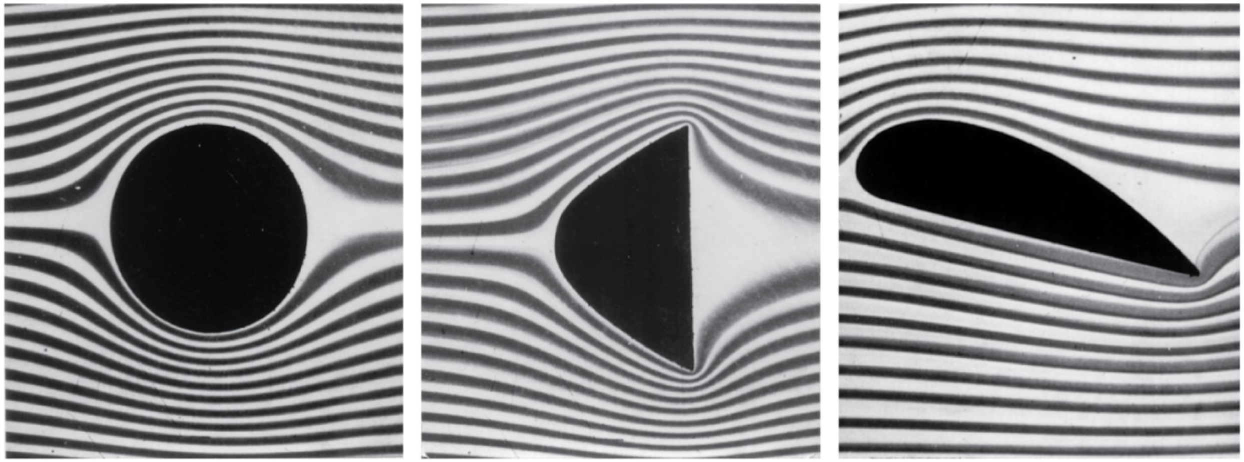
$$\sum F_y = B_{air} + T_{air} + (-m_{statue}g) = 0$$

$$T_{air} = m_{statue}g - B_{air} = m_{statue}g = 147\text{ N}$$

Fluid Flow

- Fluid flow can be extremely complex.
- Idealized Models: **ideal fluid** is a fluid that is *incompressible* and has *no internal friction*
  - Its density cannot change
  - Viscosity: laminar flow





Continuity Equation

- The figure at the right shows a flow tube with changing cross-sectional area from A_1 to A_2
- The mass flowing into the tube across A_1 in time dt equal to the mass flowing out across A_2 in time dt

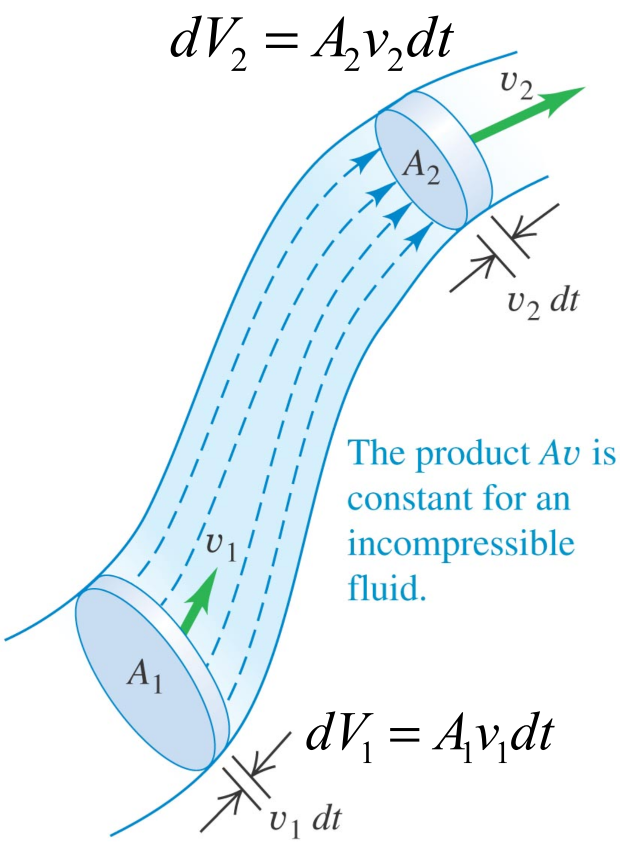
$$\rho A_1 v_1 dt = \rho A_2 v_2 dt$$

- Continuity equation for an incompressible fluid:

$$A_1 v_1 = A_2 v_2$$

- Continuity equation for a compressible fluid:

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$



Bernoulli's Equation

- Bernoulli's equation relates the pressure, flow speed, and height for flow of an ideal, incompressible fluid
- The net work dW done on this fluid element by the surrounding fluid during this displacement is

$$dW = p_1 A_1 ds_1 - p_2 A_2 ds_2 = (p_1 - p_2) dV$$

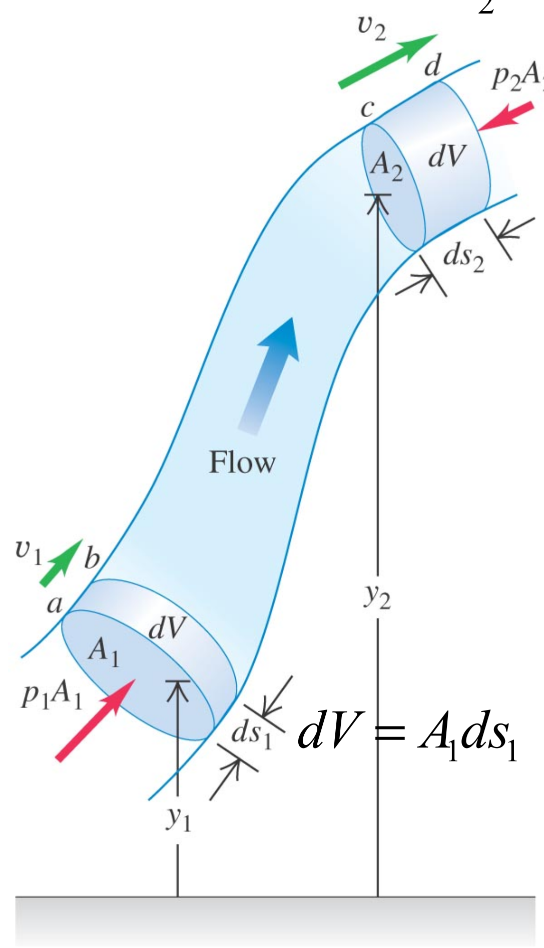
- The net change in Kinetic energy dK during time dt is

$$dK = \frac{1}{2}\rho dV (v_2^2 - v_1^2)$$

- The net change in potential energy dU during time dt is

$$dU = \rho dV g (y_2 - y_1)$$

$$dV = A_2 ds_2$$



$$dV = A_1 ds_1$$

Bernoulli's Equation

- The work-energy theorem gives $dW = dK + dU$

$$(p_1 - p_2)dV = \frac{1}{2}\rho dV(v_2^2 - v_1^2) + \rho g dV(y_2 - y_1)$$

- The work done on a unit volume of fluid by the surrounding fluid is equal to the sum of the changes in kinetic and potential energies per unit volume that occur during the flow

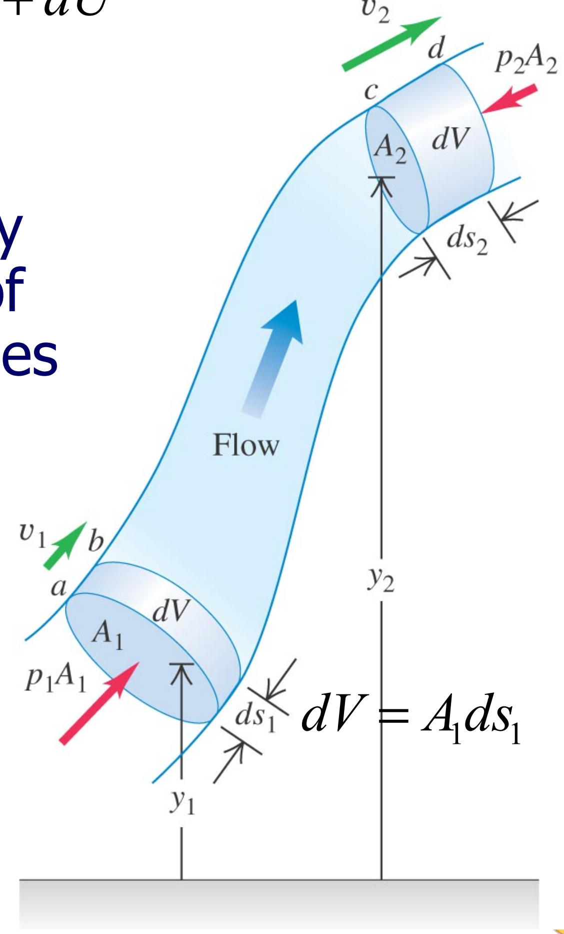
$$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)$$

- Bernoulli's equation

$$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$p + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$$

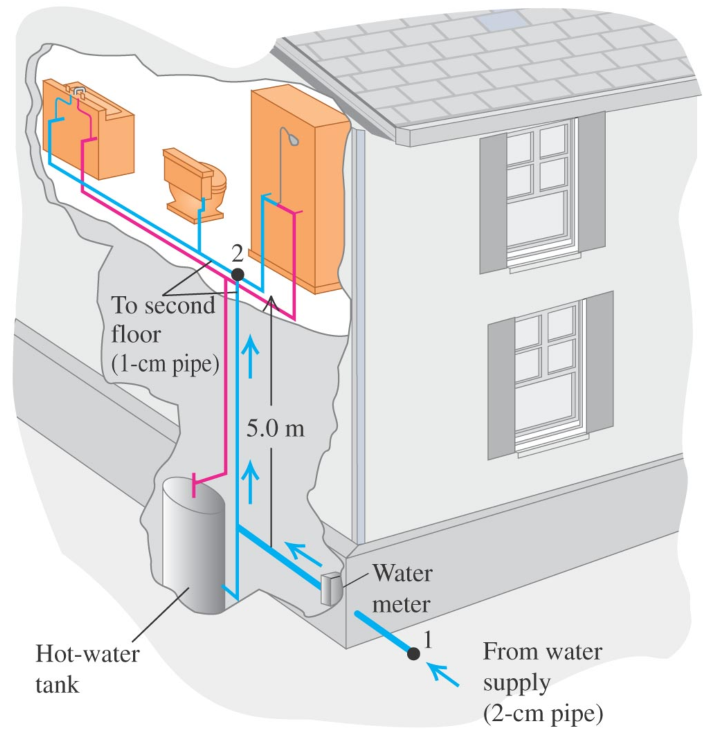
$$dV = A_2 ds_2$$



$$dV = A_1 ds_1$$

Water Pressure in the Home

- Water enters a house through a pipe with an inside diameter of 2 cm at an pressure of 4×10^5 Pa. A 1-cm-diameter pipe leads to the 2nd floor bathroom 5 m above. When the flow speed at the inlet pipe is 1.5 m/s. Find 1) flow speed, 2) volume flow rate, and 3) pressure in the bathroom.



$$v_2 = \frac{A_1}{A_2} v_1 = 6.0 \text{ m/s}$$

$$\frac{dV}{dt} = A_2 v_2 = \pi (0.5\times10^{-2}\text{m})^2 (6.0 \text{ m/s}) = 4.7\times10^{-4}\text{m}^3/\text{s} = 0.47 \text{ L/s}$$

$$p_2 = p_1 - \frac{1}{2}\rho(v_2^2 - v_1^2) - \rho g (y_2 - y_1) = 3.3\times10^5 \text{Pa}$$

Practice Problems

Density of a Neutron Star

A neutron star is the collapsed core of a massive star when it burns down all its fuel – this process comes with a supernova explosion. A typical neutron star has a radius of 10 km, but weights twice that of our Sun. How many times denser are neutron stars comparing to our Earth? The Sun's mass is 2 x 10^{30} kg, and the Earth's mean density is 5500 kg/m^3.

(A) 2 (B) 200 (C) 2×10^{10} (D) 2×10^{14} (E) 2×10^{18}

Air Pressure

- A laptop of size 30 cm x 20 cm is sitting on the table. What is the magnitude of downward force that the surface of the laptop feels, in N, exerted by the air in the room? The air pressure in the room is 1 atm, or 10^5 N/m^2.

A. 60 B. 600 C. 6000.

D. 60000 E. 60000000

Measuring Density

- I would like to check if the goldbrick I bought (for \$100) is really made of gold, so I did an experiment: It weights 5 N in the air and 4 N when it is immersed in the water (density 1000 kg/m³). What is its density in kg/m³?

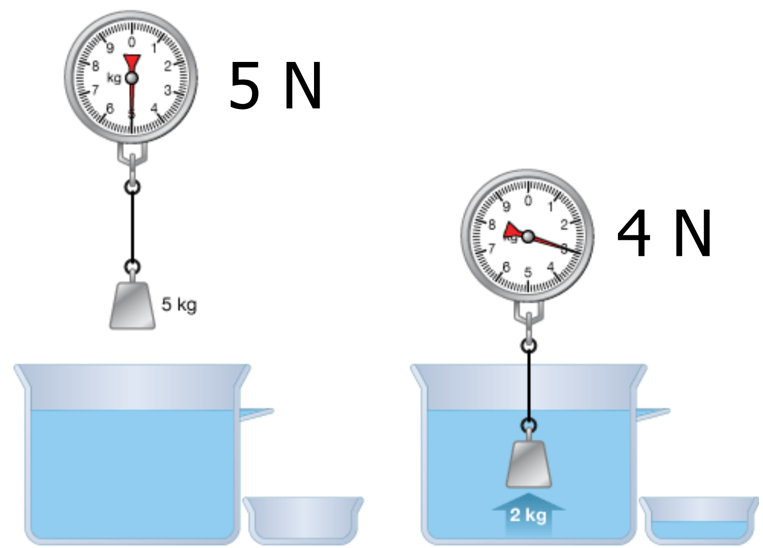
A. 1250

B. 2500

C. 5000

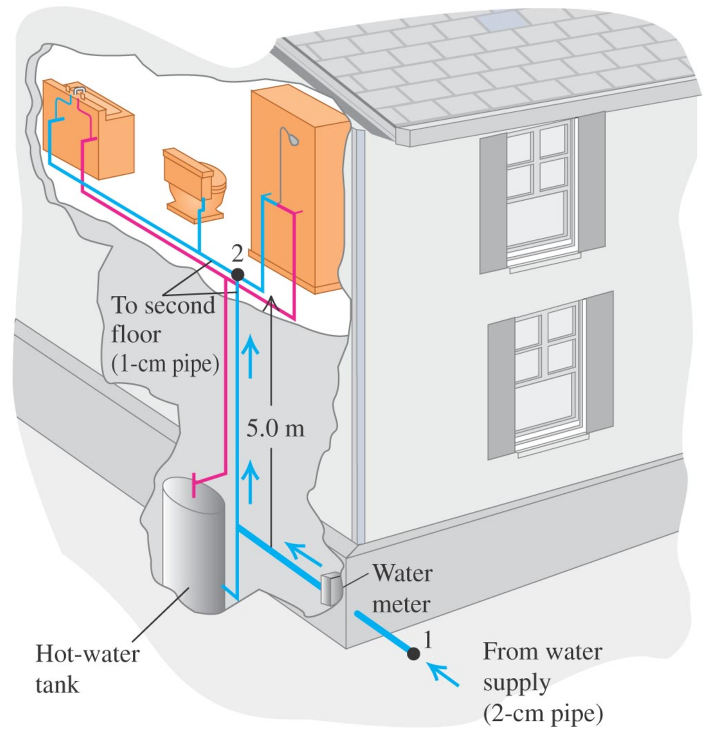
D. 10000

E. 20000



Water Pressure in the Home

- Water enters a house through a pipe with an inside diameter of 2.0 cm at an pressure of 4×10^5 Pa. A 1.0-cm-diameter pipe leads to the 2nd floor bathroom 5 m above. When the flow speed at the inlet pipe is 1.5 m/s, Find the flow speed in the bathroom.



A) 3.0 m/s

B) 0.75 m/s

C) 0.375 m/s

D) 6.0 m/s

E) 1.5 m/s

$$v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi(1.0\text{ cm})^2}{\pi(0.5\text{ cm})^2}(1.5\text{ m/s}) = 6.0\text{ m/s}$$