Phys 321: Lecture 3

The Interaction of Light and Matter

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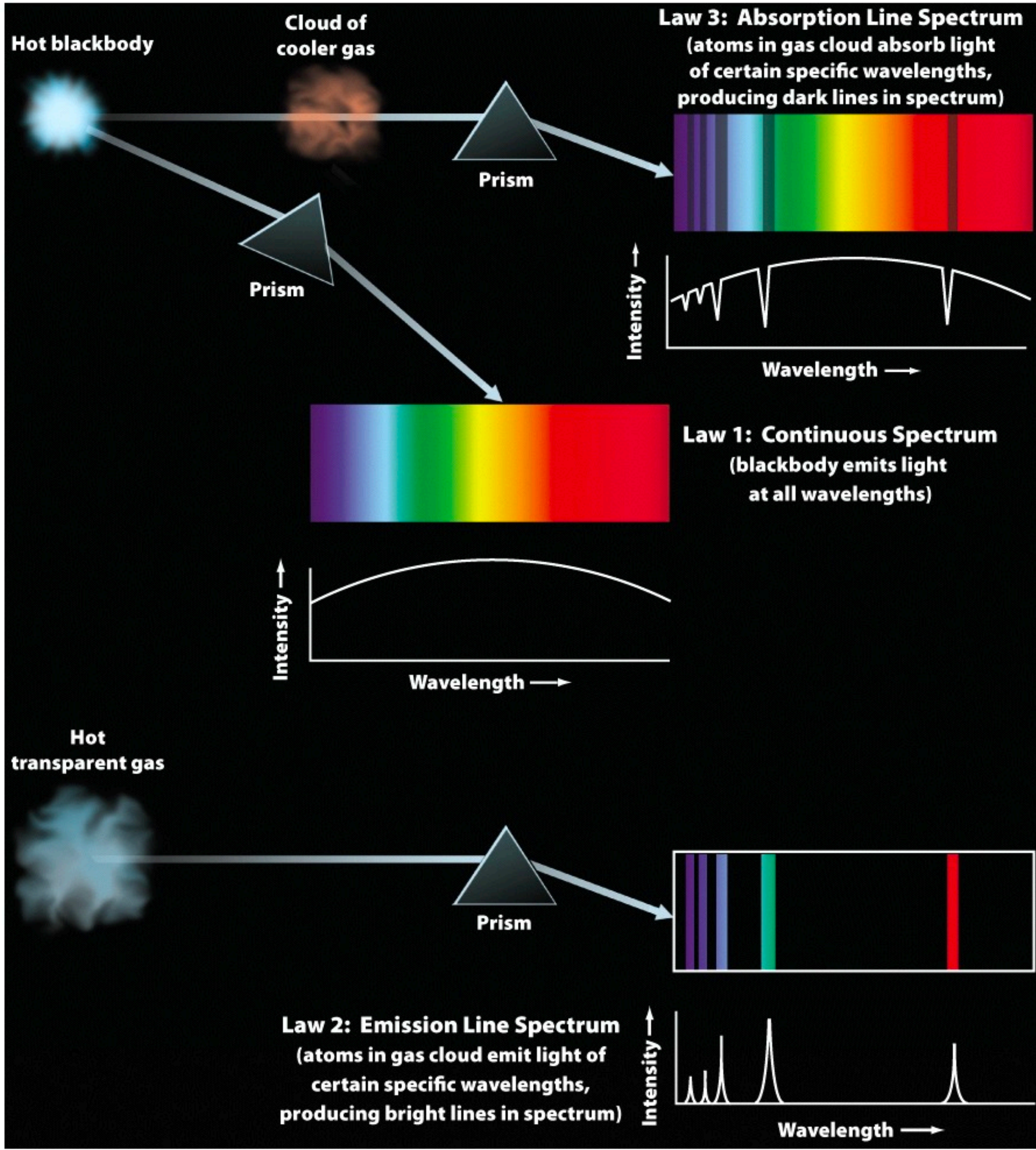
Fraunhofer Lines

- By 1814, German optician Joseph von Fraunhofer cataloged 475 of dark lines in the solar spectrum, known today as the "**Fraunhofer Lines**"
- A video story

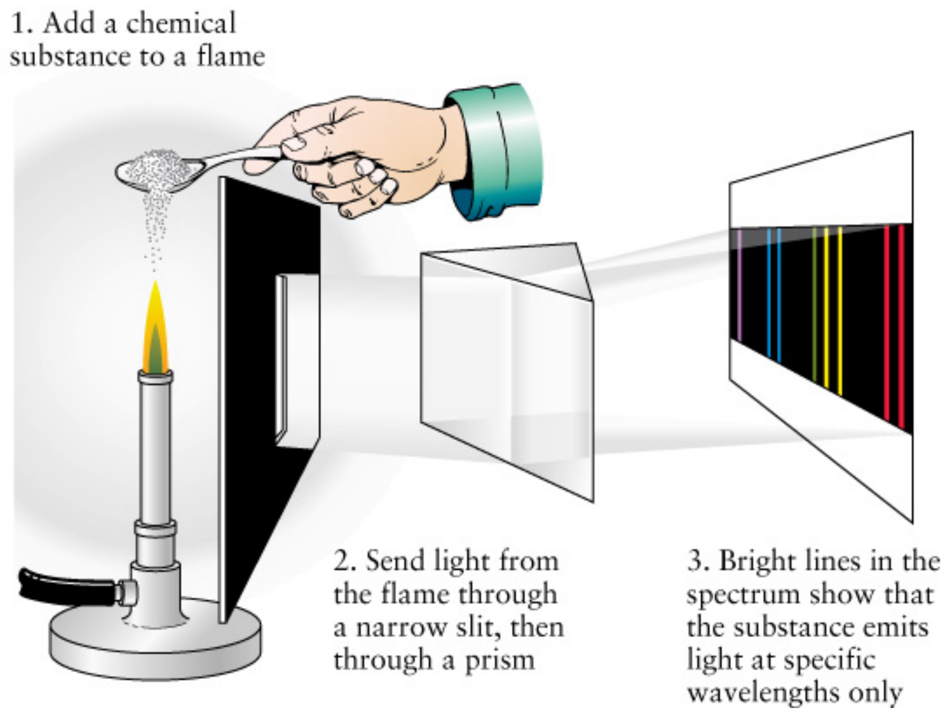
Kirchhoff's Laws

1. A **hot, dense** gas or hot solid object produces a **continuous spectrum**
2. A **hot, diffuse** gas produces bright spectral lines (**emission lines**)
3. A **cool, diffuse** gas in front of a source of a continuous spectrum produces dark spectral lines (**absorption lines**) in the continuous spectrum

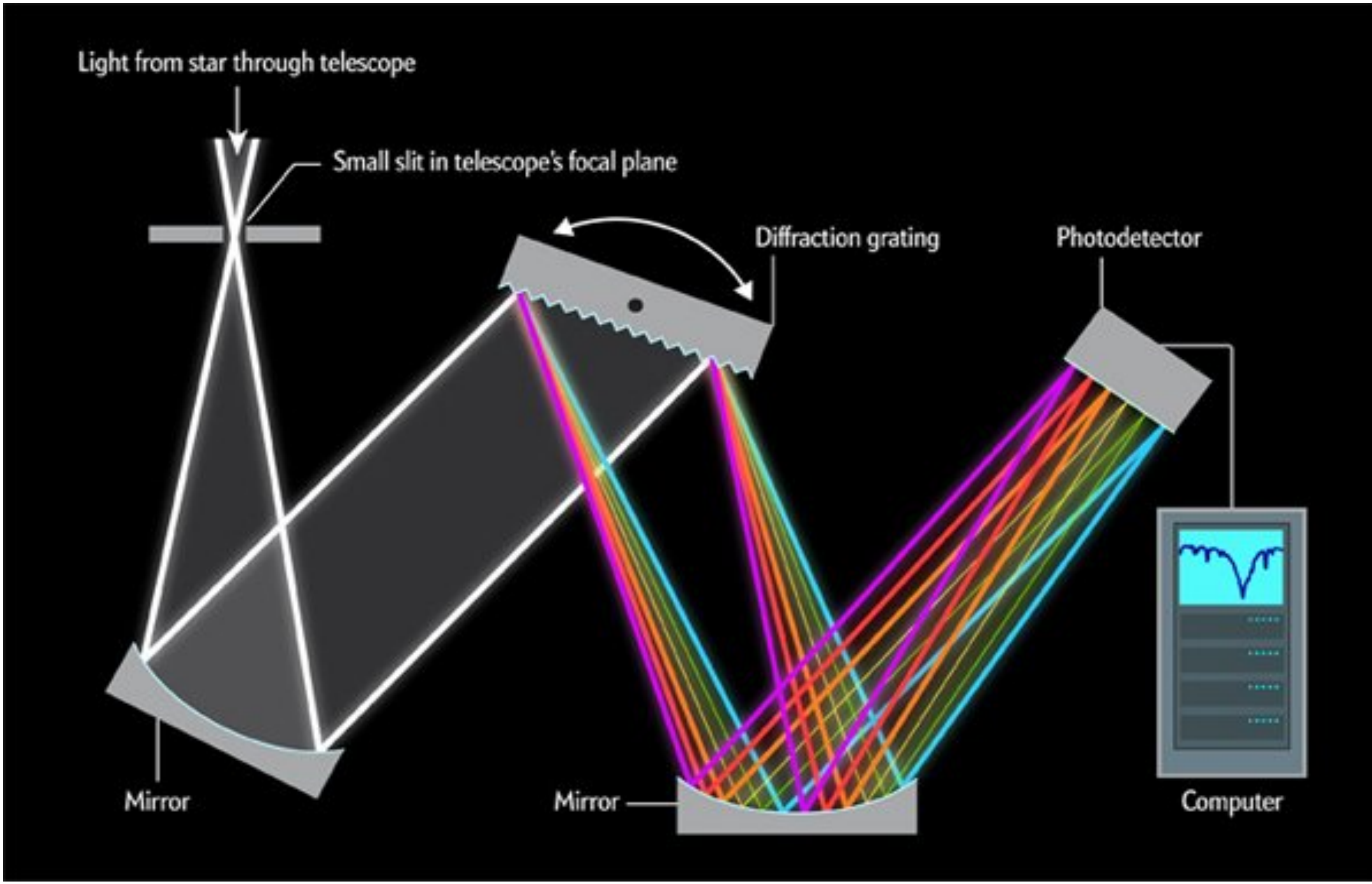


Measuring Spectra: Prism

Astronomers use ***spectrographs*** to separate the light in different wavelengths and measure the spectra of stars and galaxies

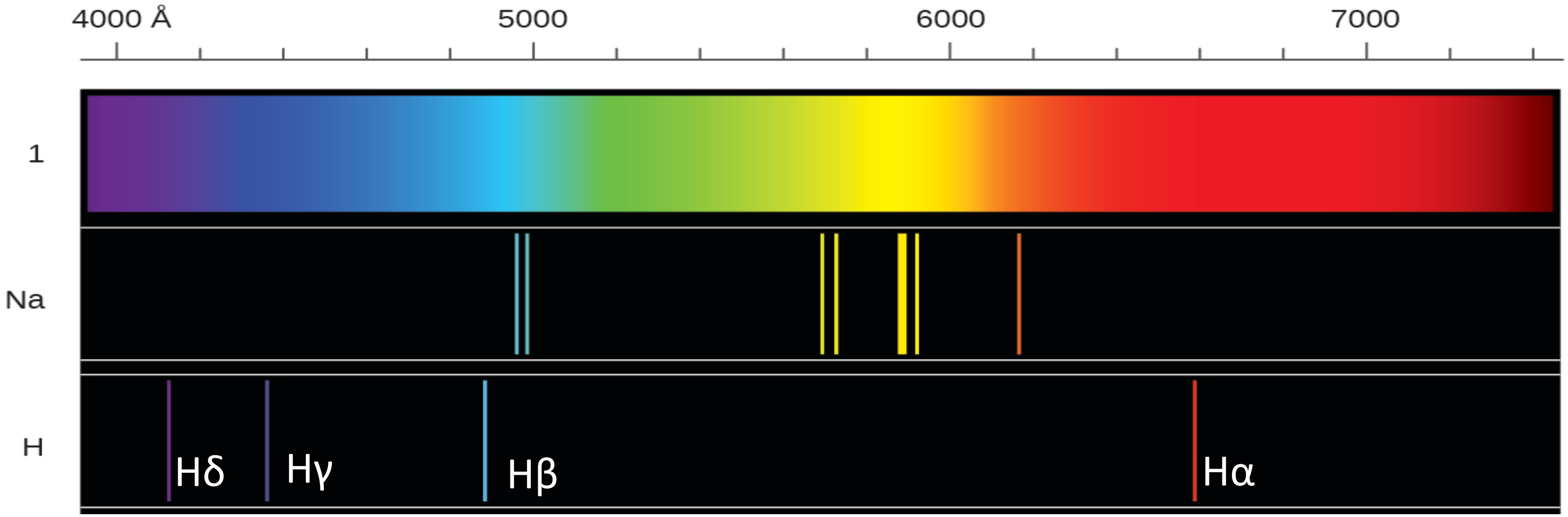


Measuring Spectra: Diffraction Grating

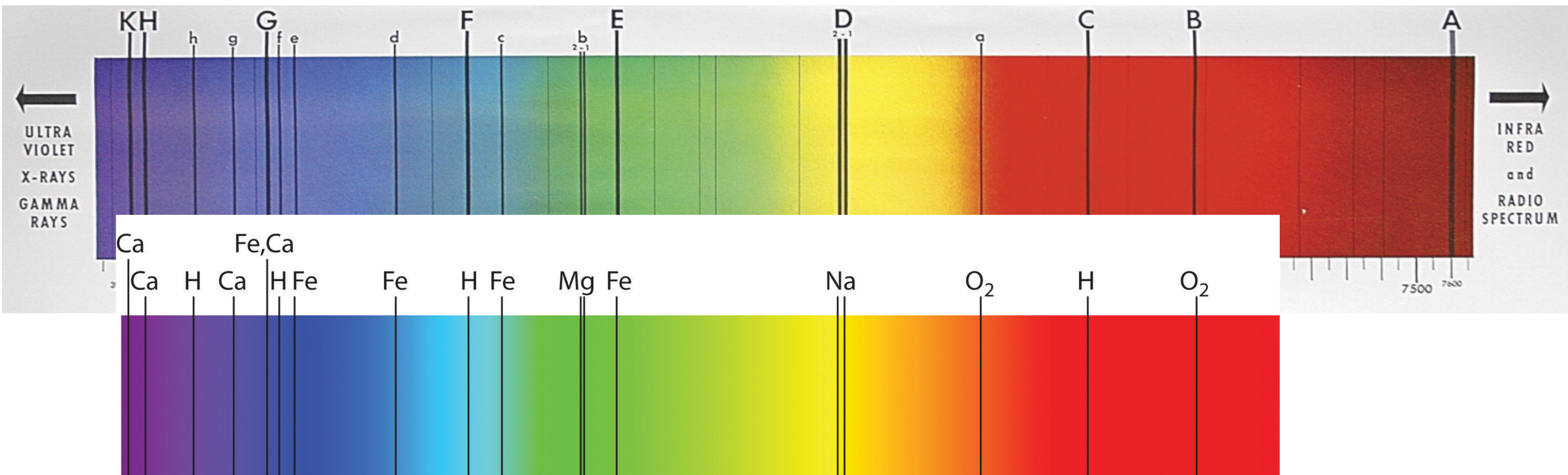


Spectra from different elements

Lab measurements



Fraunhofer lines in Solar Spectrum



Spectroscopy: Applications

- Identify elements on distant astronomical bodies: *Chemical Composition*
- The element *Helium* is first discovered from the solar spectrum, not Earth.
- Measure physical properties: temperature, density, pressure, magnetic fields, high-energy particles, etc.
- Measure velocities of distant objects via ***Doppler shift***

Doppler Shift

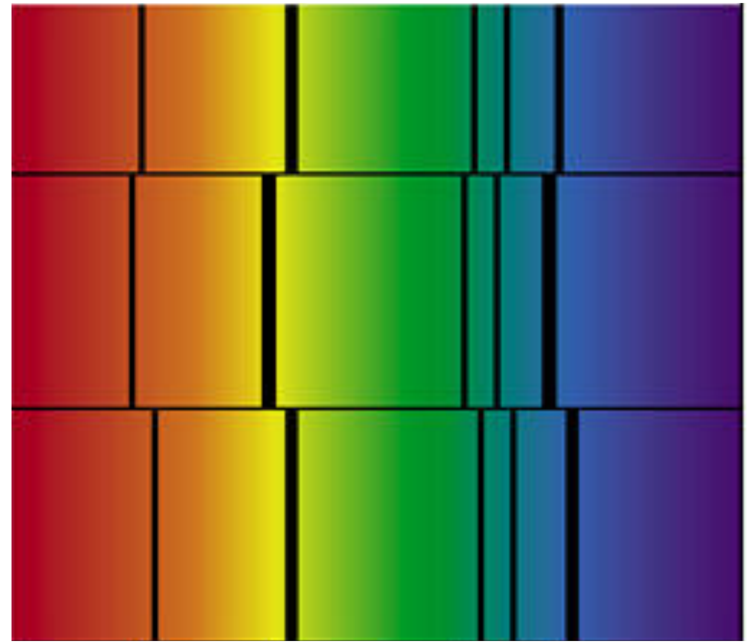
"It's the apparent change in the frequency of a wave caused by relative motion between the source of the wave and the observer." --- Sheldon Cooper

A short animated explanation (Youtube)

Reference spectrum in lab

Observed spectrum from Star A

Observed spectrum from Star B



Question: Which star is moving towards us?

A. Star A

B. Star B

Doppler Shift

$\lambda_{obs} > \lambda_{rest}$: **Redshift**ed -> Object moving **away** from observer

$\lambda_{obs} < \lambda_{rest}$: **Blueshif**ted -> Object moving **towards** observer

$$\frac{\lambda_{\rm obs} - \lambda_{\rm rest}}{\lambda_{\rm rest}} = \frac{\Delta\lambda}{\lambda_{\rm rest}} = \frac{v_r}{c}$$

v_r is the radial component of the objects velocity relative to the observer. **Positive** means the object is **moving away** and *vice versa*.

Radial Velocity of Vega

- The rest wavelength of Hα is 656.281 nm as measured in the lab. From a ground-based telescope, the Hα line from star Vega in constellation Lyra is measured at 656.251 nm. What is its radial velocity?

$$v_r = \frac{c\,(\lambda_{\rm obs} - \lambda_{\rm rest})}{\lambda_{\rm rest}} = -13.9\ \rm km\ s^{-1}$$

Back to nature of light discussion…

Young's double-slit experiment

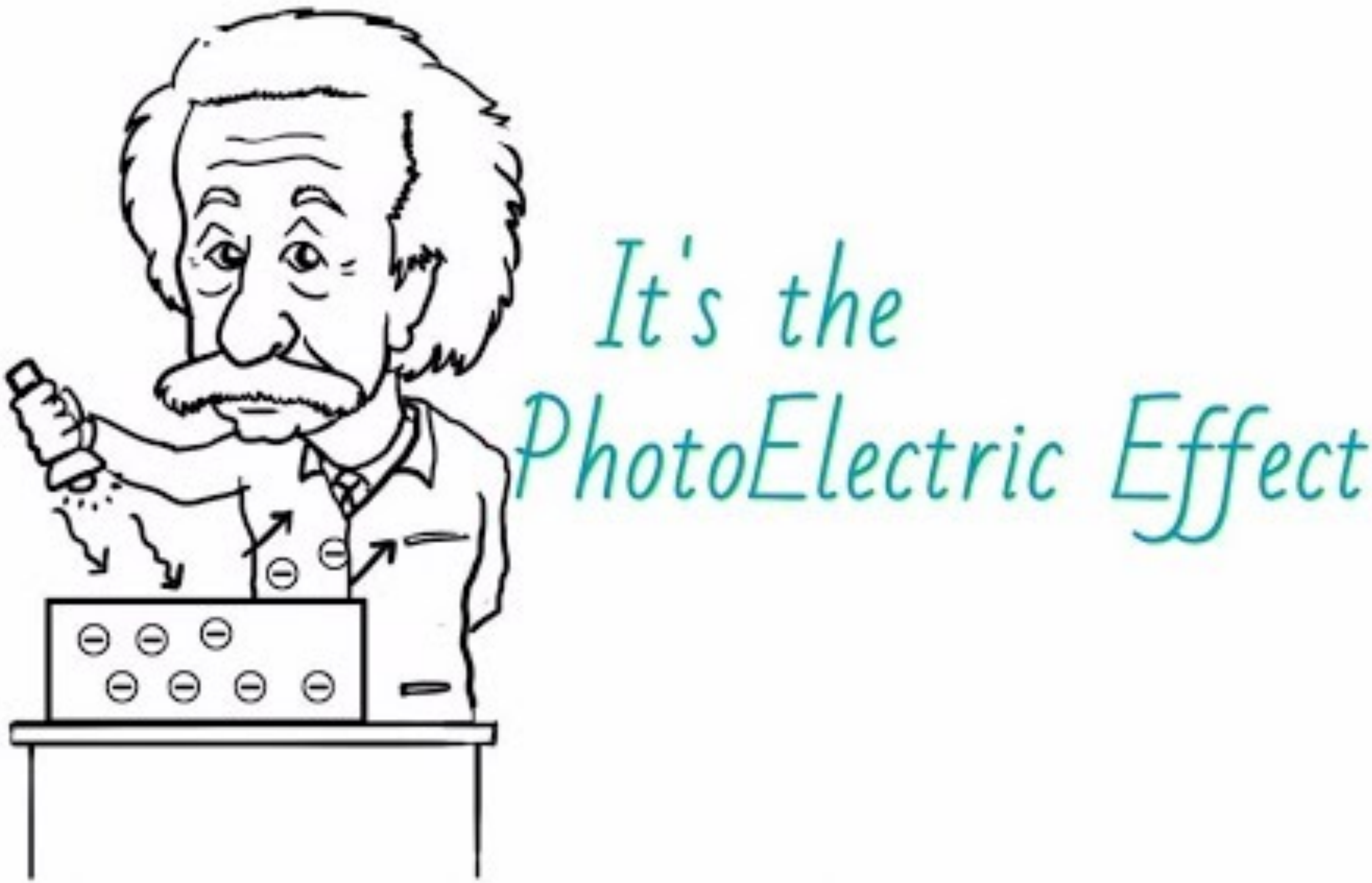
Maxwell's Equations

→ Light = Waves

Ultraviolet Catastrophe of blackbody radiation

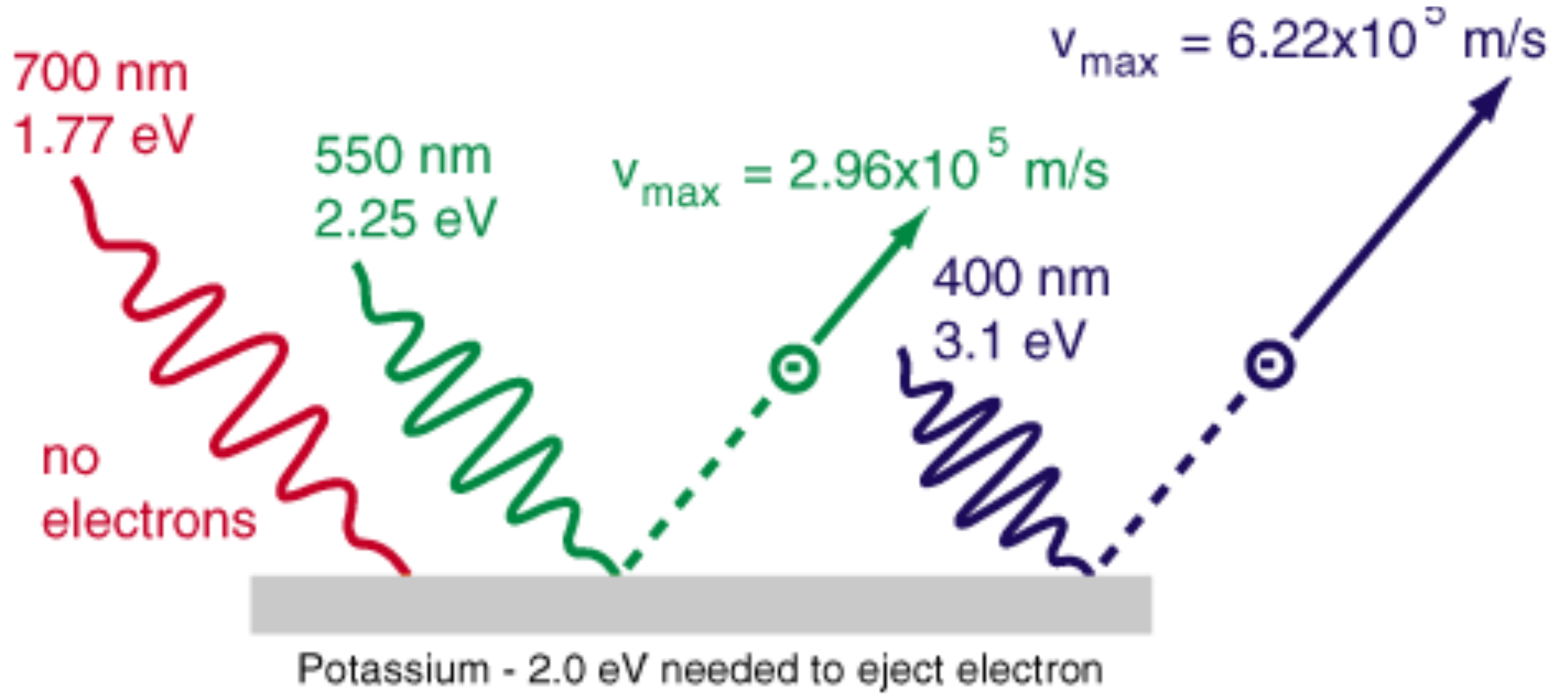
Discrete emission/absorption lines

→ **Something is wrong!**



(Link to a Youtube video)

Photoelectric Effect

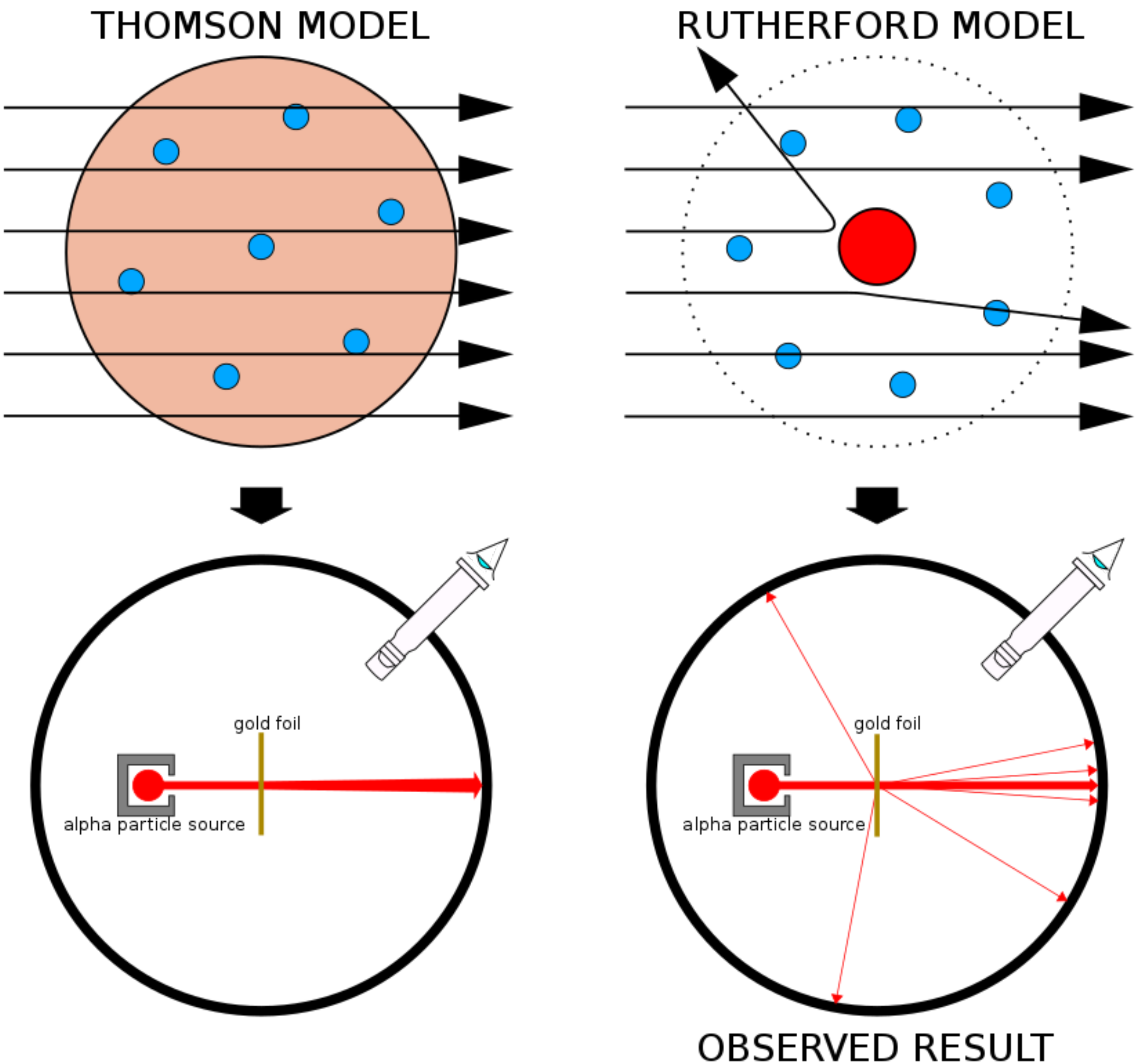


- Whether or not an electron is produced depends **only** on the **frequency** of the light, **not** the **flux** of the light
- Condition for producing an electron: $E_{photon} > \phi$
- A single photon has energy: $E_{photon} = h\nu$ ← Planck's quantized energy of light waves!

LIGHT IS A

particle!

Atomic Structure



Most of the space within an atom is empty!

The radius the of nucleus was 10,000 times smaller than the radius of the atom

Hydrogen Spectral Lines

- In 1885, a Swiss teacher, Johann Balmer found an empirical formula to reproduce the wavelengths of hydrogen lines (in visible wavelengths), today known as the **Balmer series**

$$\frac{1}{\lambda} = R_H \left(\frac{1}{4} - \frac{1}{n^2} \right)$$

where $n = 3, 4, 5, \ldots,$ and $R_H = 1.09677583 \times 10^7 \pm 1.3 \text{ m}^{-1}$

is the experimentally determined Rydberg constant for hydrogen

Hydrogen Spectral Lines

More generally, $$\frac{1}{\lambda} = R_H \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$

- **Lyman series**: m = 1
- **Balmer series**: m = 2
- **Paschen series**: m = 3

Series Name	Symbol	Transition	Wavelength (nm)
Lyman	Lyα	$2 \leftrightarrow 1$	121.567
	Lyβ	$3 \leftrightarrow 1$	102.572
	Lyγ	$4 \leftrightarrow 1$	79.254
	$\mathrm{Ly}_{\mathrm{limit}}$	$\infty \leftrightarrow 1$	91.18
Balmer	Hα	$3 \leftrightarrow 2$	656.281
	Hβ	$4 \leftrightarrow 2$	486.134
	Hγ	$5 \leftrightarrow 2$	434.048
	Hδ	$6 \leftrightarrow 2$	410.175
	Hϵ	$7 \leftrightarrow 2$	397.007
	H_8	$8 \leftrightarrow 2$	388.905
	$\mathrm{H}_{\mathrm{limit}}$	$\infty \leftrightarrow 3$	364.6
Paschen	Paα	$4 \leftrightarrow 3$	1875.10
	Paβ	$5 \leftrightarrow 3$	1281.81
	Paγ	$6 \leftrightarrow 3$	1093.81
	$\mathrm{Pa}_{\mathrm{limit}}$	$\infty \leftrightarrow 3$	820.4

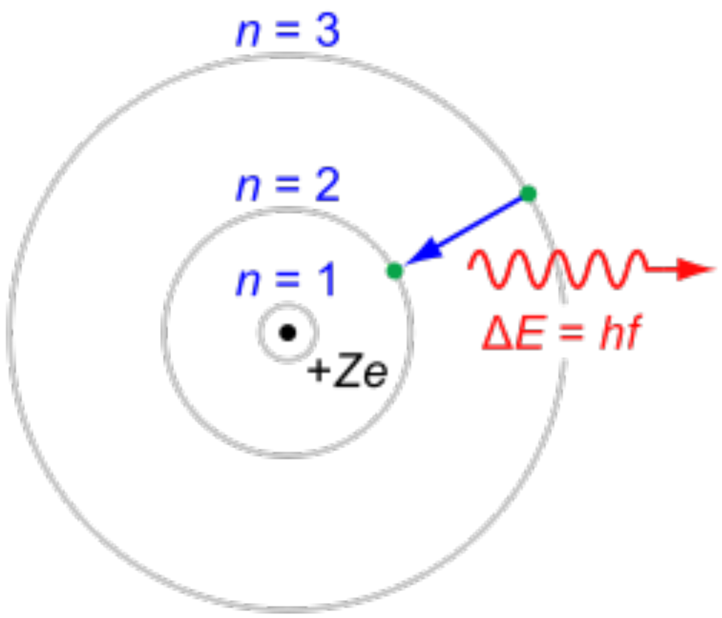
Bohr's Semiclassical Atomic Model

Model:

- Nucleus is a point mass sitting at the center of the atom
- Electrons move in circular orbits

Bohr's Postulations:

- Only a discrete number of orbits of the electrons (those with angular momentum that is an integral multiple of $\hbar = h/2\pi$) is allowed
- Radiation of a discrete quantum of energy is emitted/absorbed only when the electrons jumps from one orbit to another
- The radiated energy equals to the energy difference between the orbits



The unit of Planck's constant

Planck's constant has units of

$$\mathrm{J\,s} = [\mathrm{kg\,m^2\,s^{-2}}]\,\mathrm{s} = \mathrm{kg}\,[\mathrm{m\,s^{-1}}]\,\mathrm{m}$$

which are the units of angular momentum (mvr).

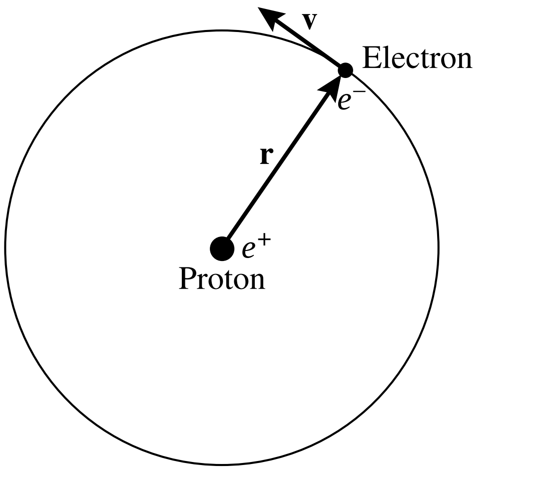
Bohr's Model of Hydrogen Atom

Electric force provides centrifugal acceleration

$$\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\,\hat{\mathbf{r}} = -\mu\,\frac{v^2}{r}\,\hat{\mathbf{r}},$$

Kinetic energy

$$K = \frac{1}{2}\mu v^2 = \frac{1}{8\pi\epsilon_0}\frac{e^2}{r}$$



Electrical potential energy

$$U = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = -2K$$

Total energy

$$E = K + U = K - 2K = -K = -\frac{1}{8\pi\epsilon_0}\frac{e^2}{r}.$$

Bohr's Model of Hydrogen Atom

Now use Bohr's quantization of angular momentum

$$L = \mu v r = n\hbar$$

To rewrite kinetic energy

$$\frac{1}{8\pi\epsilon_0}\frac{e^2}{r} = \frac{1}{2}\mu v^2 = \frac{1}{2}\frac{(\mu v r)^2}{\mu r^2} = \frac{1}{2}\frac{(n\hbar)^2}{\mu r^2}$$

We have the radius of allowed the electron orbits

$$\boxed{r_n = \frac{4\pi\epsilon_0\hbar^2}{\mu e^2}n^2 = a_0 n^2,}$$

Inserting this to the expression of total energy

$$\boxed{E_n = -\frac{\mu e^4}{32\pi^2\epsilon_0^2\hbar^2}\frac{1}{n^2} = -13.6\ \mathrm{eV}\frac{1}{n^2}.}$$

n is the **principal quantum number**

Hydrogen spectral lines derived

Wavelength of the emitted photon

$$\boxed{E_{\text{photon}} = E_{\text{high}} - E_{\text{low}}}$$

Or

$$\frac{hc}{\lambda} = \left(-\frac{\mu e^4}{32\pi^2\epsilon_0^2\hbar^2}\frac{1}{n_{\text{high}}^2}\right) - \left(-\frac{\mu e^4}{32\pi^2\epsilon_0^2\hbar^2}\frac{1}{n_{\text{low}}^2}\right)$$

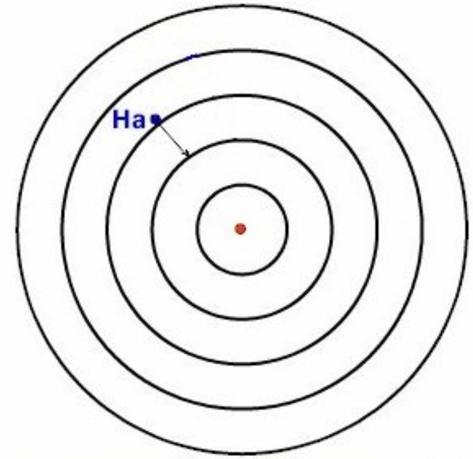
Which gives

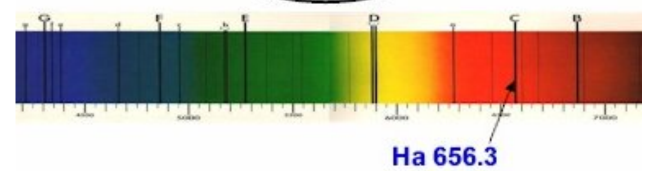
$$\frac{1}{\lambda} = \frac{\mu e^4}{64\pi^3\epsilon_0^2\hbar^3 c}\left(\frac{1}{n_{\text{low}}^2} - \frac{1}{n_{\text{high}}^2}\right) \qquad R_H = \frac{\mu e^4}{64\pi^3\epsilon_0^2\hbar^3 c} = 10967758.3\ \text{m}^{-1}$$

Compare with Balmer's empirical expression

$$\frac{1}{\lambda} = R_H\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

Example: Hα line





- Hα line is produced when the electron in the hydrogen atom makes the transition from the n = 3 to the n = 2 orbit. What is the wavelength?

$$\frac{1}{\lambda} = \frac{\mu e^4}{64\pi^3\epsilon_0^2\hbar^3 c}\left(\frac{1}{n_{\text{low}}^2} - \frac{1}{n_{\text{high}}^2}\right)$$

$$R_H = \frac{\mu e^4}{64\pi^3\epsilon_0^2\hbar^3 c} = 10967758.3\ \text{m}^{-1}$$

$$E_{\text{photon}} = E_{\text{high}} - E_{\text{low}}$$

$$\frac{hc}{\lambda} = -13.6\ \text{eV}\ \frac{1}{n_{\text{high}}^2} - \left(-13.6\ \text{eV}\ \frac{1}{n_{\text{low}}^2}\right)$$

$$= -13.6\ \text{eV}\ \left(\frac{1}{3^2} - \frac{1}{2^2}\right).$$

$$\lambda = 656.469\ \text{nm}$$

Comparing to the measured wavelength in the air: $\lambda_{air} = 656.275$ nm

Why?

Which of the following transitions of the hydrogen atom has the **shortest** wavelength?

A. n = 3 −> n = 2 (Hα)

B. n = 4 -> n = 2 (Hβ)

C. n = 2 -> n = 1 (Lyman α)

D. n = 5 -> n = 3 (Paschen γ)

E. n = 4 -> n = 3 (Paschen α)

Hydrogen spectral lines and Kirchhoff's Law

1. A **hot, dense** gas or hot solid object produces a **continuous spectrum**
2. A **hot, diffuse** gas produces bright spectral lines (**emission lines**)
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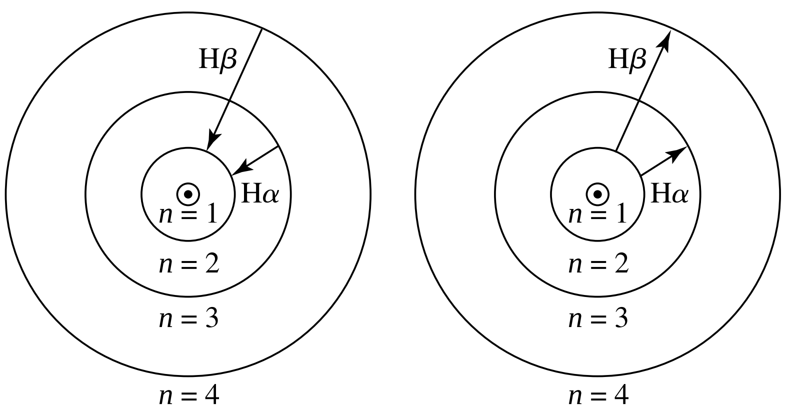
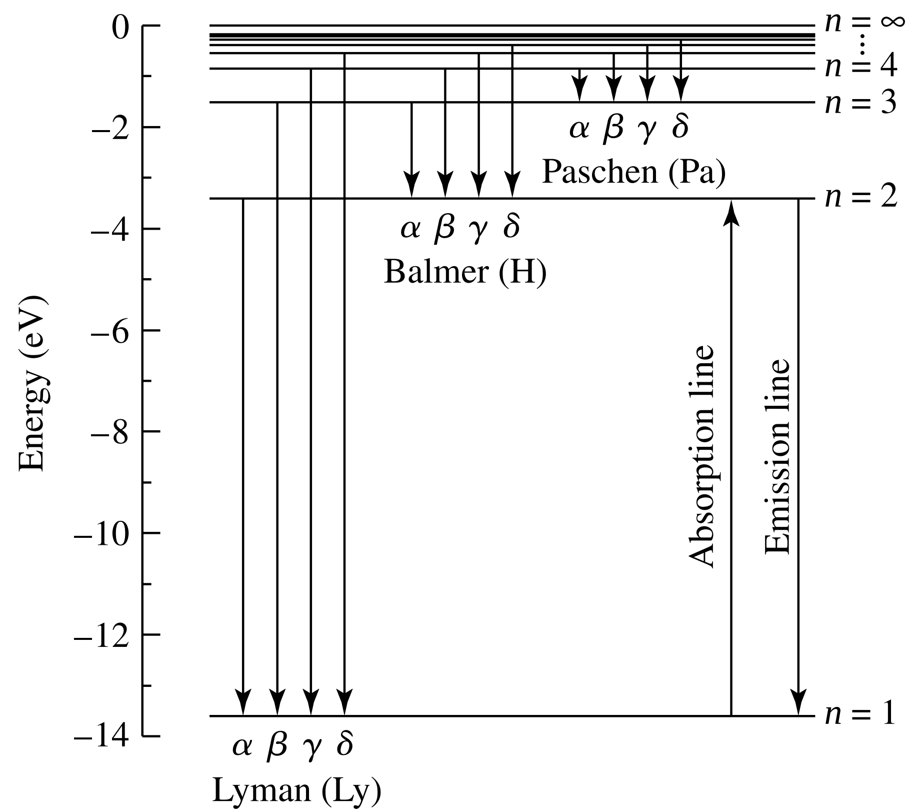


Emission Lines Absorption Lines



More quantum physics…

Useful for later lectures

Wave-particle duality of *all* particles

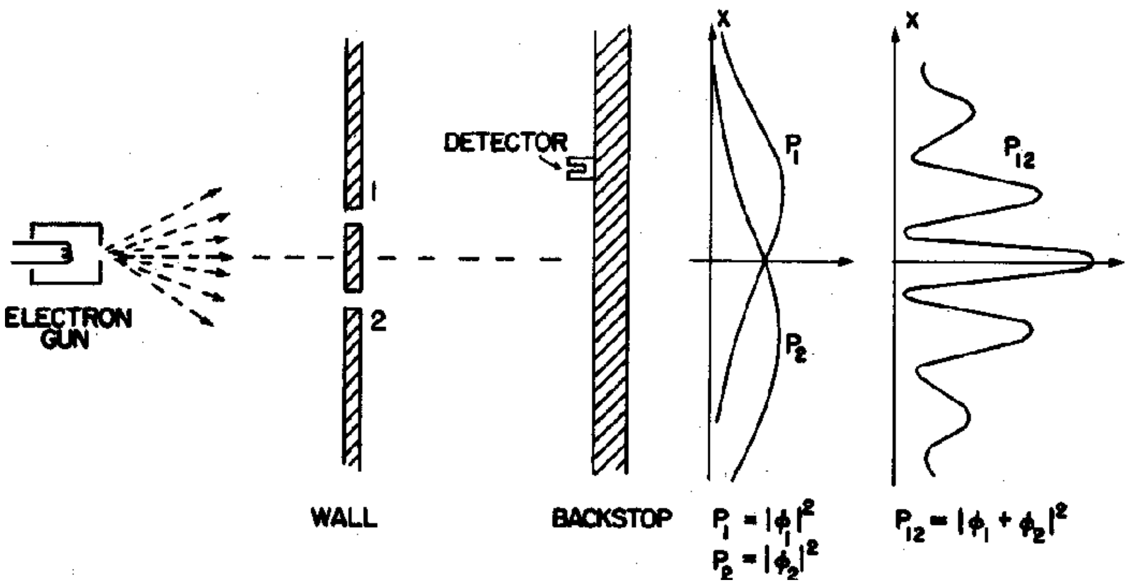
- In his 1927 Ph.D. thesis, de Broglie's extended the wave-particle duality to ***all*** particles
- de Broglie wavelength

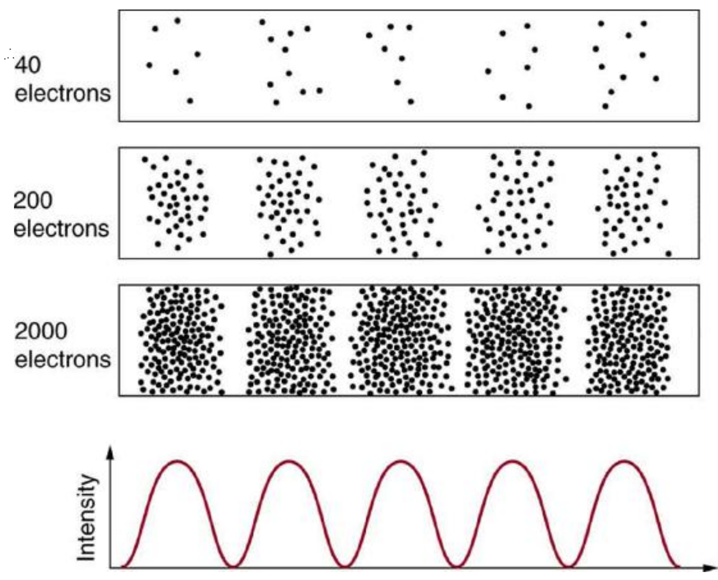
$$\lambda = \frac{h}{p}$$

- ***Everything*** exhibits wave properties!

If a light-wave could also act like a particle, why shouldn't matter-particles also act like waves?

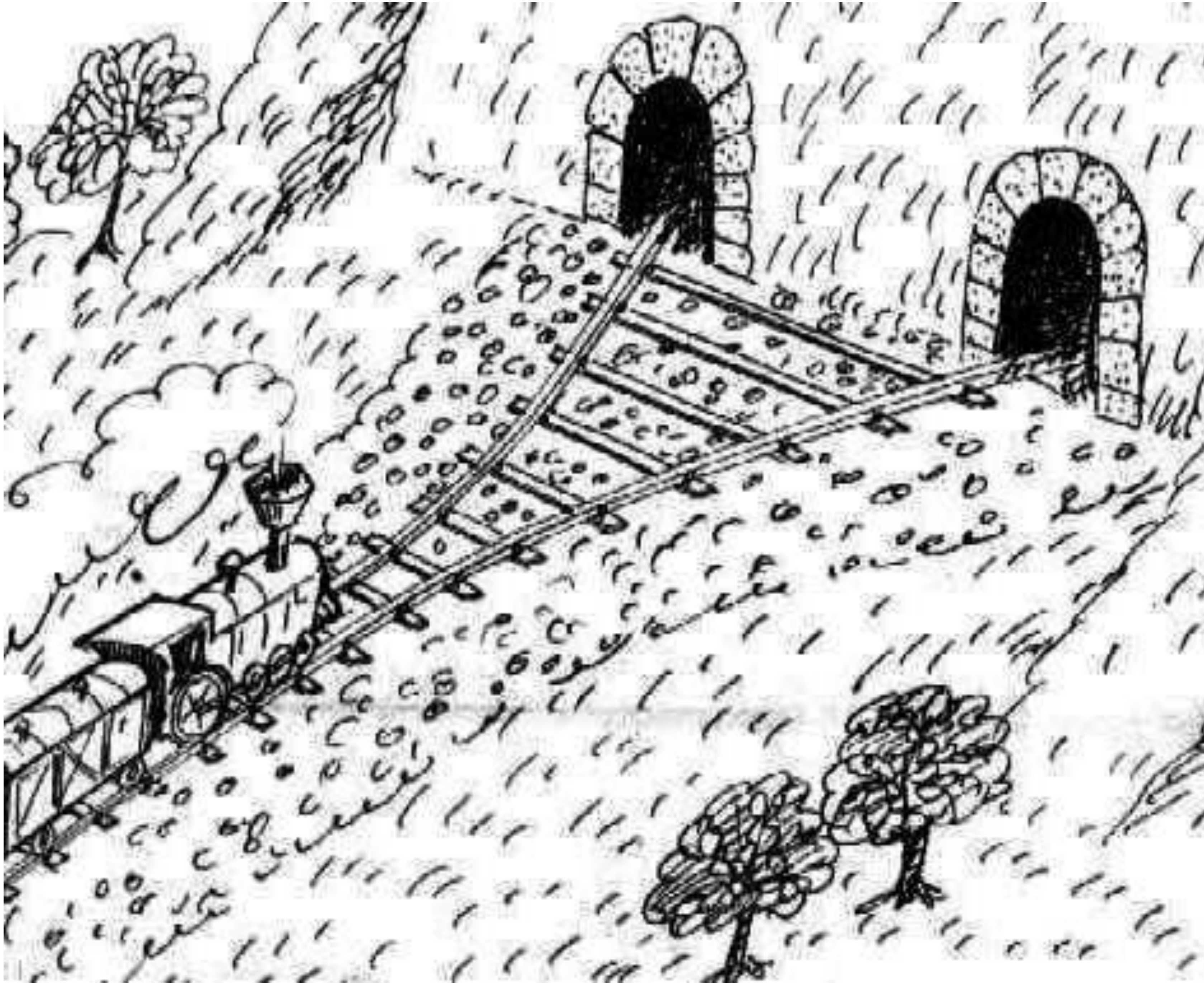
Electron double-slit experiment





Electrons exhibit wave-particle duality

Each electron passes ***both*** slit

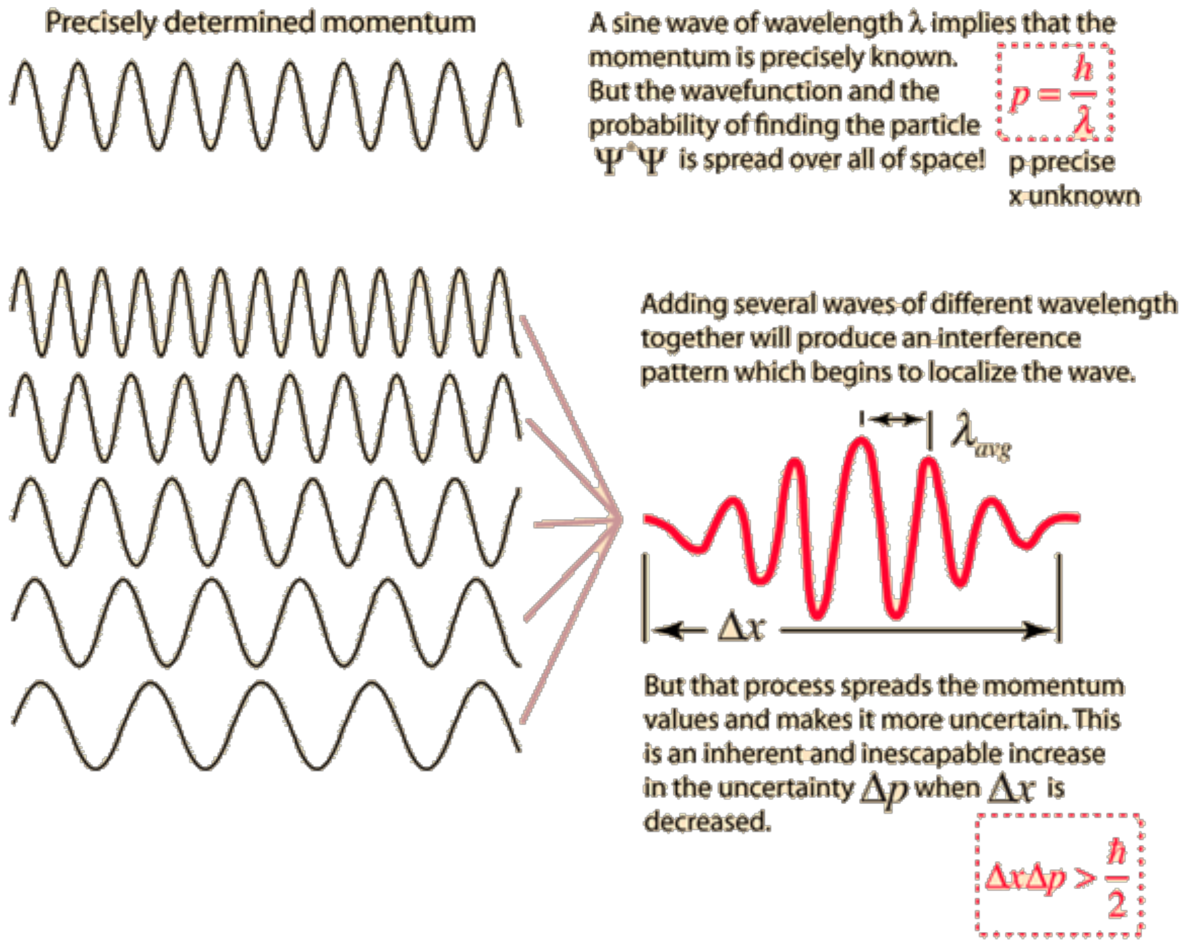


Example: Wavelength of a jogger

- What is the wavelength of a 70-kg man jogging at 3 m/s?

$$\lambda = \frac{h}{p} = \frac{h}{m_{\text{man}} v} = 3.16 \times 10^{-36}\ \text{m}$$

Heisenberg's Uncertainty Principle



Forms often used: $\Delta x \, \Delta p \approx \hbar.$ $\Delta E \, \Delta t \approx \hbar.$

More quantum numbers

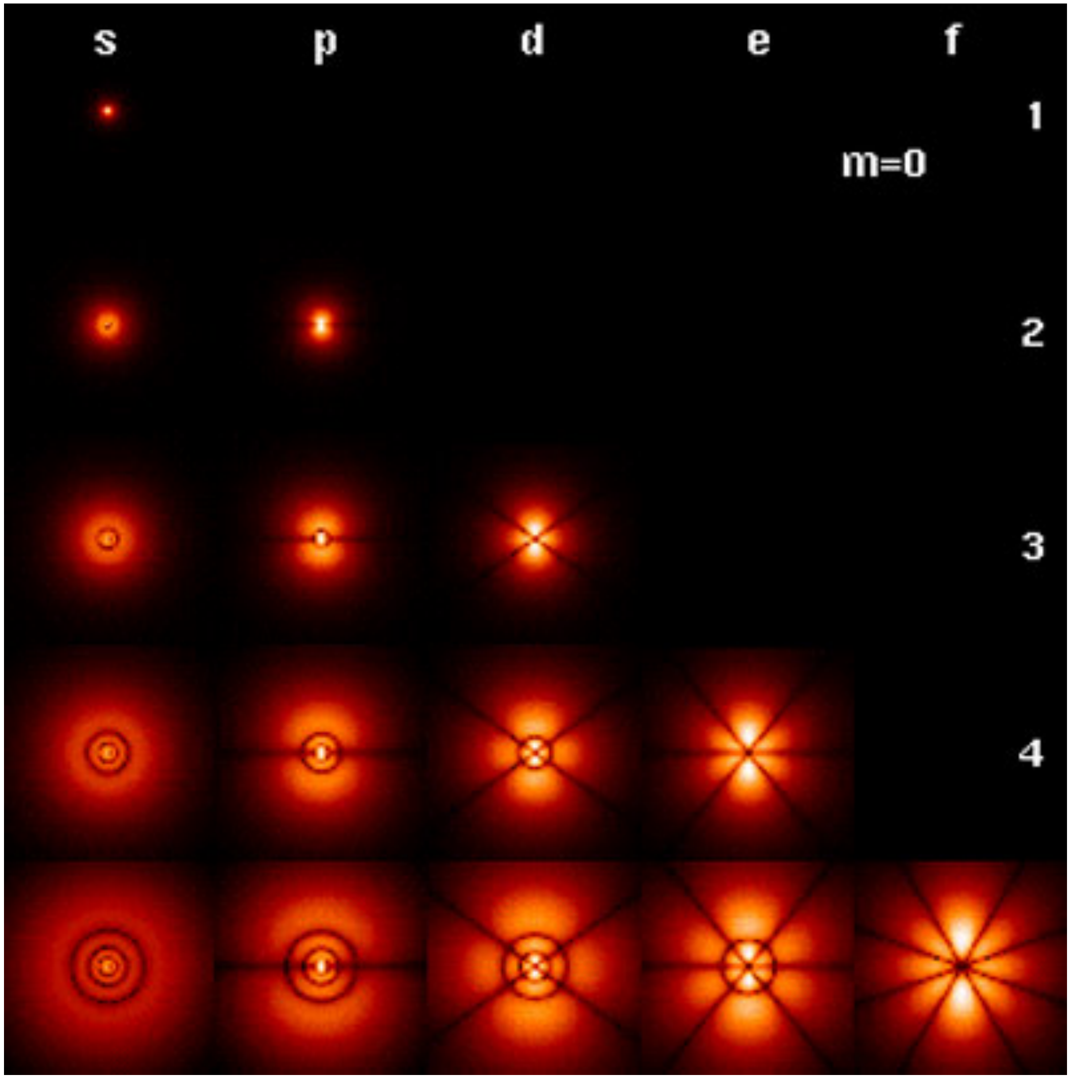
- Heisenberg's uncertainty principle implies that electrons cannot have well-defined circular orbits and angular momentum at the same time

  ⇨ Bohr's model must be updated!

- Electron orbits are "fuzzy"
- More than one quantum number
  - Principle quantum number ***n***: n = 1, 2, 3 …
    - ***Energy*** and ***Size*** of the orbital
  - Angular momentum quantum number ***l***: *l* = 0, …, *n-1*
    - ***Shape*** of the orbital
  - Magnetic quantum number $\boldsymbol{m_l}$: ml= -*l*, …, 0, … +*l*
    - ***Orientation*** of the orbital
  - Spin quantum number $\boldsymbol{m_s}$: m_s = + ½ or - ½
    - ***Orientation of the spin axis*** of the electron

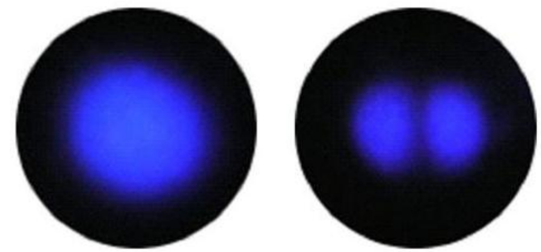
Atomic Orbitals: Probability Map

l = 0 l = 1 l = 2 l = 3 l = 4



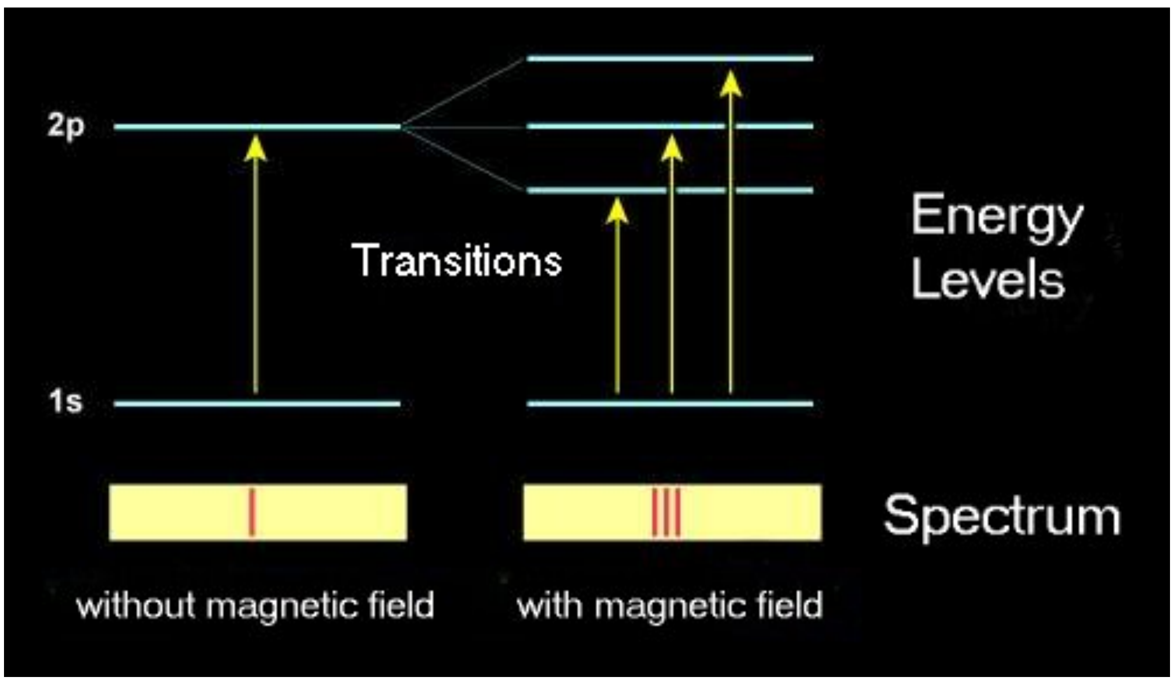
n = 1
n = 2
n = 3
n = 4

Comparing with experimental results



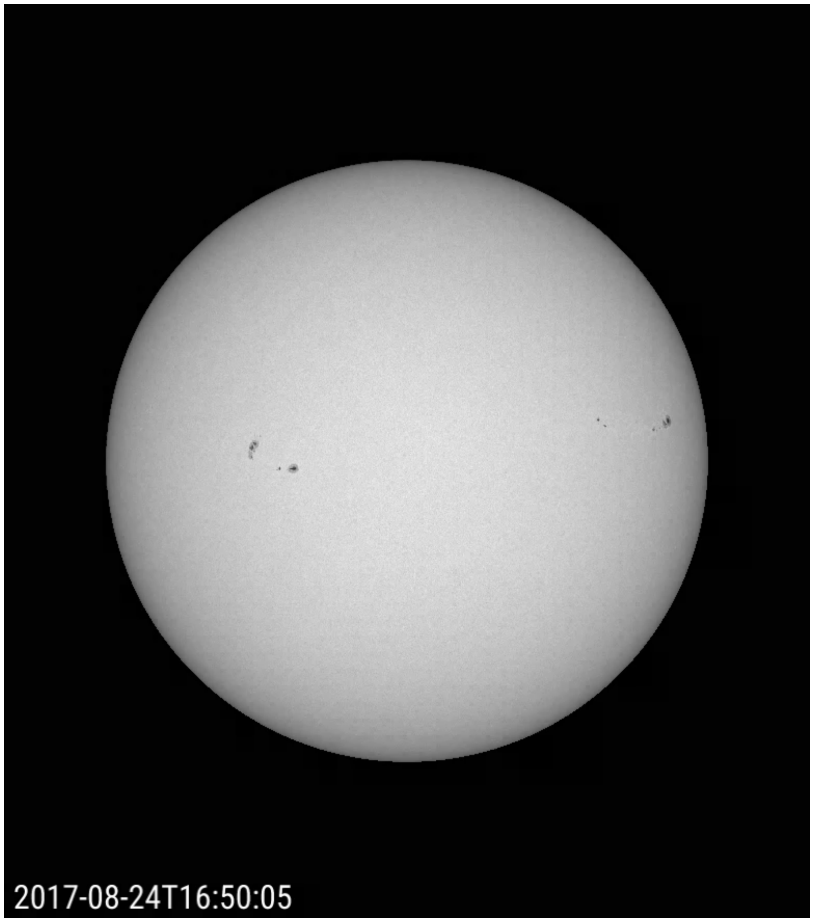
Zeeman Effect

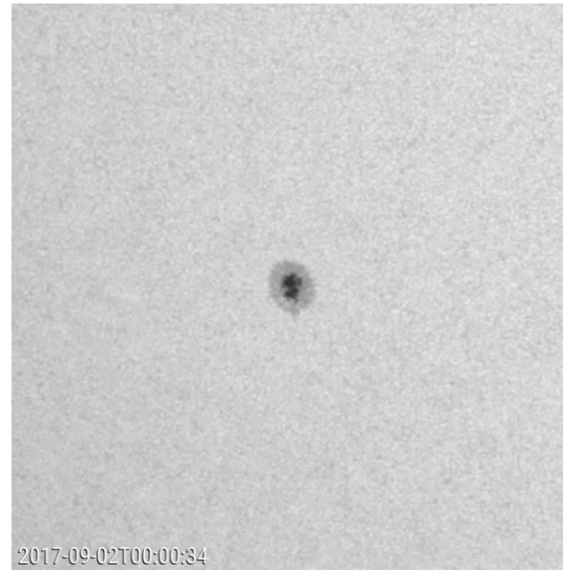
- Energy states correspond to the magnetic quantum number $\boldsymbol{m_l}$ (which describes the orientation of the electron's motion) are **separated** under the presence of **external magnetic field**

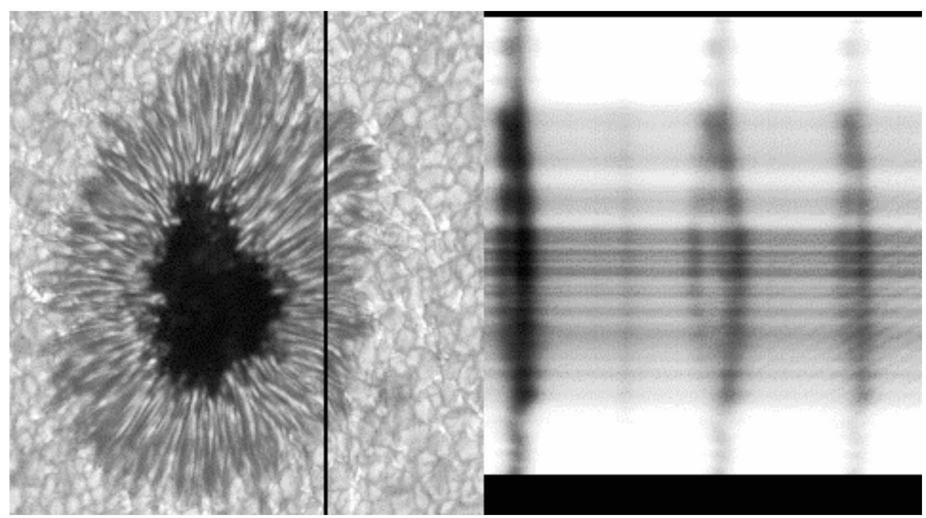


$$\nu = \nu_0 \qquad \text{and} \qquad \nu_0 \pm \frac{eB}{4\pi\mu},$$

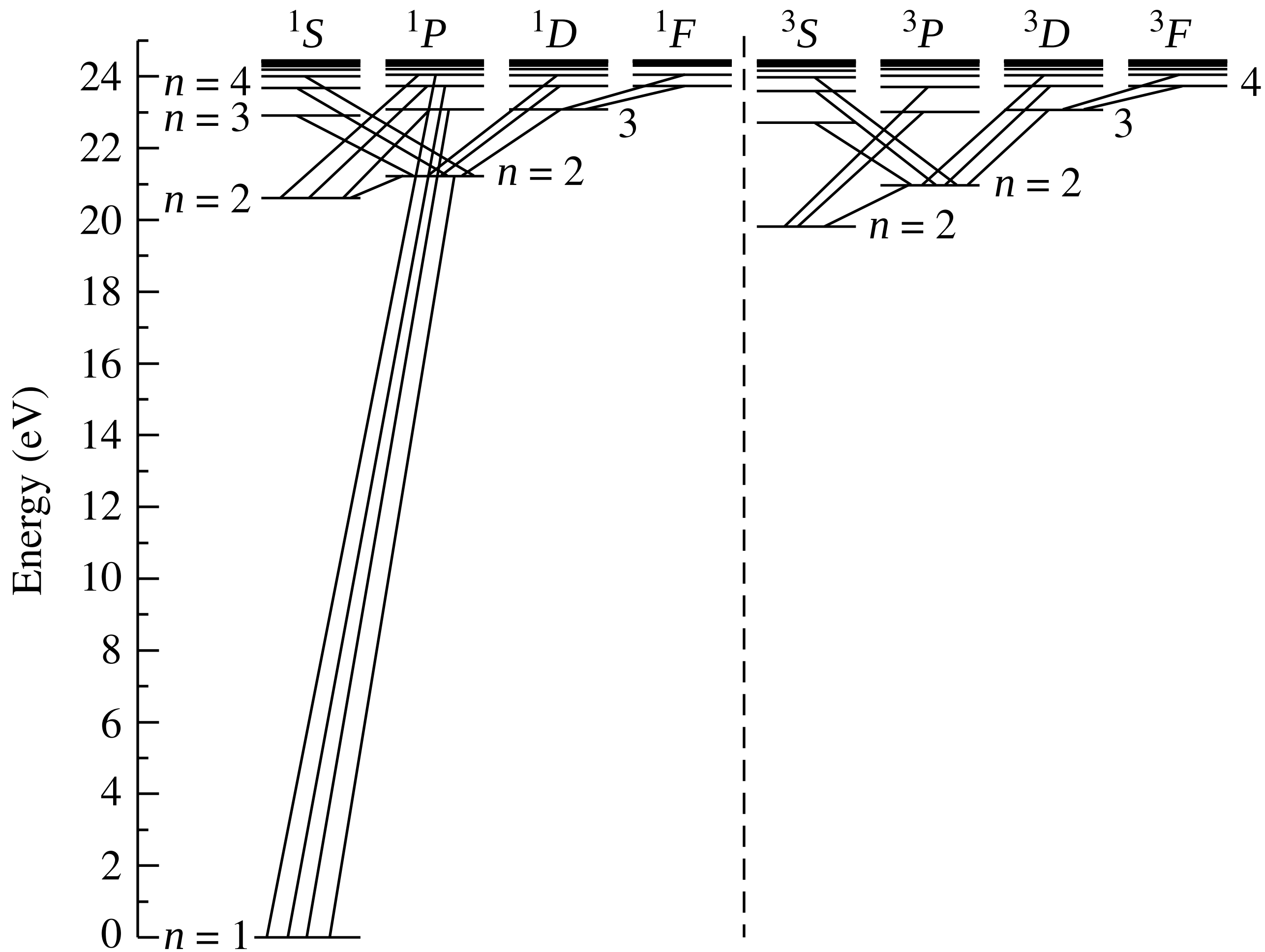
Zeeman Effect and the Sun’s magnetic field







The Complex Spectra of Atoms



Some of the electronic energy levels of the helium atom and possible allowed transitions