Phys 321: Lecture 2

Light, Blackbody Radiation, Color Index

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The Wave Nature of Light

- In 1600s, Isaac Newton believed that light consists of a rectilinear stream of particles
- Christian Huygens proposed that the light must consist of waves
- If light = waves:
  - It has a wavelength of λ and frequency of ν
  - Light travels at $c = 2.99792458 \times 10^8$ m s^{-1}

$$\boxed{c = \lambda \nu.}$$

Young's double-slit experiment

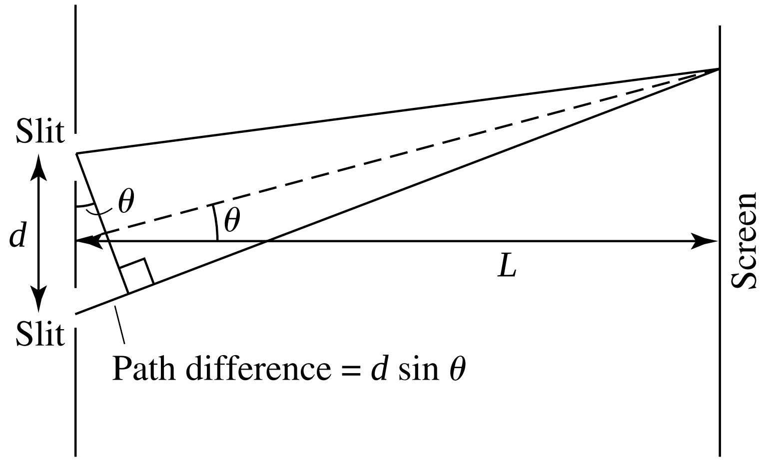
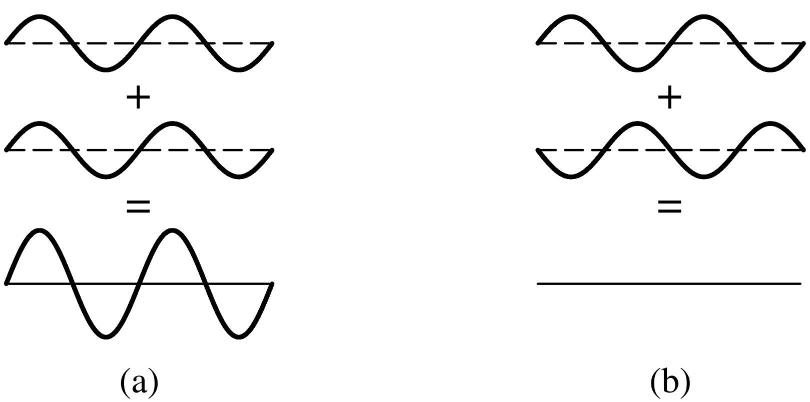


FIGURE 3 Double-slit experiment.

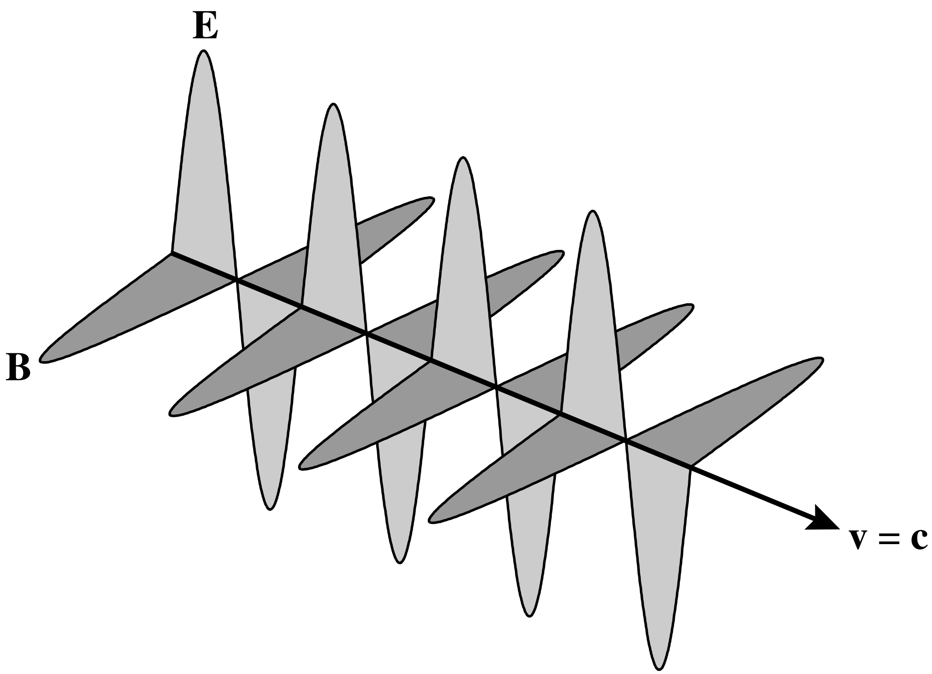


Bright Fringes

$$d \sin \theta = \begin{cases} n\lambda \\ \left(n - \frac{1}{2}\right)\lambda \end{cases}$$

Dark Fringes

Maxwell: Light = Electromagnetic Waves



E ⊥ B

Poynting Flux: $\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}$,

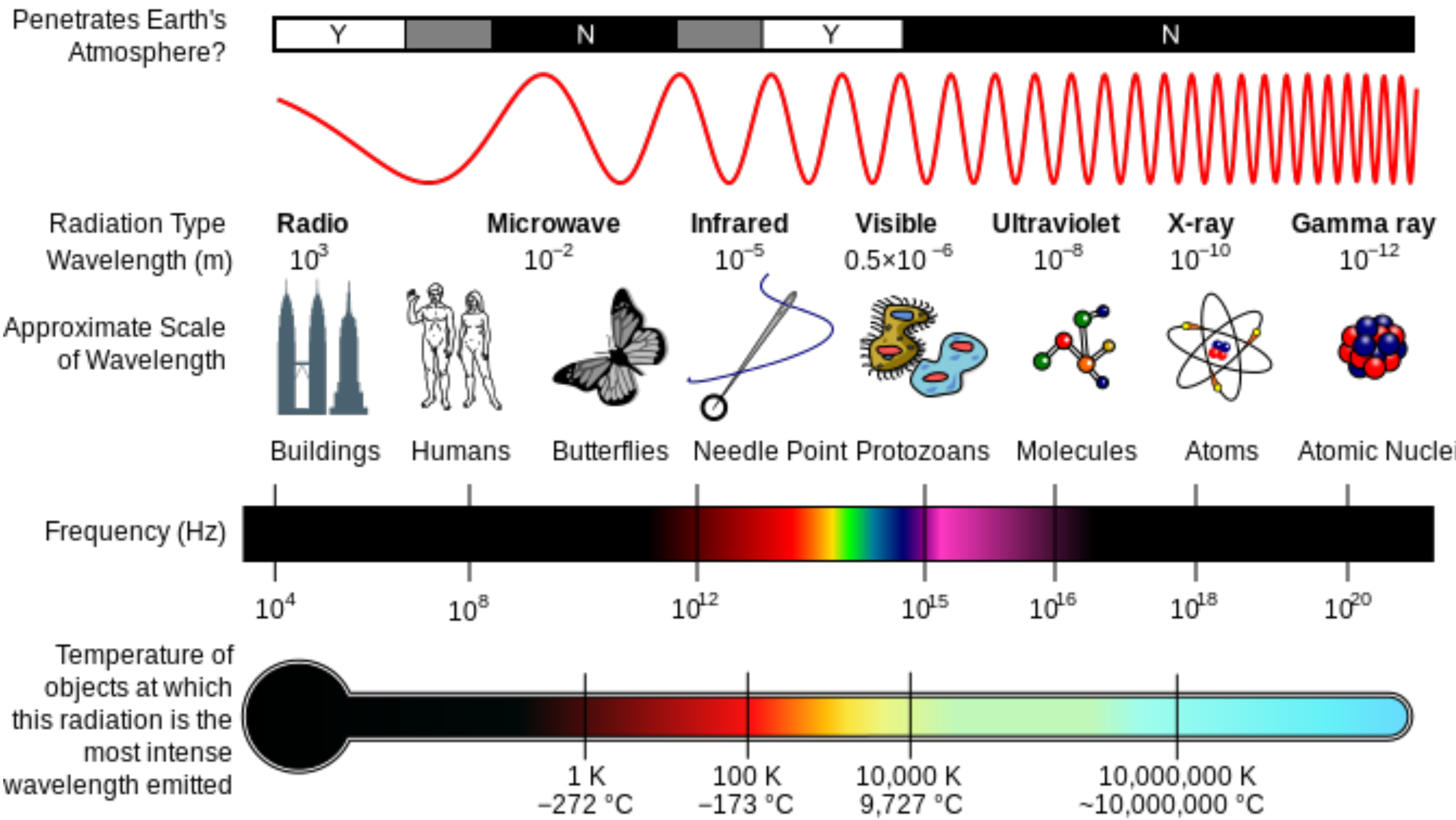
Time-averaged **Poynting Flux**:

$$\langle S \rangle = \frac{1}{2\mu_0} E_0 B_0$$

This is the **flux** (unit: W m^{-2}) carried by a light wave at a *single wavelength* (monochromatic wave)

The flux **F** we discussed before is the result of ***integration over all wavelengths***

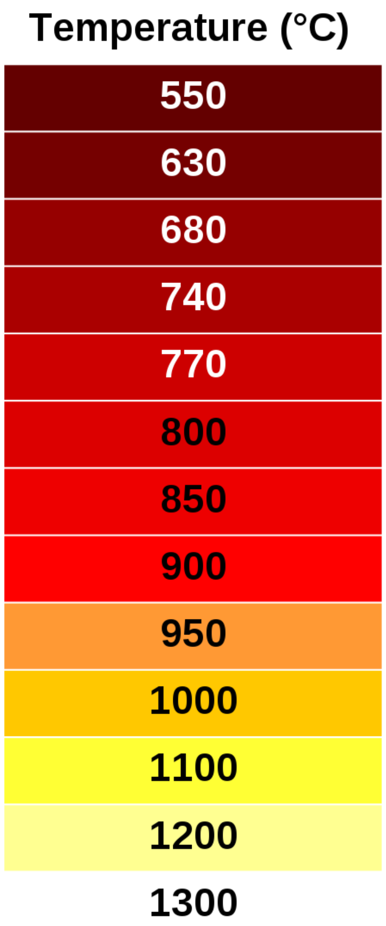
The Electromagnetic Spectrum



From Wikipedia

Color and Temperature

A “red-hot” nail



Which part of the nail is hotter?

Color and Temperature: Stars

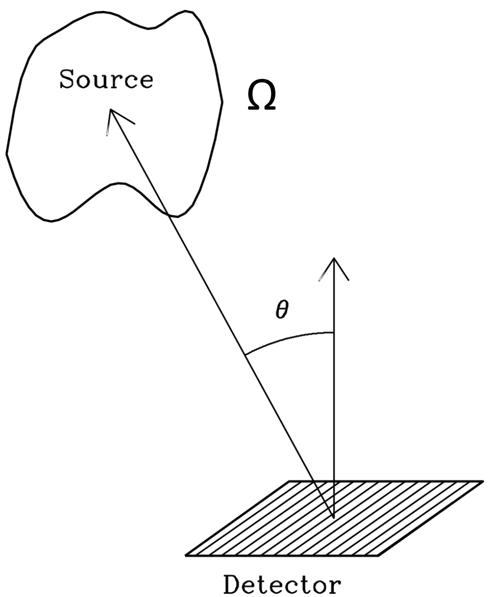
Constellation Orion

Which star is the coolest?

Color	Example	Surface Temperature (°C)
	Rigel (Orion)	28,000–11,000
	Sirius (Canis Major)	11,000–7,500
	Sun & Capella (Auriga)	6,000–5000
	Aldebaran (Taurus)	5,000–3,600
	Betelgeuse (Orion)	3,600–2,000

Red to **blue** **-> increasing** temperature

Intensity vs. Flux



- Flux ***F***: energy received in a unit time through a unit surface area from all directions
  - unit W m^{-2}
- Intensity ***I*** accounts for both the direction and size of the source, so it is the energy received per unit time per unit surface area (along the direction of the light ray) per unit solid angle, in language of calculus

$$dE = I\cos\theta dA dt d\Omega$$

  - It is **independent of distance** (why?)
  - It is **the same at the source and detector**
- The flux ***F*** is an integration of intensity ***I*** over solid angle

$$F = \int_{source} I(\theta,\phi)\cos\theta\, d\Omega = \int_{source} I(\theta,\phi)\cos\theta\sin\theta\, d\theta\, d\phi$$

Specific intensity and flux density

- ***I*** and ***F*** are quantities **integrated over all wavelengths**
- Light emitted from stars distribute over the entire electromagnetic spectrum
- **Specific intensity** I_λ or I_ν and **flux density** F_λ or F_ν are intensity and flux within given a wavelength range or frequency range (λ to $\lambda + d\lambda$, or ν to $\nu + d\nu$)

Specific intensity (per wavelength)
Unit: J s^{-1} m^{-2} sr^{-1} m^{-1}

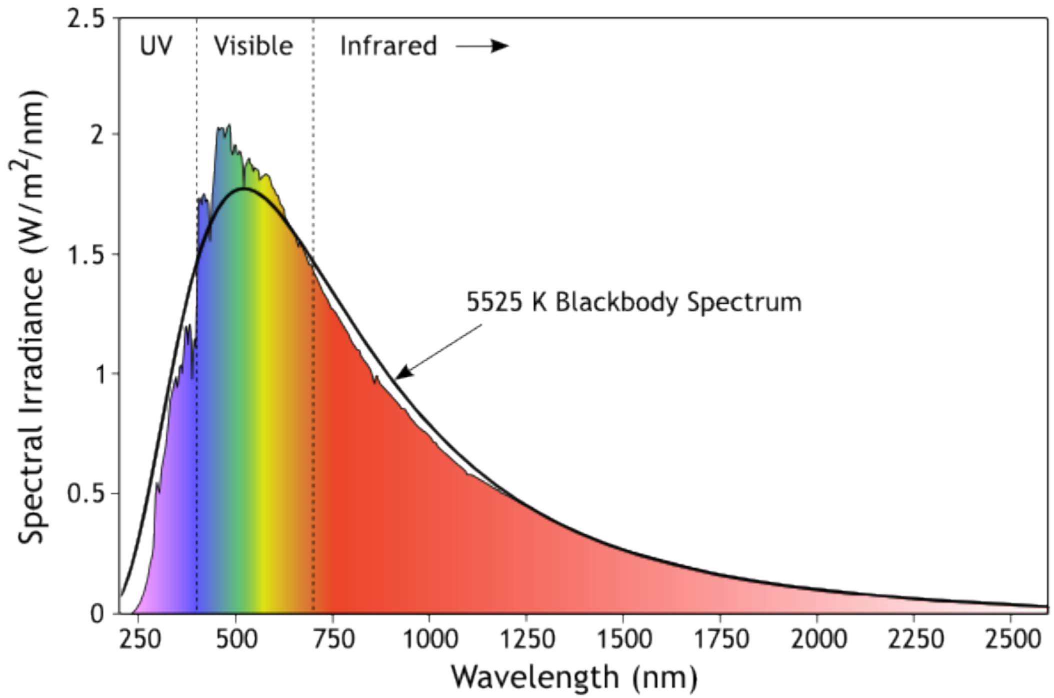
$$dE = I_\lambda \cos\theta dA dt d\Omega d\lambda$$

Flux density (per wavelength)
Unit: J s^{-1} m^{-2} m^{-1}

$$F_\lambda = \int_{source} I_\lambda(\theta,\phi)\cos\theta\, d\Omega$$

Blackbody

- Is an idealized physical body that absorbs all incident electromagnetic radiation, regardless of frequency and angle of incidence, and reradiates all the energy with a characteristic spectrum.
- Is black hole a blackbody?
  - Yes, if Stephen Hawking is correct on his “Hawking Radiation”

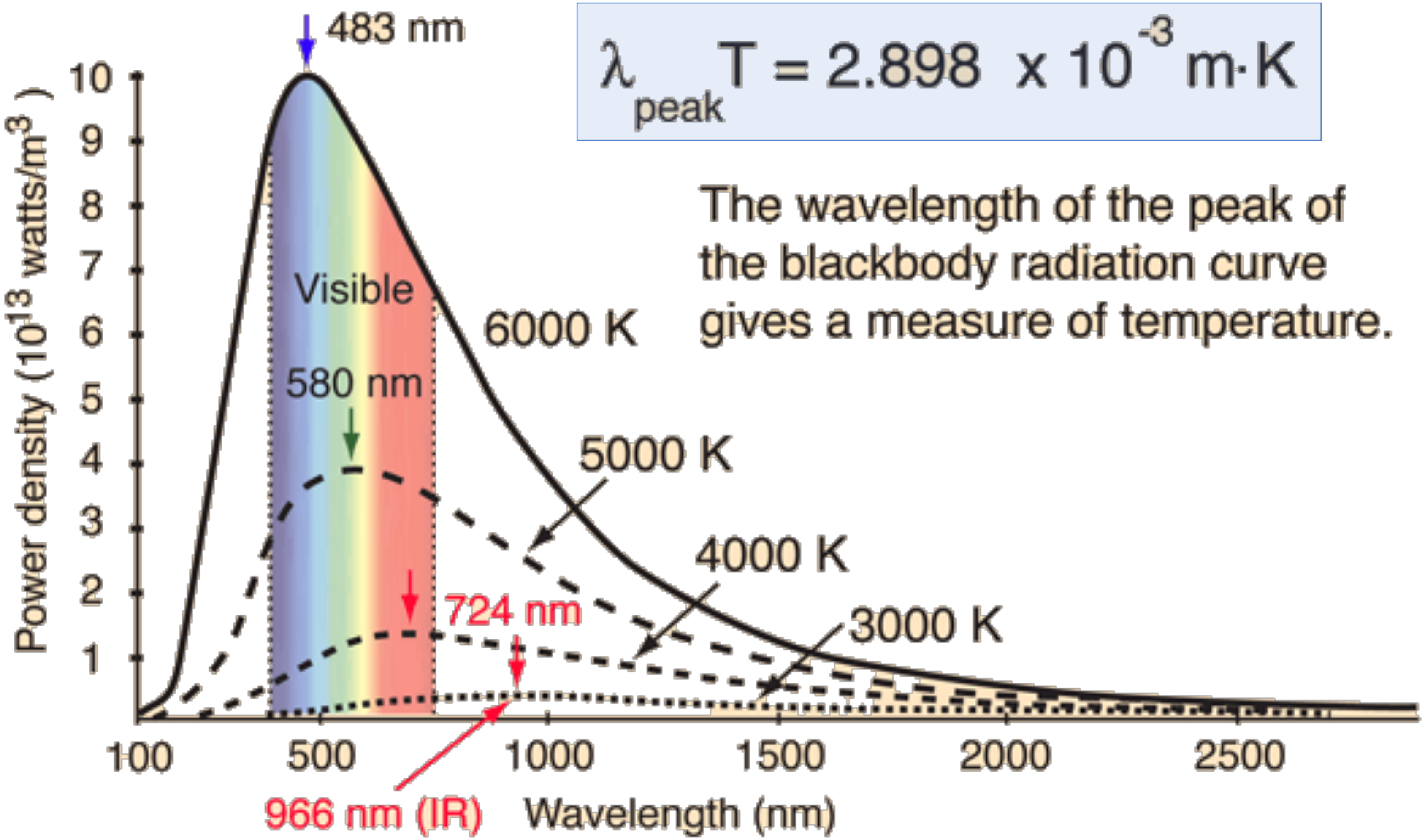


Solar irradiance at Earth (no atmosphere)

Wien's Displacement Law

$$\lambda_{peak} T = 2.898 \times 10^{-3}\ m \cdot K$$

The wavelength of the peak of the blackbody radiation curve gives a measure of temperature.



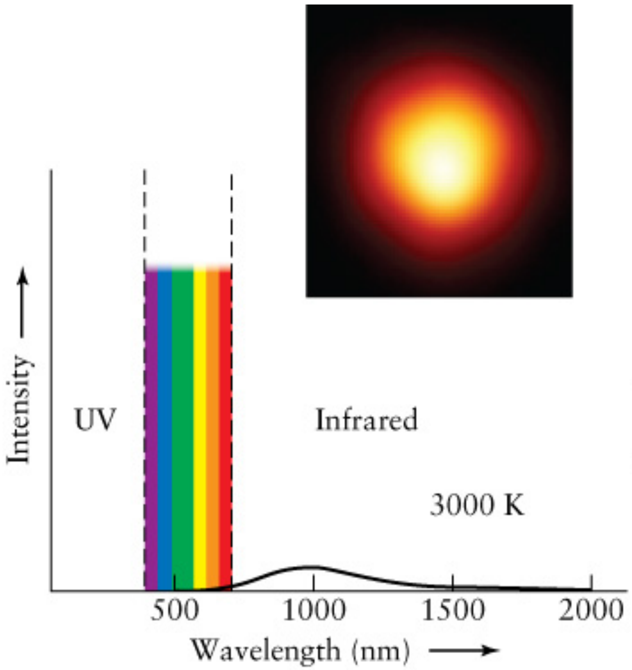
Example: Peak wavelength of Betelgeuse and Rigel

- Betelgeuse and Rigel have surface temperature of 3,600 K and 13,000 K. What are their peak wavelengths? In which region of the electromagnetic spectrum?

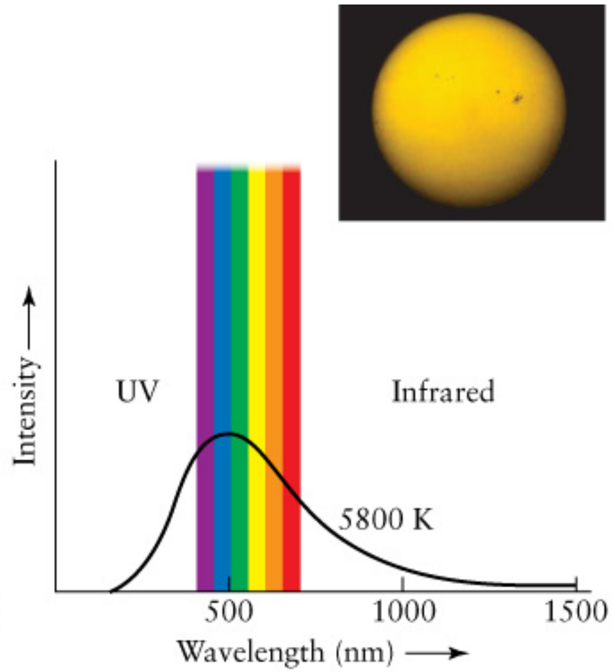
Betelgeuse: $\lambda_{\max} \simeq \dfrac{0.0029 \text{ m K}}{3600 \text{ K}} = 8.05 \times 10^{-7} \text{ m} = 805 \text{ nm}$ Infrared

Rigel: $\lambda_{\max} \simeq \dfrac{0.0029 \text{ m K}}{13{,}000 \text{ K}} = 2.23 \times 10^{-7} \text{ m} = 223 \text{ nm}$ Ultraviolet

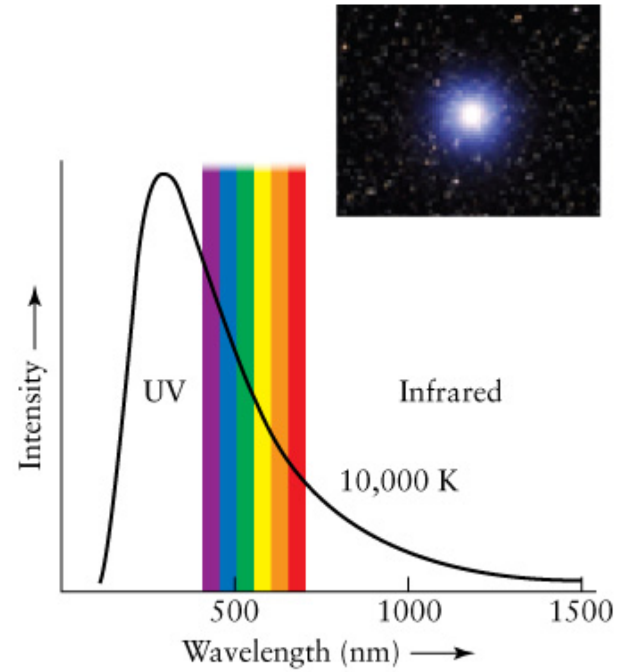
Understanding color of stars



(a) A **cool star** with surface temperature 3000 K emits much more red light than blue light, and so appears red.



(b) A **warmer star** with surface temperature 5800 K (like the Sun) emits similar amounts of all visible wavelengths, and so appears yellow-white.

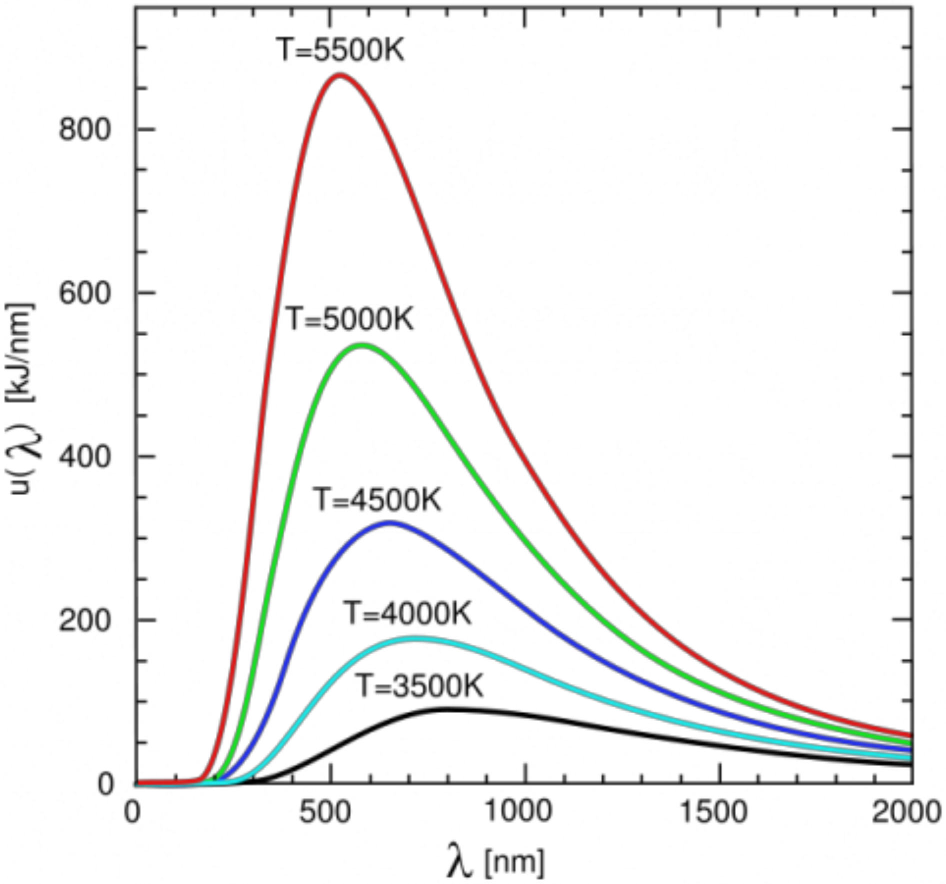


(c) A **hot star** with surface temperature 10,000 K emits much more blue light than red light, and so appears blue.

According to **Wien's law** …

- A ***low*** temperature star emits most of its energy at ***long*** wavelengths
- A ***high*** temperature star emits most of its energy at ***short*** wavelengths
- Thus, a ***cool*** star appears ***red***, while a ***hot*** one appears ***blue***

Stefan-Boltzmann Equation



- T increases -> intensity increases at all wavelengths
- Experiment show that luminosity depends on temperature T and area A

Stefan-Boltzamann Equation

$$L = A\sigma T^4.$$

Where

$$\sigma = 5.670400 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}.$$

Is the Stefan-Boltzmann constant

Example

The Sun's luminosity and radius are

$$L_\odot = 3.839 \times 10^{26}\ \mathrm{W}$$

$$R_\odot = 6.95508 \times 10^{8}\ \mathrm{m}$$

What is the surface temperature of the Sun? What is the Sun's peak wavelength?

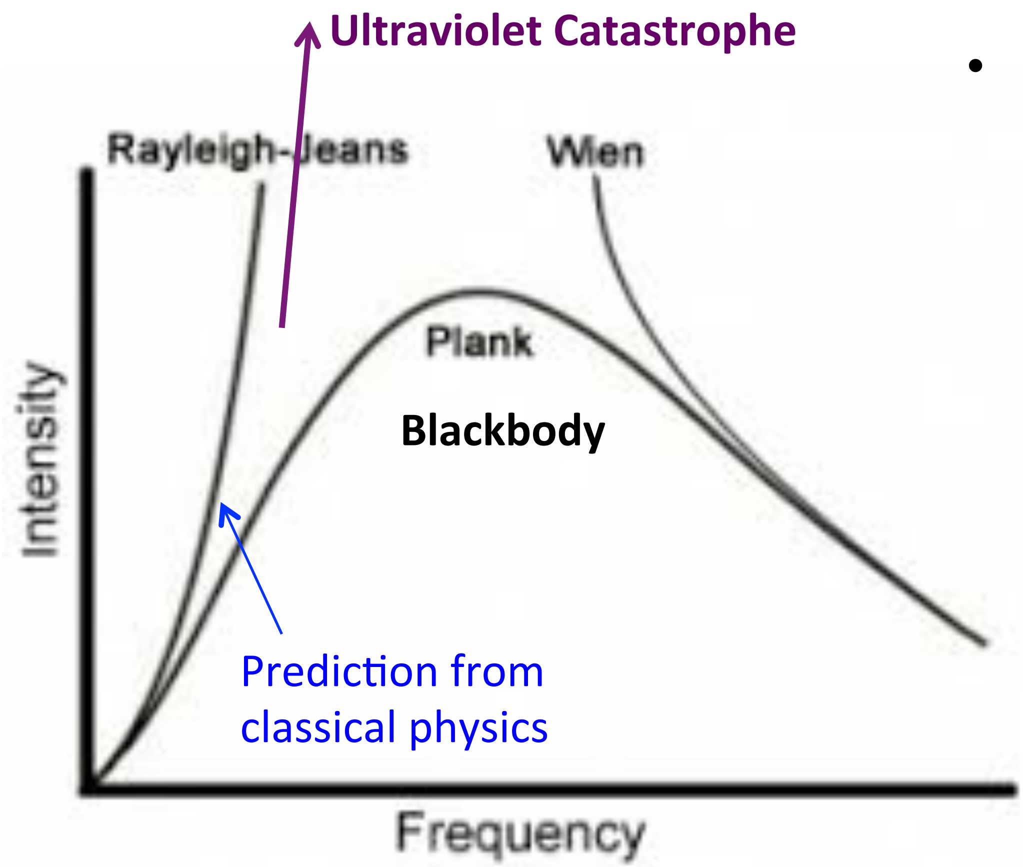
$$T_\odot = \left(\frac{L_\odot}{4\pi R_\odot^2 \sigma}\right)^{\frac{1}{4}} = 5777\ \mathrm{K}$$

$$\lambda_{\mathrm{max}} \simeq \frac{0.0029\ \mathrm{m\ K}}{5777\ \mathrm{K}} = 5.016 \times 10^{-7}\ \mathrm{m} = 501.6\ \mathrm{nm}$$

What is the luminosity of a star with a surface temperature of 12,000 K and a radius of $R_{sun}/2$? The Sun's surface temperature is approximately 6,000 K and its luminosity is L_{sun}

A. $L_{sun}/4$
B. $L_{sun}/2$
C. L_{sun}
D. $2\ L_{sun}$
E. $4\ L_{sun}$

The Nature of Light: Dawn of a New View



- Classical physics (thermal mechanics and Maxwell’s equation) -> **Rayleigh-Jeans Law**

$$B_\lambda(T) \approx \frac{2ckT}{\lambda^4} \qquad B_\nu(T) \approx \frac{2\nu^2 kT}{c^2}$$

B_λ : specific intensity from blackbody

$k = 1.38 \times 10^{-23}$ J K^{-1}: Boltzmann’s constant

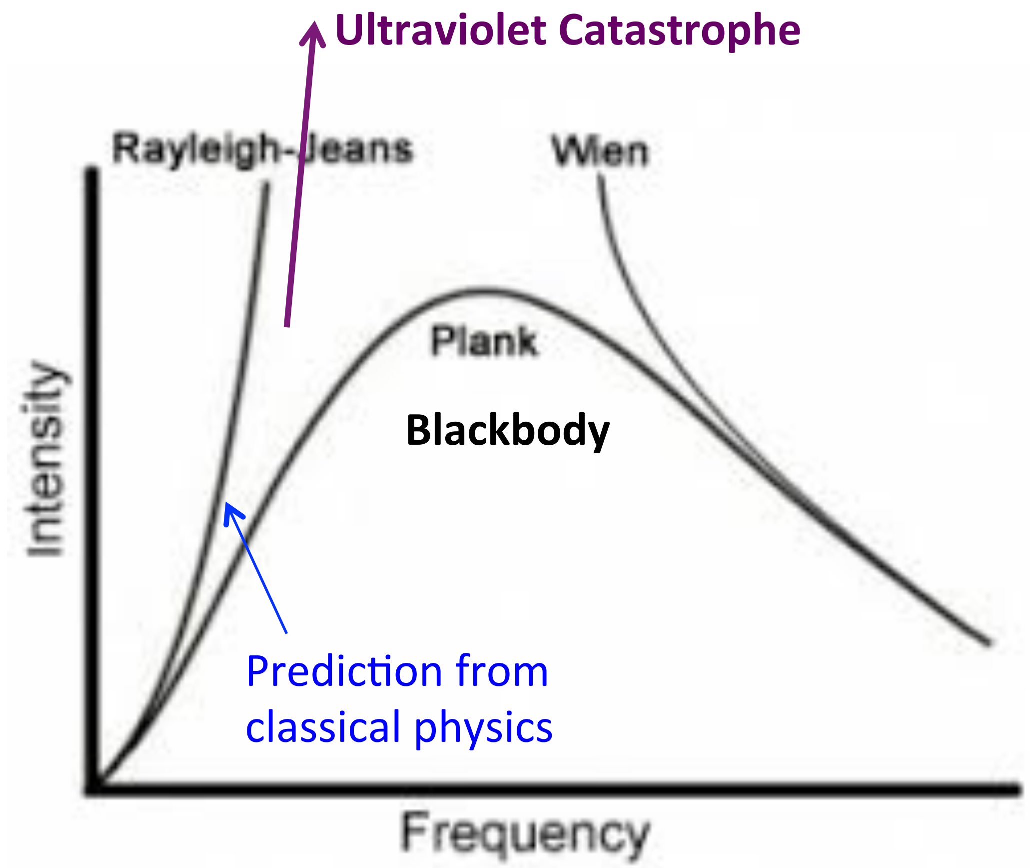
Only agrees well with the very low end of the blackbody spectrum (from experiments). At shorter wavelengths, B_λ increases without limit

Ultraviolet Catastrophe

At high frequencies, an *empirical* relation is found from experiments named **Wien’s Approximation**:

$$B_\lambda(T) \approx a\lambda^{-5} e^{-b/\lambda T}$$

Classical physics goes terribly wrong at higher frequencies/shorter wavelengths!



Photons: quantization of energy

- By late 1900, Max Planck tried to make sense of the blackbody spectra
- He assumes the allowed energy carried by any wave cannot be infinitely small, but should be an integer number of $h\nu$, or hc/λ, where h is a constant, i.e., the energy is **quantized**.
- This smallest, quantized energy E is carried by a type of elementary particles, known as **photons**. A single photon carries

$$\boxed{E = h\nu}$$

We will come back to this next week

The Planck Function

- With quantized energy of light waves, Max Planck got the famous **Planck Function**:

Diff. in wavelength

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}.$$

Diff. in frequency

$$B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}.$$

Rayleigh-Jeans Law

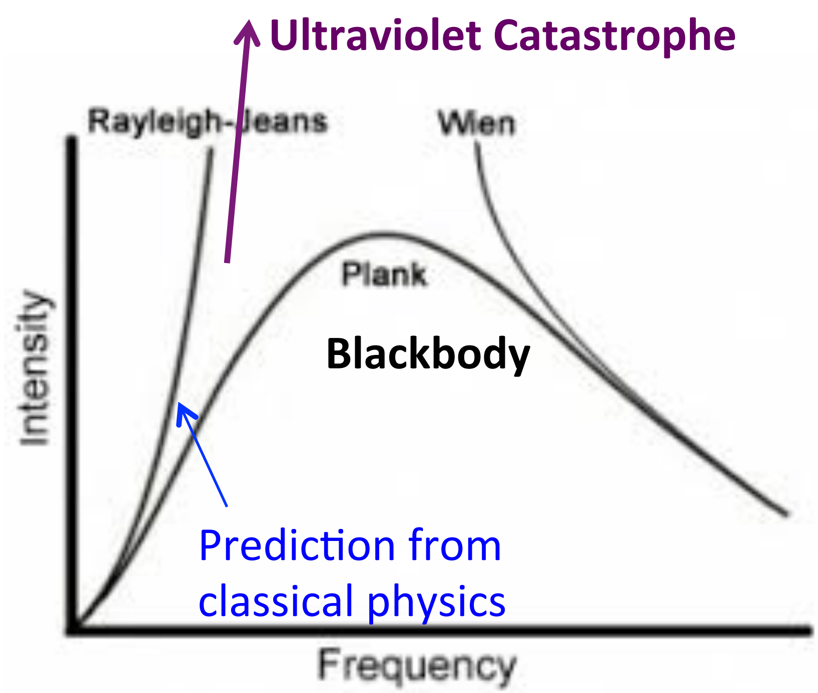
Long wavelength limit:

$$B_\lambda(T) \approx \frac{2ckT}{\lambda^4}$$

Wien's Approximation

Short wavelength limit:

$$B_\lambda(T) \approx a\lambda^{-5}e^{-b/\lambda T}$$



The Planck Function and Astrophysics

- Most stellar spectra can be approximated by blackbody in general
- They emit nearly *isotropically*, i.e., independent of angle
- So the star's luminosity in a given frequency range

$\nu \rightarrow \nu + d\nu$

Radius of the star

$$L_\nu d\nu = \int_{star} B_\nu \, d\nu \, dA \cos\theta \sin\theta \, d\theta \, d\phi = 4\pi^2 R^2 B_\nu d\nu$$

- Integrating $L_\nu d\nu$ over all frequencies, we recover the Stefan-Boltzmann Equation:

$$L = A\sigma T^4 = 4\pi R^2 \sigma T^4$$

A star's **luminosity** depends only on **radius *R*** and **temperature *T*** (if it is well approximated by a blackbody)

The Planck Function and Astrophysics

- Flux received at a distance

Recall from Lecture 1 $$F = \frac{L}{4\pi d^2}$$ Where d is the distance to the light source

Substitute Stefan-Boltzmann's Equation for stellar luminosity

$$L = 4\pi R^2 \sigma T^4$$

We have

$$F = \sigma T^4 \left(\frac{R}{d}\right)^2$$

The flux density (or monochromatic flux)

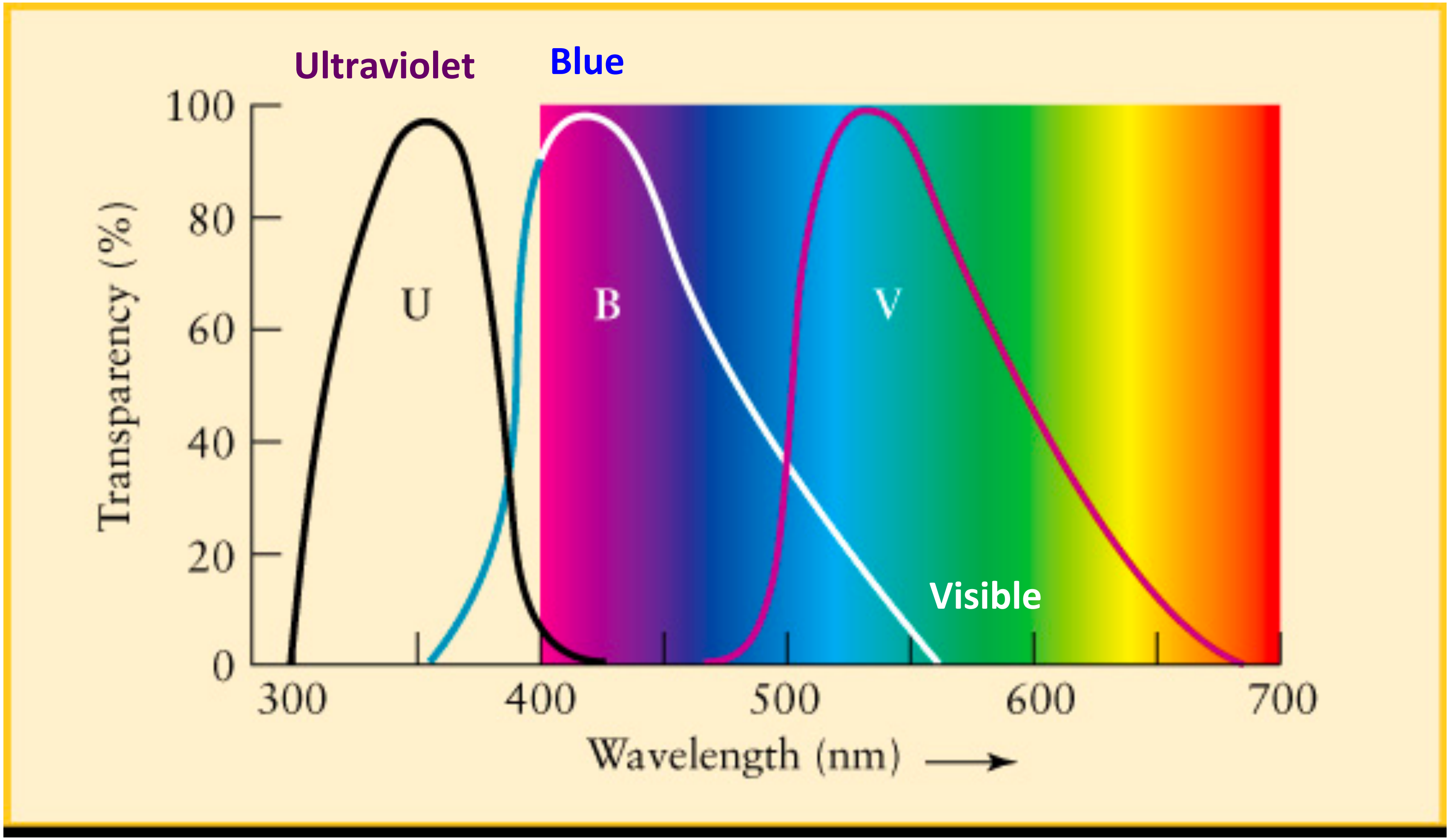
$$F_\lambda \, d\lambda = \frac{L_\lambda}{4\pi r^2} \, d\lambda = \frac{2\pi hc^2/\lambda^5}{e^{hc/\lambda kT} - 1} \left(\frac{R}{r}\right)^2 d\lambda$$

Revisit magnitudes

$$m_1 - m_2 = -2.5 \log_{10}\left(\frac{F_1}{F_2}\right) \qquad 100^{(m-M)/5} = \frac{F_{10}}{F} = \left(\frac{d}{10\ \mathrm{pc}}\right)^2$$

- The apparent and absolute magnitudes in lecture 1 are values integrated over *the entire spectrum*, known as **bolometric magnitudes**, denoted by m_{bol} and M_{bol}
- However, in practice, we usually only measure the flux of a star within certain wavelength ranges

Color filters and UBV magnitude System



UBV magnitudes

- **Apparent** bolometric magnitude

$$m_{\mathrm{bol}} = -2.5\log_{10}\left(\int_0^\infty F_\lambda \, d\lambda\right) + C_{\mathrm{bol}}$$

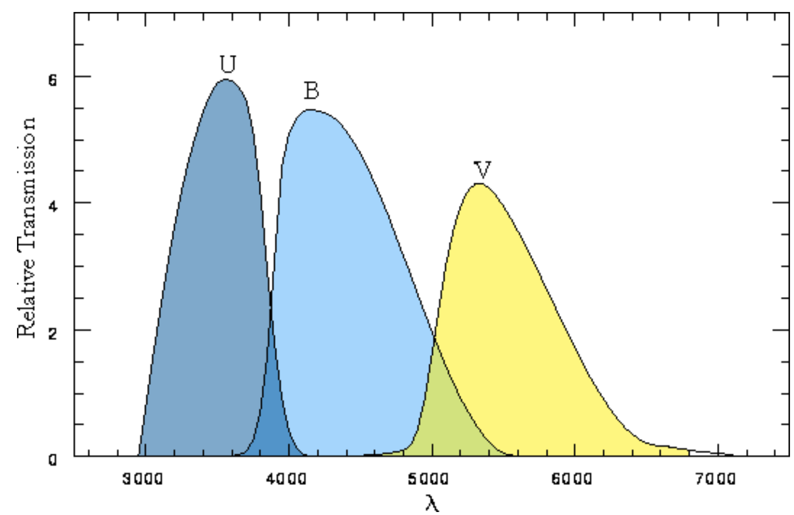
- **Apparent** UBV magnitudes

$S_U \quad S_B \quad S_V$

$$m_U = U = -2.5\log_{10}\left(\int_0^\infty F_\lambda S_U \, d\lambda\right) + C_U$$

$$m_B = B = -2.5\log_{10}\left(\int_0^\infty F_\lambda S_B \, d\lambda\right) + C_B$$

$$m_V = V = -2.5\log_{10}\left(\int_0^\infty F_\lambda S_V \, d\lambda\right) + C_V$$



Sensitivity Functions

Constants C_u, C_B, C_V are chosen so that U, B, V of star Vega are all zero

Color Indices

- A star's color index is the difference between the apparent magnitudes of two different color filters

$$U - B = m_U - m_B \qquad = M_U - M_B$$

$$B - V = m_B - m_V \qquad = M_B - M_V$$

These are ***independent of distance***!

$$m - M = 5\log_{10}\left(\frac{d}{10\,pc}\right)^2$$

Bolometric correction

- Difference between a star's bolometric magnitude and its visual magnitude

$$\boxed{BC = m_{\text{bol}} - V = M_{\text{bol}} - M_V\,.}$$

All wavelengths

Visual

Color Index and Surface Temperature

Color indices tell us the ***colors*** of the star, which are related to their ***surface temperatures***

e.g., the ***smaller*** B-V index, the ***bluer*** the star, the ***hotter*** the star