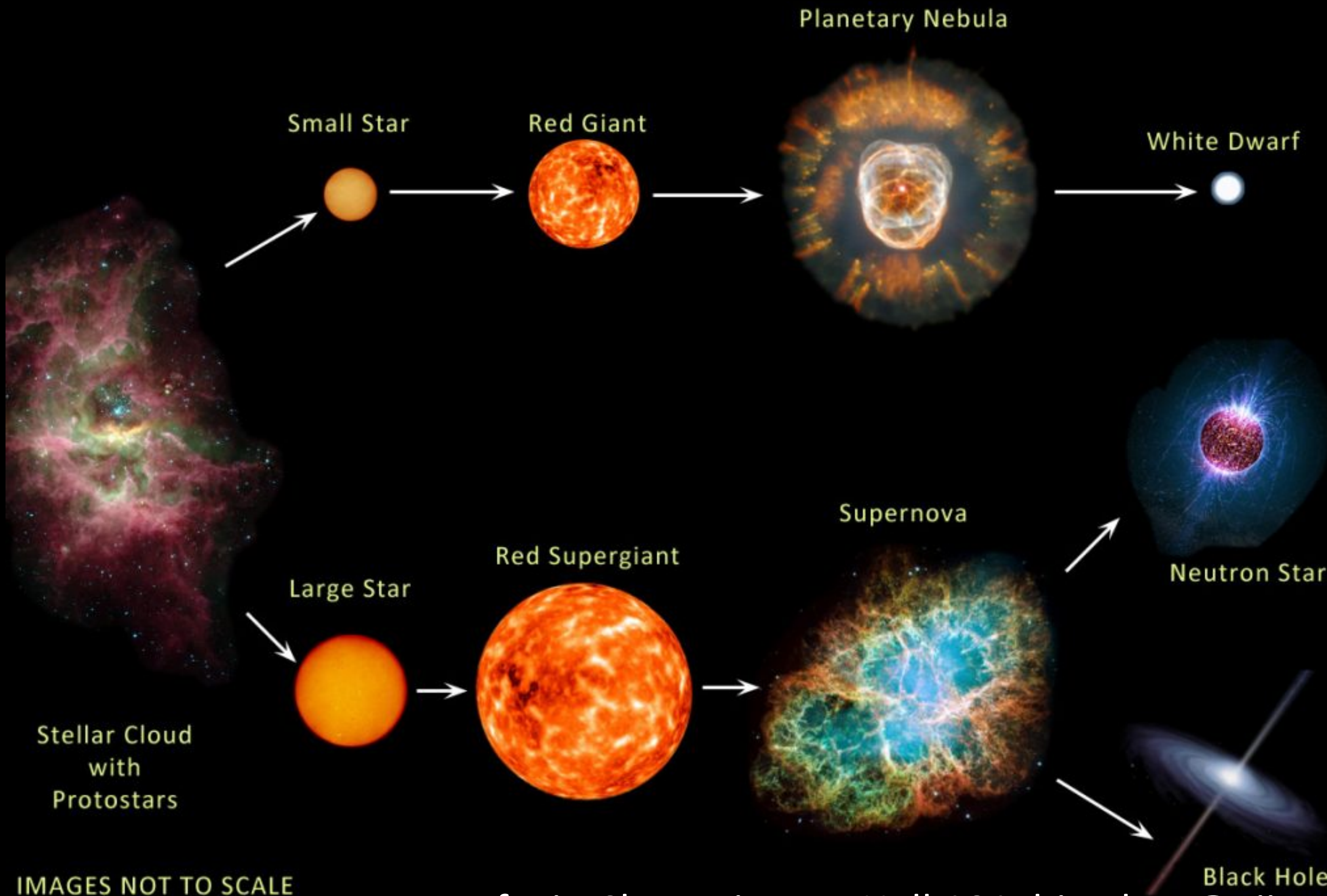
Phys 321: Lecture 8

Stellar Remnants



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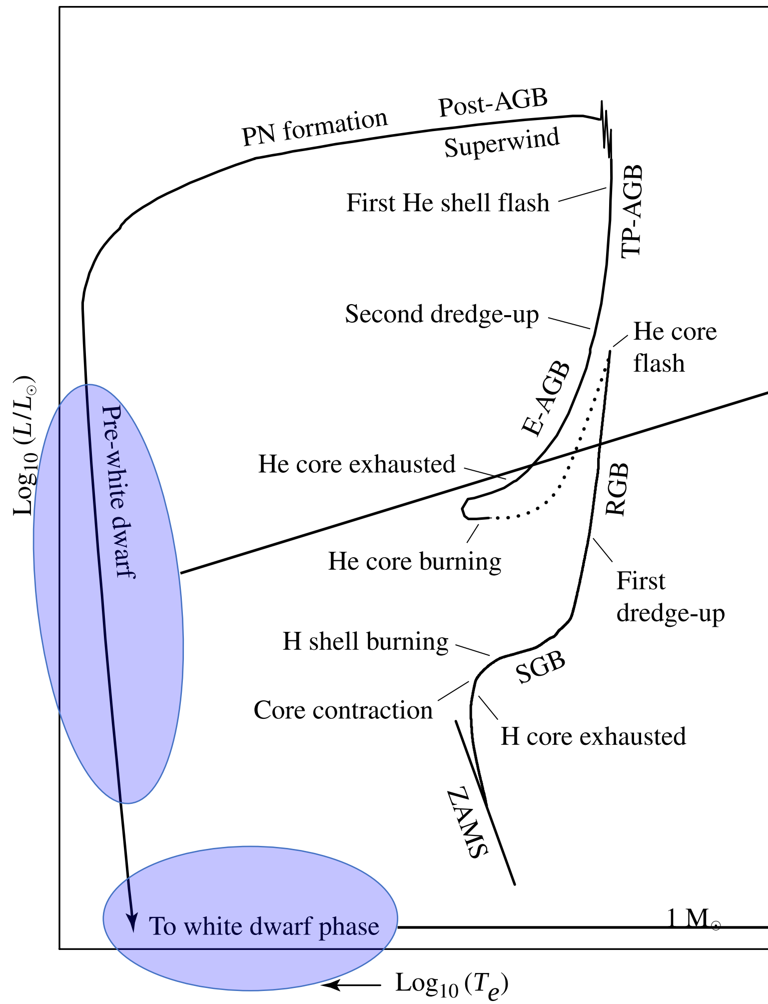
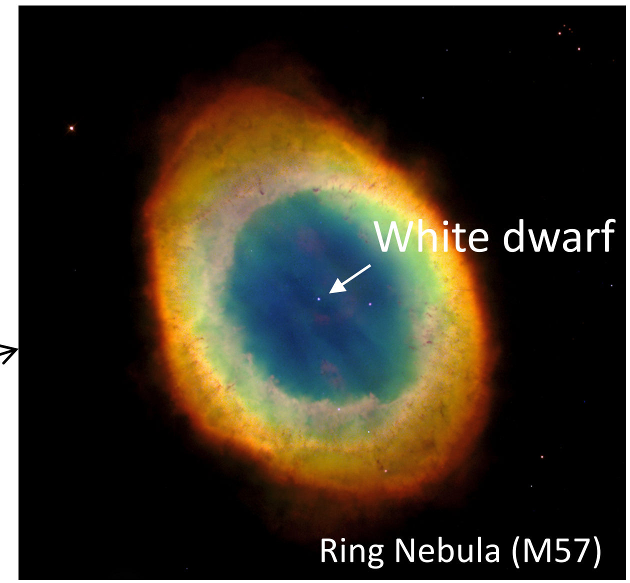
Evolution of a low-mass star

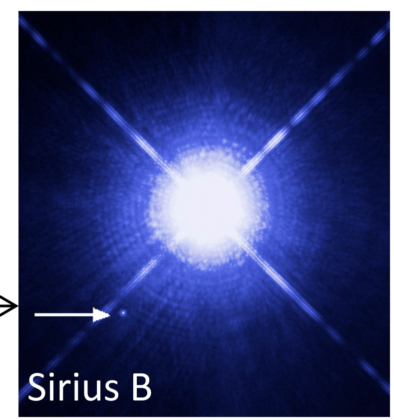
Evolution track of an 1 solar-mass star



Planetary Nebula



Red: Nitrogen, Green: Oxygen, Blue: Helium

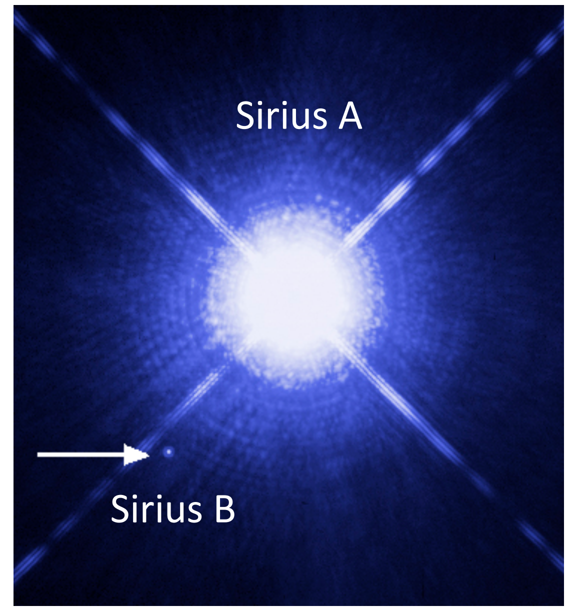


Death of low-mass stars

- [Crash course by Phil Plait]

The discovery of Sirius B

- In 1844, Friedrich Bessel found the brightest star in the night sky, Sirius, has a companion, using precise parallax observations for ten years
- In 1862, Alvan G. Clark discovered the “Pup” at its predicted position using his father’s new 18-inch refractor
- The Pup is very faint (~0.03 L_{sun}) so, probably cool and red?
- In 1915, Walter Adams discovered that, however, the surface temperature is ~27,000 K! A hot, blue-white star!



Estimating radius of Sirius B

- Luminosity of Sirius B is 0.03 L_{sun}
- Effective temperature is 27,000 K
- Mass is ~1.05 solar mass
- What is its radius?

$$1\ \mathrm{L}_\odot = 3.839(5) \times 10^{26}\ \mathrm{W}$$

$$\begin{aligned}\sigma &\equiv 2\pi^5 k^4/(15c^2h^3) \\ &= 5.670400(40) \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}\end{aligned}$$

Solar Radius: $1\ \mathrm{R}_\odot = 6.95508(26) \times 10^{8}\ \mathrm{m}$

Earth Radius: $1\ \mathrm{R}_\oplus = 6.378136 \times 10^{6}\ \mathrm{m}$

Using **Stefan-Boltzman's Law**: $L = 4\pi R^2 \sigma T^4$

We have R_{WD} ~ 5.5 x 10^6 m

This means compressing the mass of the entire Sun within a volume < Earth!

Properties of white dwarfs

Density

$$1\ \mathrm{M}_\odot \quad = \quad 1.9891 \times 10^{30}\ \mathrm{kg}$$

Sirius B $R_{WD} \sim 5.5 \times 10^6$ m

$$G \quad = \quad 6.673(10) \times 10^{-11}\ \mathrm{N\ m^2\ kg^{-2}}$$

$\boxed{\rho \sim 3 \times 10^9\ \mathrm{kg\ m^{-3}}}$

Surface gravity: Phil said 100,000x that of the Earth's gravity

$$g = G\frac{M}{R^2} \qquad \boxed{\sim 4.6 \times 10^6\ \mathrm{m\ s^{-2}}}$$

Central conditions of WDs

- Using the temperature gradient equation due to radiative energy transfer (p. 160 of textbook), the central temperature of WDs is ~ several times 10^7 K
- Hot enough for H fusion, sometimes also He
- Not enough for triggering carbon and oxygen fusion
- WDs consist primarily of completely ionized helium/carbon/oxygen/neon/magnesium nuclei

Initial mass of the star:

(< 0.5 Msun) => He white dwarf
(0.5 - 5 Msun) => Carbon/Oxygen white dwarf
(5 -7 Msun) => Oxygen/Neon/Magnesium white dwarf

Central pressure of WDs

From hydrostatic equilibrium, assuming a constant density (which is unrealistic)

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2} = -\frac{G\left(\frac{4}{3}\pi r^3\rho\right)\rho}{r^2} = -\frac{4}{3}\pi G\rho^2 r.$$

Integrate from r to r = R (surface), where P (r = R) = 0, the pressure becomes

$$P(r) = \frac{2}{3}\pi G\rho^2\left(R^2 - r^2\right)$$

The central pressure is

$$P_c \approx \frac{2}{3}\pi G\rho^2 R_{\mathrm{wd}}^2 \approx 3.8 \times 10^{22}\ \mathrm{N\ m^{-2}}$$

About **1.5 million times** larger than the pressure at the center of the Sun

The physics of degenerate matter

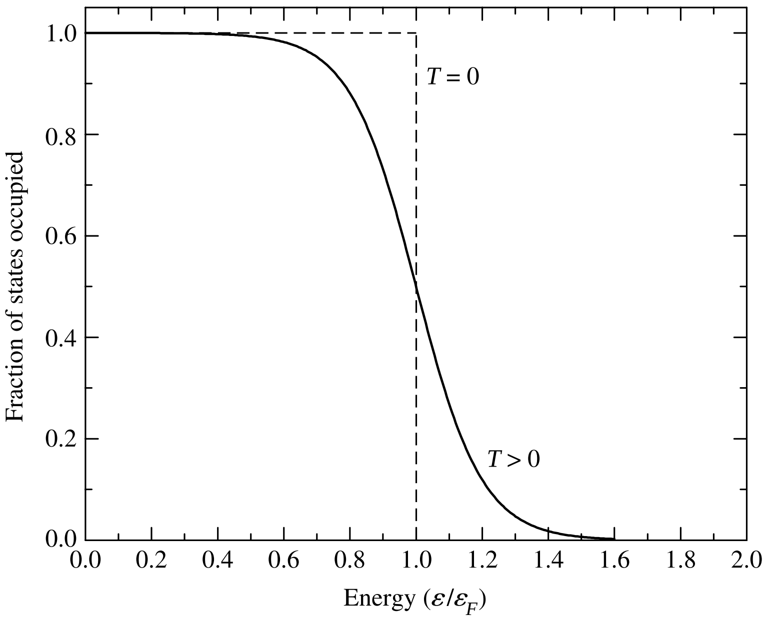
- The Pauli Exclusion Principle allows at most one fermion to occupy each quantum state
- Everyday gas at room temperature and pressure, 1 of every 10^7 quantum states is occupied by a gas particle
  - Degeneracy is insignificant
  - Pressure is dominated by thermal pressure
- When all the electrons (fermions) are squeezed together due to high density and pressure, electron degeneracy pressure becomes important

Fermi Energy

- Fermi energy ε_F : the maximum energy of any electron in a completely degenerate gas at T = 0

$$\boxed{\varepsilon_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3},}$$

Mass of electron Number density of electron



If expressed in mass density ρ of fully ionized gas

\# of protons per nuclei

$$\varepsilon_F = \frac{\hbar^2}{2m_e}\left[3\pi^2\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{2/3} \quad \text{with} \quad n_e = \left(\frac{\#\text{ electrons}}{\text{nucleon}}\right)\left(\frac{\#\text{ nucleons}}{\text{volume}}\right) = \left(\frac{Z}{A}\right)\frac{\rho}{m_H}$$

\# of nucleons per nuclei

Condition for degeneracy

If the average thermal energy of an electron 3/2kT, is smaller than the Fermi Energy

$$\frac{3}{2}kT < \varepsilon_F$$

An average electron is unable to make a transition to an unoccupied quantum state to "break" the degeneracy, and the electron gas will stay degenerate

$$\frac{3}{2}kT < \frac{\hbar^2}{2m_e}\left[3\pi^2\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{2/3},$$

or

$$\frac{T}{\rho^{2/3}} < \frac{\hbar^2}{3m_e k}\left[\frac{3\pi^2}{m_H}\left(\frac{Z}{A}\right)\right]^{2/3} = 1261 \text{ K m}^2 \text{ kg}^{-2/3}$$

For Z/A = 0.5

The condition for degeneracy becomes $\boxed{\frac{T}{\rho^{2/3}} < \mathcal{D}.}$ where $\mathcal{D} \equiv 1261 \text{ K m}^2 \text{ kg}^{-2/3}$

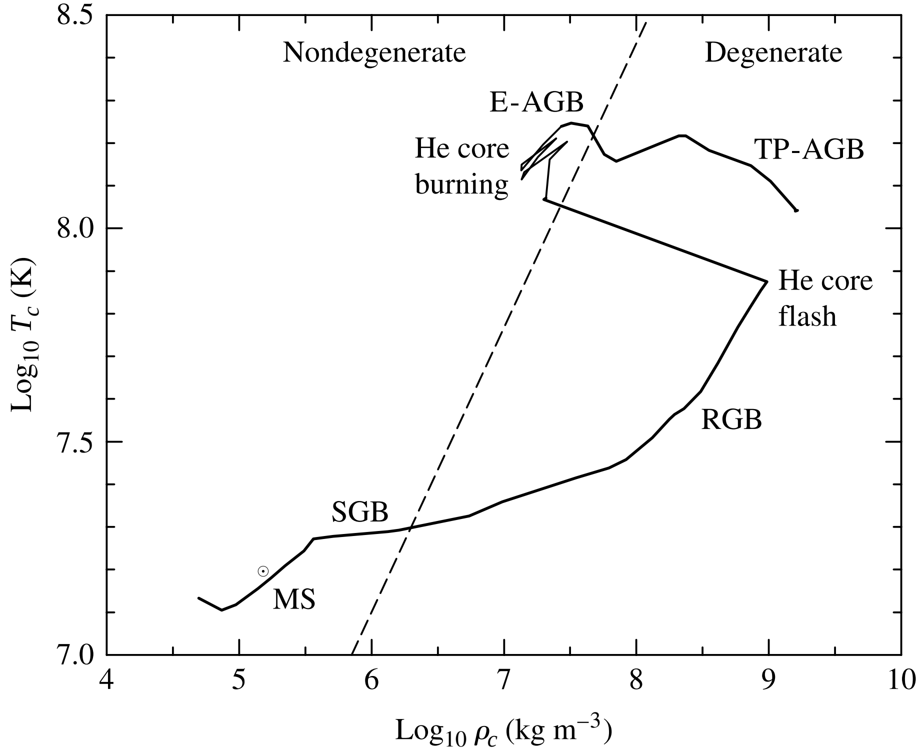
Degenerate or not

Example 3.1. How important is electron degeneracy at the centers of the Sun and Sirius B? At the center of the standard solar model, $T_c = 1.570 \times 10^7$ K and $\rho_c = 1.527 \times 10^5$ kg m^{-3}. Then

$$\frac{T_c}{\rho_c^{2/3}} = 5500 \text{ K m}^2 \text{ kg}^{-2/3} > \mathcal{D}.$$

For Sirius B:

$$\frac{T_c}{\rho_c^{2/3}} = 37 \text{ K m}^2 \text{ kg}^{-2/3} \ll \mathcal{D},$$



Degeneracy in the Sun's center as it evolves

Electron degeneracy pressure

1. The Pauli exclusion principle, which allows at most one electron in each quantum state; and
2. Heisenberg's uncertainty principle in the form of,

$$\Delta x \, \Delta p_x \approx \hbar,$$

Pressure in a gas associated with particle's momentum

$$P \approx \frac{1}{3} n_e p v$$

$$n_e = \left(\frac{\text{\# electrons}}{\text{nucleon}}\right)\left(\frac{\text{\# nucleons}}{\text{volume}}\right) = \left(\frac{Z}{A}\right)\frac{\rho}{m_H}$$

For a degeneracy gas, electrons are packed as tightly as possible, so the separation between electrons is just $n_e^{1/3}$. Heisenberg's uncertainty principle gives

1D: $$p_x \approx \Delta p_x \approx \frac{\hbar}{\Delta x} \approx \hbar n_e^{1/3}$$

3D: $$p^2 = p_x^2 + p_y^2 + p_z^2 = 3p_x^2$$

or $$p = \sqrt{3} p_x$$

Electron degeneracy pressure

$$P \approx \frac{1}{3} n_e p v$$

$$n_e = \left(\frac{\text{\# electrons}}{\text{nucleon}}\right)\left(\frac{\text{\# nucleons}}{\text{volume}}\right) = \left(\frac{Z}{A}\right)\frac{\rho}{m_H}$$

$$p = \sqrt{3} p_x \qquad p_x \approx \Delta p_x \approx \frac{\hbar}{\Delta x} \approx \hbar n_e^{1/3}$$

$$P \approx \frac{\hbar^2}{m_e}\left[\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{5/3}$$

$$p \approx \sqrt{3}\hbar\left[\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{1/3}$$

$$\begin{aligned} v &= \frac{p}{m_e} \\ &\approx \frac{\sqrt{3}\hbar}{m_e} n_e^{1/3} \\ &\approx \frac{\sqrt{3}\hbar}{m_e}\left[\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{1/3} \end{aligned}$$

For nonrelativistic case

Exact calculation gives:

$$\boxed{P = \frac{(3\pi^2)^{2/3}}{5}\frac{\hbar^2}{m_e}\left[\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{5/3}.}$$

$$P \sim \rho^{5/3}$$

Mass-Volume Relation

By setting the central pressure to equal to electron degenerate pressure:

$$\frac{2}{3}\pi G\rho^2 R_{\rm wd}^2 = \frac{(3\pi^2)^{2/3}}{5}\frac{\hbar^2}{m_e}\left[\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{5/3}$$

Use $\rho = M_{\rm wd}/\frac{4}{3}\pi R_{\rm wd}^3$

$$R_{\rm wd} \approx \frac{(18\pi)^{2/3}}{10}\frac{\hbar^2}{G m_e M_{\rm wd}^{1/3}}\left[\left(\frac{Z}{A}\right)\frac{1}{m_H}\right]^{5/3}$$

we have a mass-volume relation:

$$\boxed{M_{\rm wd} V_{\rm wd} = \text{constant.}}$$

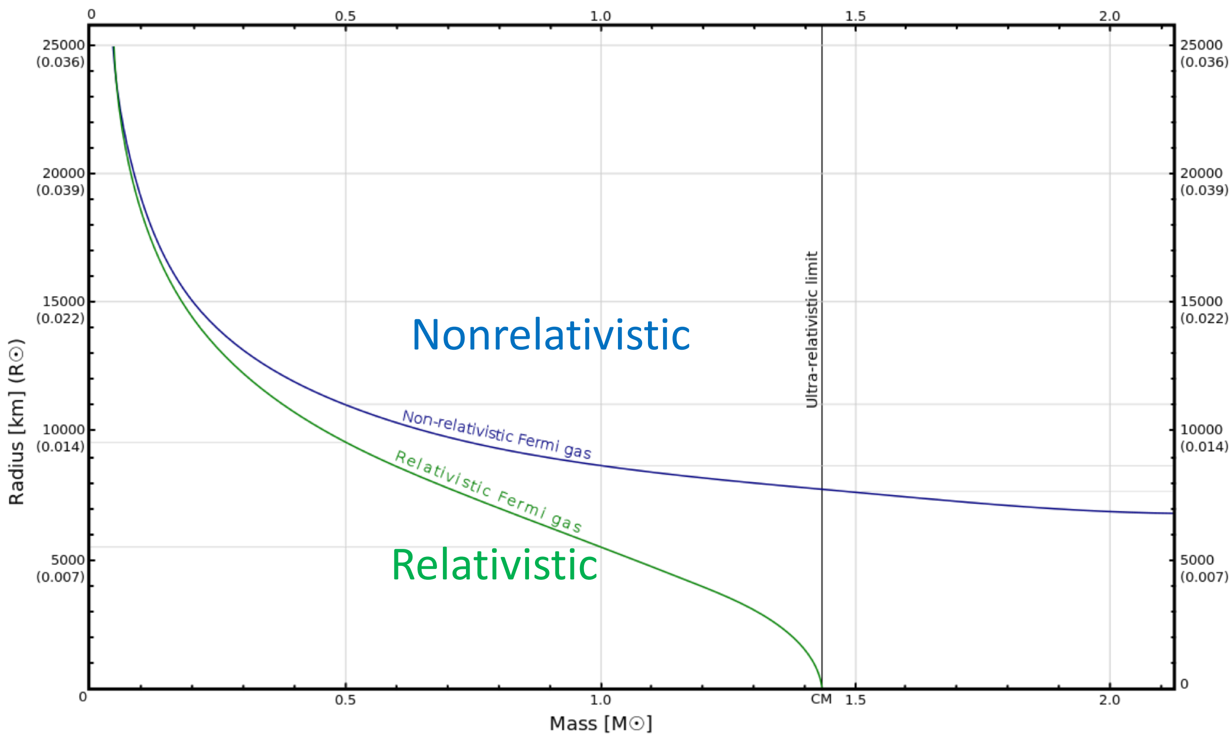
More **massive** white dwarfs are **smaller**!

The Chandrasekhar Limit

- Packing more mass onto the WD will decrease the volume of the WD
- Density increases significantly

$$v \approx \frac{\hbar}{m_e}\left[\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{1/3}$$

- v goes toward the speed of light -> relativistic effect kicks in
- Mass-volume relation in the non-relativistic limit breaks



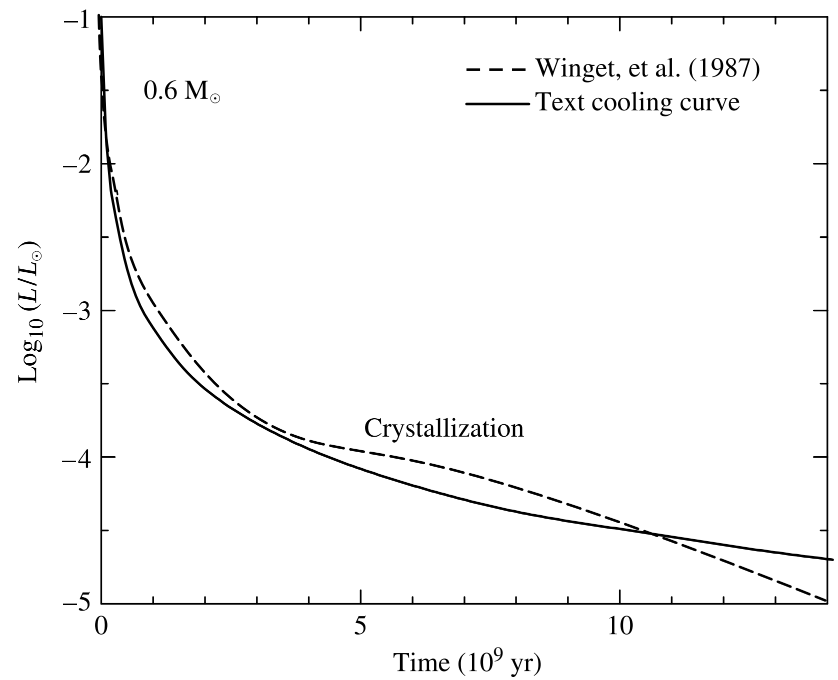
WDs collapse to zero volume at M ~ 1.4 M_{sun}

Chandrasekhar Limit

No WDs > 1.4 M_{sun} have been found so far

Fate of WDs

- No energy source in WDs
- The luminosity of WDs is from their residual internal energy
- In time, WDs cool down, and luminosity decreases
- Reaches its “eternal death”: a cool, dark, Earth-sized crystallized carbon/oxygen



“Diamonds in the sky”

Supernovae and neutron stars

- At the end of the evolution of high mass stars ($M > 8\ M_{sun}$), stars will burn their fuel down to iron (recall Lecture 7)
- The core's mass exceeds $M > 1.4\ M_{sun}$ (Chandrasekhar Limit), electron degeneracy is no longer enough to keep gravity at bay
- Matter is crushed to form neutrons

Photodisintegration

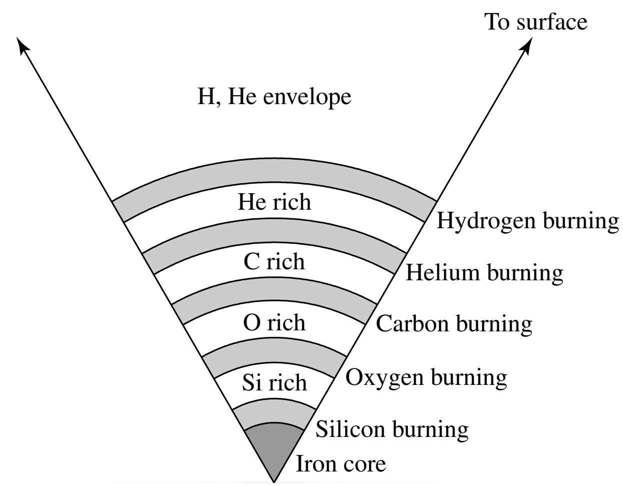
$$^{56}_{26}\text{Fe} + \gamma \rightarrow 13\,^{4}_{2}\text{He} + 4n$$

$$^{4}_{2}\text{He} + \gamma \rightarrow 2p^{+} + 2n.$$

Neutron formation

$$p^{+} + e^{-} \rightarrow n + \nu_e.$$

- Supernova occurs, left with a bare **neutron star**



Neutron Stars

- A 1.4 solar-mass neutron star consists of 10^{57} neutrons packed together by gravity and supported by the neutron degeneracy pressure
- It is literally a giant nucleus with a mass number A ~ 10^{57}!
    - Uranium is a pretty heavy element, right?
    - A = 235 for U 235 and A = 238 for U 238
- Radius of neutron stars

$$R_{\rm ns} \approx \frac{(18\pi)^{2/3}}{10} \frac{\hbar^2}{G M_{\rm ns}^{1/3}} \left(\frac{1}{m_H}\right)^{8/3}$$

Homework problem

For a 1.4 solar-mass neutron star, this yields 4.4 km. The actual radius is ~10 km

Neutron stars

- For a 1.4 solar-mass neutron star with 10-km radius:
  - Average density: 6.65 x 10^{17} kg m^{-3}
  - Comparing to typical density of an atomic nucleus, 2.3 x 10^{17} kg m^{-3}
  - Even denser than the atomic nucleus!
  - All neutrons are “touching” each other, bounded by gravity

For white dwarfs

How much does it weigh on neutrons stars (in tons)?

Comparing it to the weight of Mt. Everest ~ 1 billion tons

  - Surface gravity is g ~ 1.86 x 10^{12} m s^{-2}, 190 billion times stronger than Earth’s surface gravity

Chandrasekhar Limit for neutron stars

- Like white dwarfs, neutron stars also obey a mass-volume relation

$$\boxed{M_{\mathrm{ns}} V_{\mathrm{ns}} = \text{constant},}$$

- However, it fails for more massive neutrons stars as there is a point beyond which neutron degeneracy pressure can no longer support the star
  - $M_{ns} < \sim 2.2\ M_{sun}$ for static neutron star
  - $M_{ns} < \sim 2.9\ M_{sun}$ for rapidly-rotating neutron star
- More massive neutron stars would collapse and become black holes

nature
International journal of science

Letter

A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest, T. Pennucci, S. M. Ransom, M. S. E. Roberts & J. W. T. Hessels

Nature **467**, 1081–1083 (28 October 2010)
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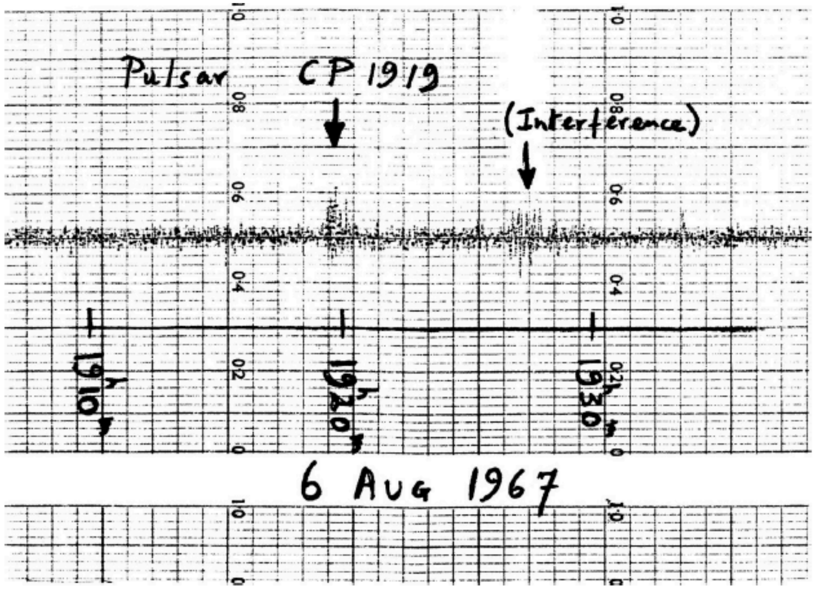
Most massive neutron star found to date is ~2 solar mass

Discovery of Pulsars

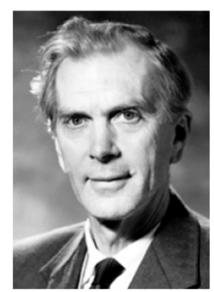


The first pulsar discovered by Jocelyn Bell

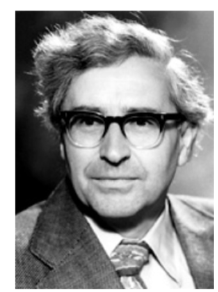
When first discovered, Bell dubbed it "Little Green Man 1"

Jocelyn Bell Burnell

The Nobel Prize in Physics 1974



Sir Martin Ryle
Prize share: 1/2



Antony Hewish
Prize share: 1/2

The Nobel Prize in Physics 1974 was awarded jointly to Sir Martin Ryle and Antony Hewish *"for their pioneering research in radio astrophysics: Ryle for his observations and inventions, in particular of the aperture synthesis technique, and Hewish for his decisive role in the discovery of pulsars"*

Pulsar characteristics

- Short periods: ~0.25–2 s
- Pulse periods are extremely well defined. Some has the accuracy that challenges the best atomic clocks
- Pulse period increase very gradually in time (slow down), typically $dP/dt \sim 10^{-15}$. The characteristic "spin-down" time $P/(dP/dt) \sim$ a few x 10^7 years
- Possible pulsar models:
    - Binary stars
    - Pulsating stars
    - Rotating stars

Was the world's first all-electronic digital watch (1970)

Pulsars as rotating neutron stars

Maximum angular velocity a fast spinning star can achieve

$$\omega_{\max}^2 R = G \frac{M}{R^2}$$

If $\omega > \omega_{max}$, the star would fly apart!

Centripetal acceleration

Gravitational acceleration

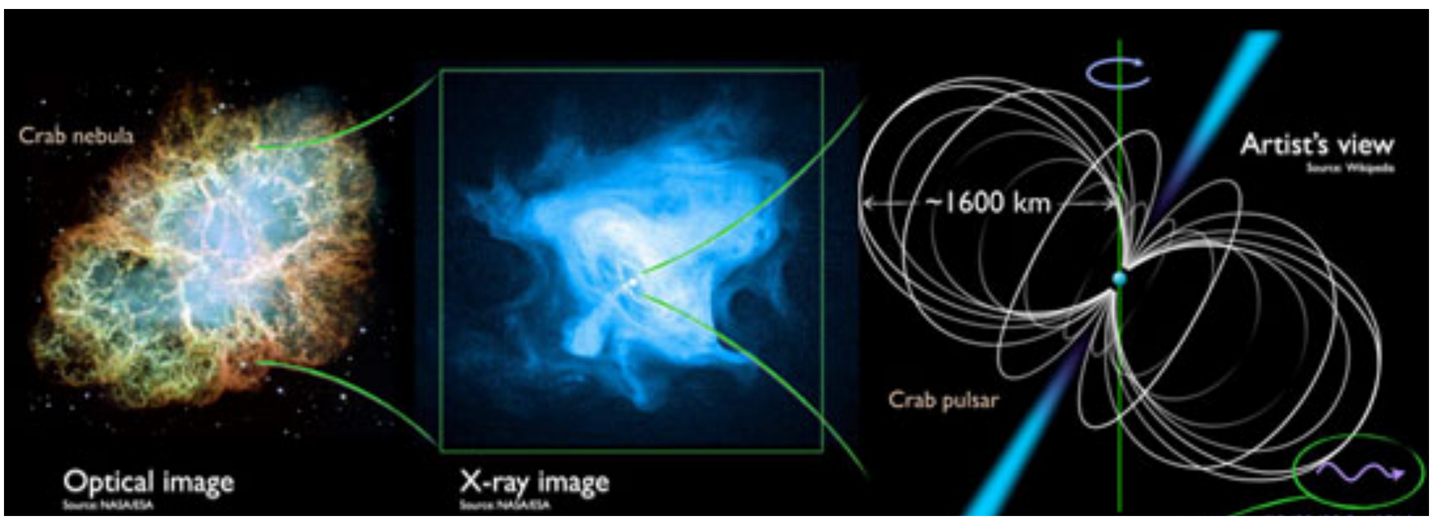
Shortest period is $$P_{\min} = 2\pi\sqrt{\frac{R^3}{GM}}.$$

- P_{min} ~ 7 s for Sirius B – too long to account for the observed pulsar periods!
  -> white dwarfs would be torn apart simply from this rapid rotation!
- P_{min} ~ 5 x 10^{-4} s for 1.4 solar-mass neutron stars -> neutron stars are safe!

Question: Why a rotating neutron star can account for the extremely precise periods?

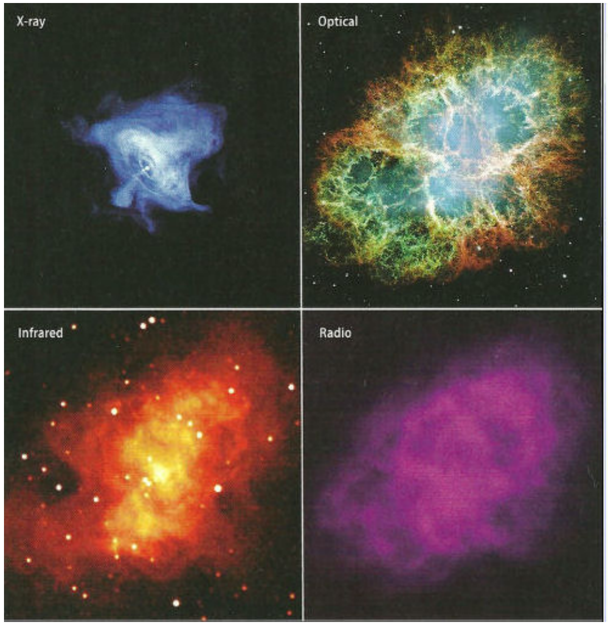
Pulsar and Crab Nebula

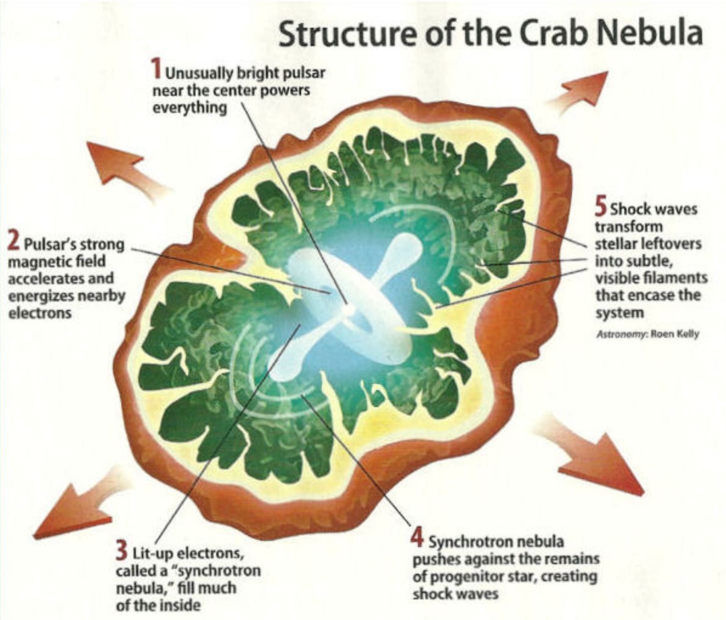
- Crab Nebula is known as a supernova remnant (recall the AD 1054 Chinese record on "Guest Star")
- A neutron star (or none) is expected as the remnant of the supernova
- A pulsar was discovered in 1968 in the Crab Nebula (PSR 0531-21) with P ~ 0.0333 s
- No other stars can survive at this rotation period other than neutron stars
- Pulsar in the Crab Nebula is a neutron star



Powering the Crab Nebula

- The Crab Nebula is bright in radio, infrared, optical, to X-rays
- Initial energy released in the supernova is not enough to account for its current emission
- The Crab Nebula needs a replenishment of magnetic field and relativistic electrons
- The total power needed for accounting for the current nebula emission is ~10^5 L_{sun}





Powering the Crab Nebula

- Crab pulsar rotation period is P = 0.0333 s, its spin-down rate is dP/dt ~ 4 x 10^{-13} s/s, or 10^{-5} s/yr
- It is losing its rotational kinetic energy!
- This lost energy goes into powering the Crab Nebula

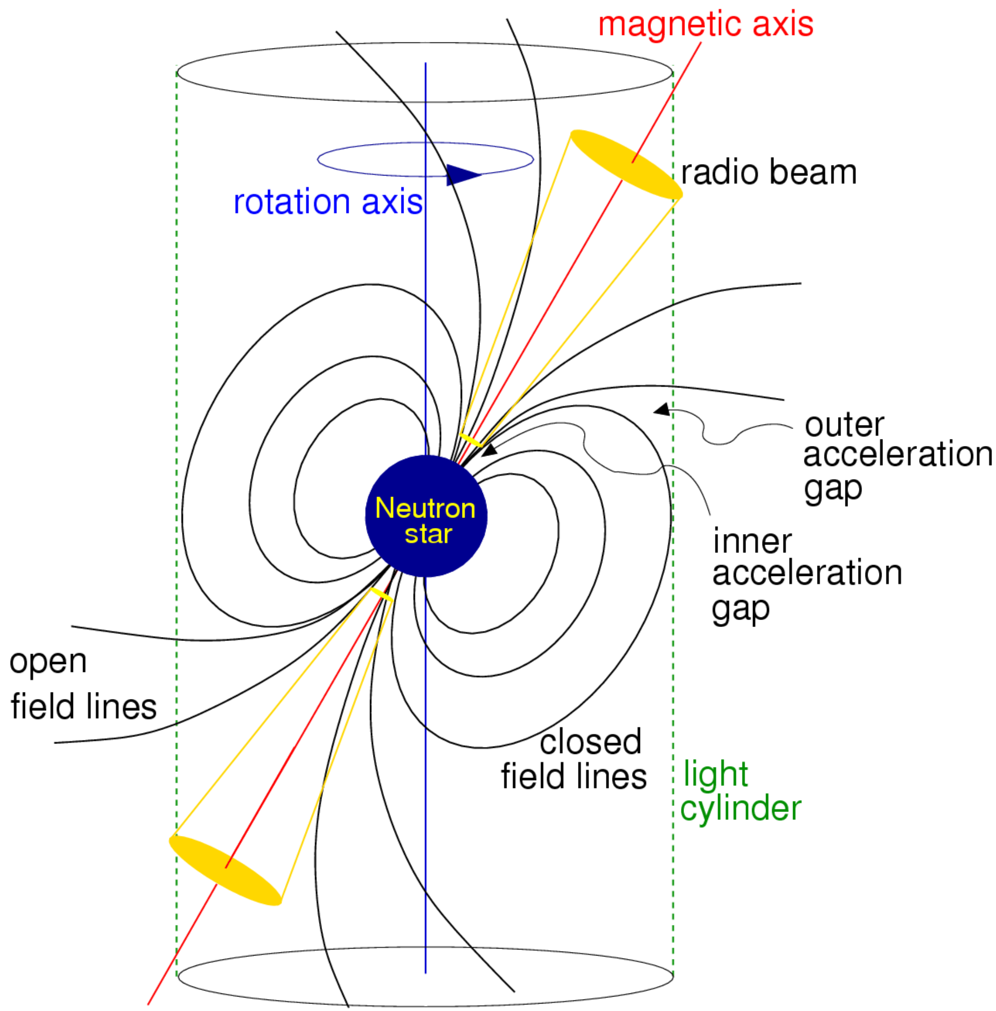
Rotational kinetic energy $$K = \frac{1}{2} I \omega^2 = \frac{2\pi^2 I}{P^2}$$

Energy loss rate $$\frac{dK}{dt} = -\frac{4\pi^2 I \dot{P}}{P^3}$$

For Crab, assuming a uniform sphere with 1.4 solar mass and 10 km $$I = \frac{2}{5} M R^2 = 1.1 \times 10^{38} \text{ kg m}^2$$

Inserting $P = 0.0333$ s and $\dot{P} = 4.21 \times 10^{-13}$, dK/dt ~ 5 x 10^{31} W ~ 10^5 L_{sun}!

Pulsar model



Black Holes

- Crash course by Phil Plait on black holes

If a collapsing stellar core exceeds 3 solar mass, nothing, even the neutron degeneracy pressure, cannot stop the core from collapsing, and it becomes a **black hole**!

Escape Velocity

Total mechanical energy of a particle

Initial state: $$E_i = \frac{1}{2}mv_i^2 - G\frac{Mm}{r}$$

If the particle can escape to infinitely large distance

Final state: $v > 0$ $r = \infty$ Gravitational potential energy is zero

$$E_f = \frac{1}{2}mv_f^2 - G\frac{Mm}{r_\infty} = \frac{1}{2}mv_f^2 > 0$$

From energy conservation $$E_i = \frac{1}{2}mv_i^2 - G\frac{Mm}{r} = E_f > 0$$

So in order for the particle to escape $$v_i > v_{esc} = \sqrt{\frac{2GM}{r}}$$

Escape Velocity on different objects

Location	Relative to	V_e (km/s)[13]
On the Sun	The Sun's gravity	617.5
On Mercury	Mercury's gravity	4.25
On Venus	Venus's gravity	10.36
On Earth	Earth's gravity	11.186
On the Moon	The Moon's gravity	2.38
On Mars	Mars' gravity	5.03
On Ceres	Ceres's gravity	0.51
On Jupiter	Jupiter's gravity	60.20

What if, the escape velocity = c?

That means only light can barely escape!

$$v_{esc} = \sqrt{\frac{2GM}{r}} = c$$

Wrong derivation, but correct result

$$r = R_s = 2GM / c^2 = 3km(M / M_\odot)$$

Schwarzschild Radius

If an one solar-mass star packs all its material under the Schwarzschild Radius (3 km), even light cannot escape from its surface! This is smaller than the radius of a neutron star.

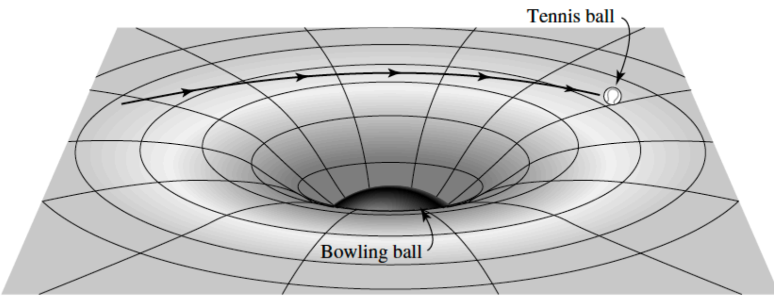
For this reason, a star that has collapsed down below R_s is called a **black hole**

It is enclosed by the event horizon, the spherical surface at $r = R_s$, within which **nothing** can escape, and is **completely out of reach**

Schwarzschild radius

Radius for Black Hole of a Given Mass		
Object	**Mass**	**Black Hole Radius**
Earth	5.98×10^{27} g	0.9 cm
Sun	1.989×10^{33} g	2.9 km
5 Solar Mass Star	9.945×10^{33} g	15 km
Galactic Core	10^{9} Solar Masses	3×10^{9} km

The curvature of spacetime



Curved space by massive objects

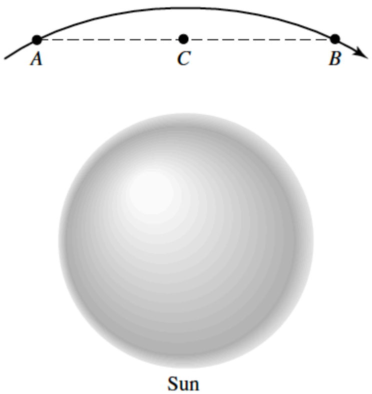


FIGURE 3 A photon's path around the Sun is shown by the solid line. The bend in the photon's trajectory is greatly exaggerated.

The shortest path connecting two points is not necessarily a straight line

Space is stretched under the influence of a massive object

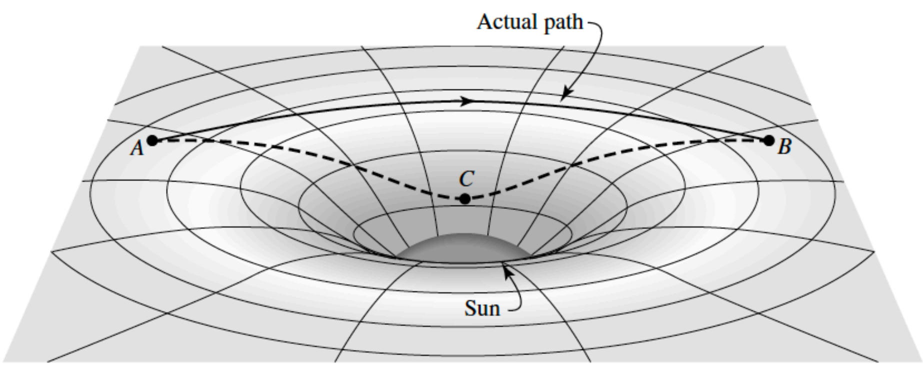
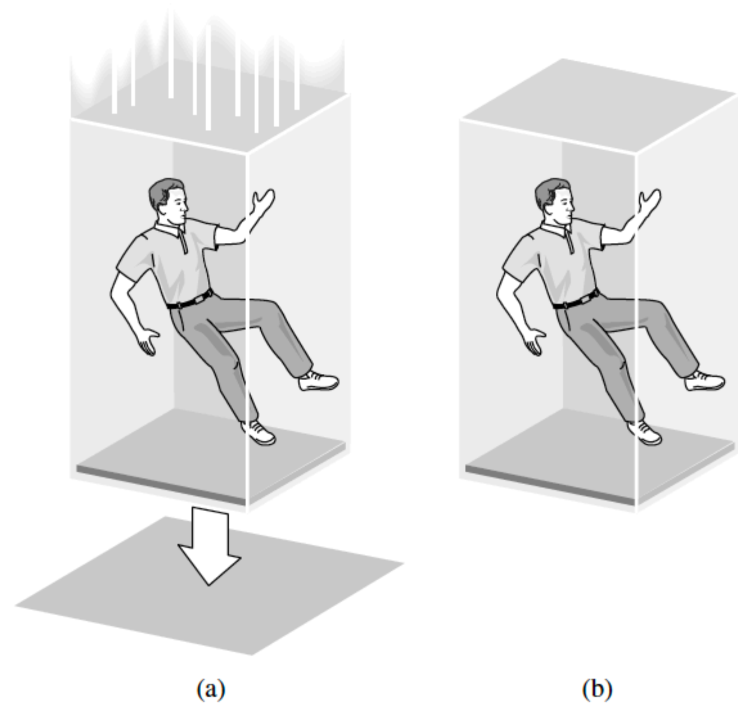


FIGURE 4 Comparison of two photon paths through curved space between points A and B. The projection of the path ACB onto the plane is the straight line depicted in Fig. 3.

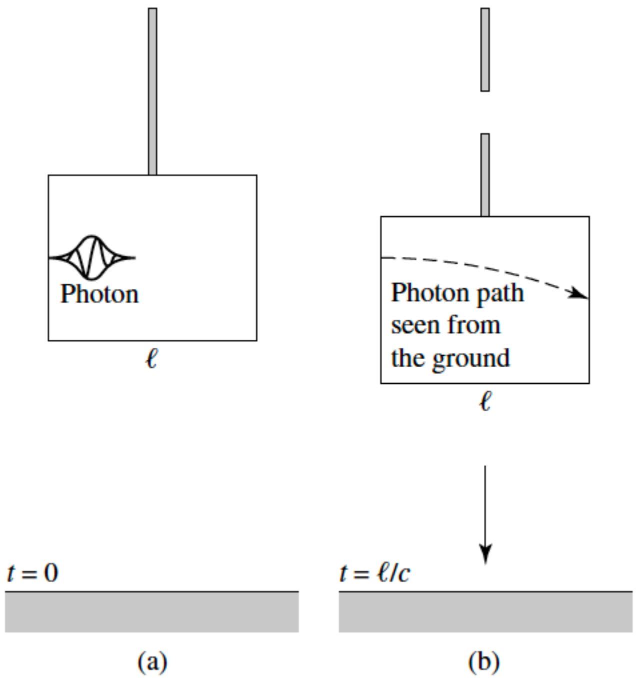
The Principle of Equivalence

All local, freely falling, nonrotating laboratories are fully equivalent for the performance of all physical experiments

In other words, one cannot tell the difference between a free-falling reference frame from one “at rest”

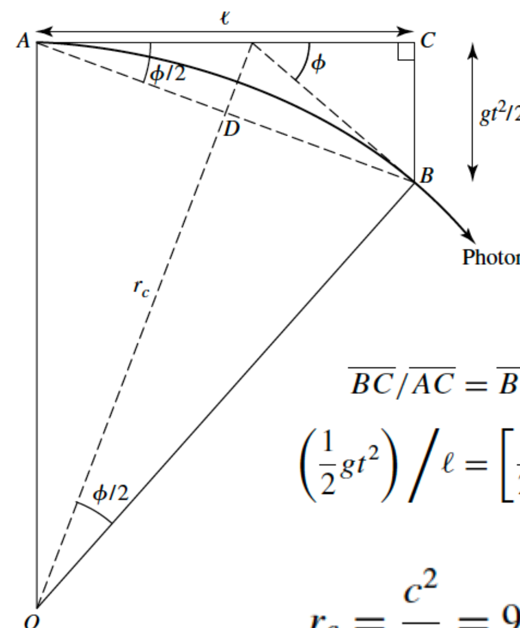


Space is curved under gravity



An elevator at rest   A free-falling elevator

- Light likes to take quickest path possible
- Light path is curved according to the observer on the ground
- Space(time) is curved!



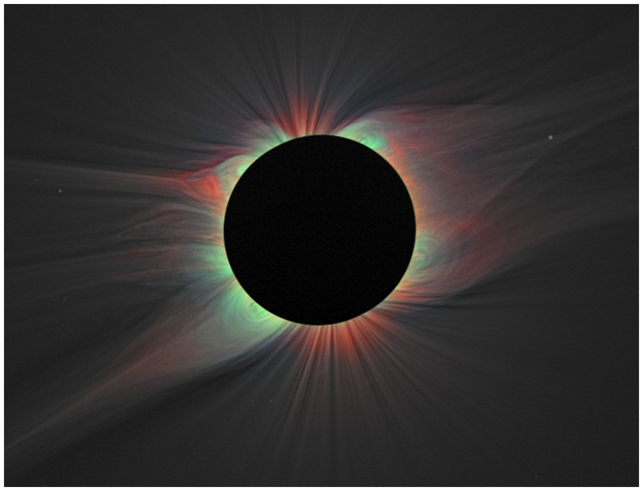
$$\overline{BC}/\overline{AC} = \overline{BD}/\overline{OD}$$

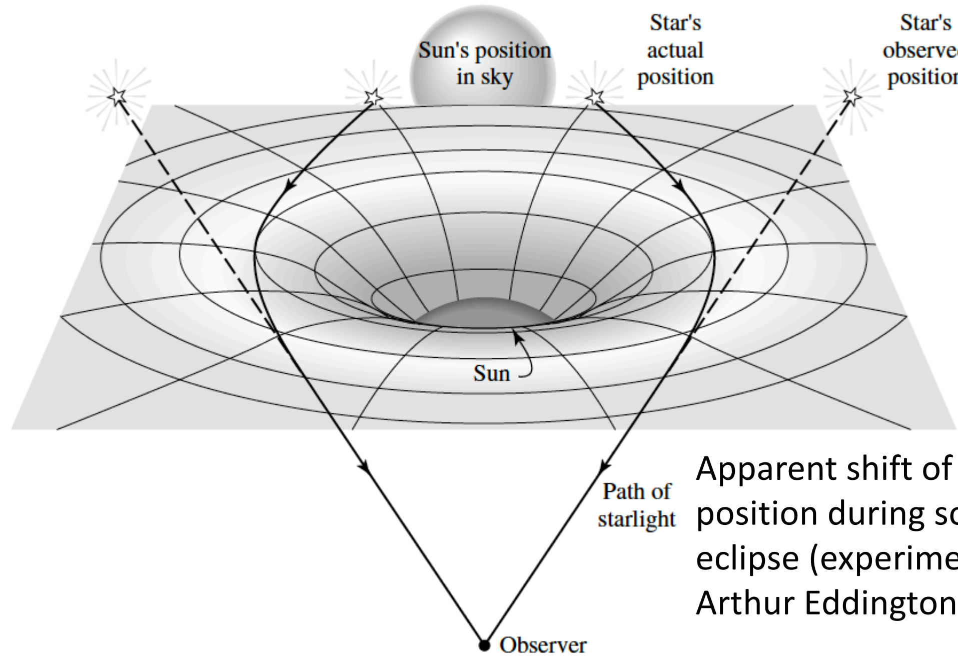
$$\left(\frac{1}{2}gt^2\right)\Big/\ell = \left[\frac{\ell}{2\cos(\phi/2)}\right]\Big/\overline{OD}.$$

$$r_c = \frac{c^2}{g} = 9.17\times 10^{15}\ \text{m}$$

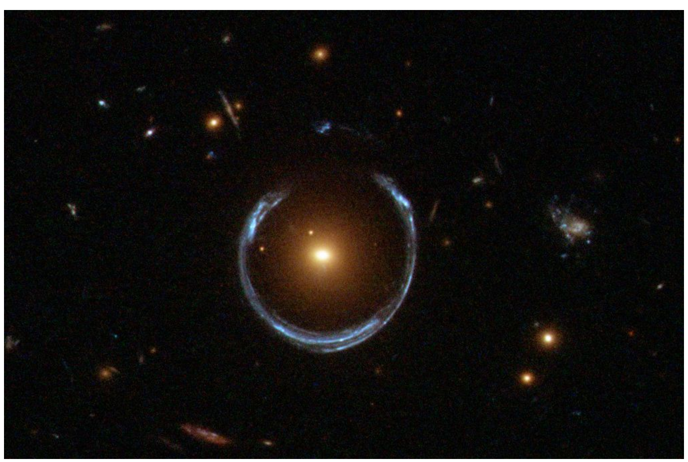
The spacetime near Earth is only slightly curved, but not the case near black holes!

Proofs that the spacetime is curved



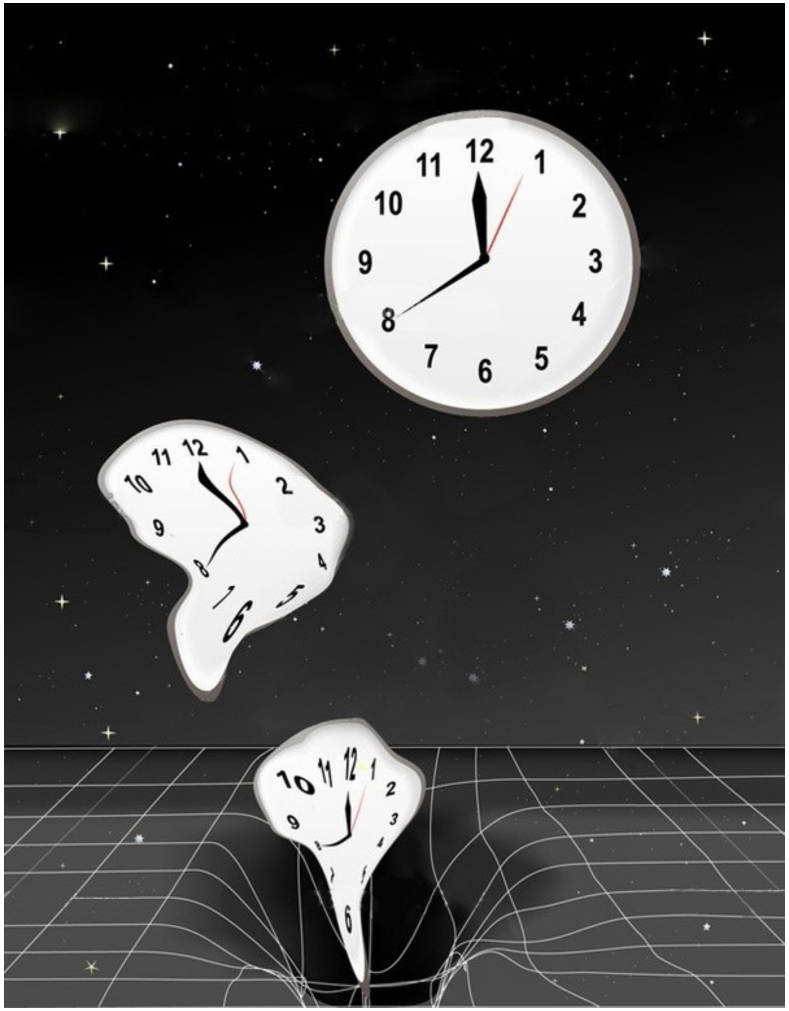


Apparent shift of star's position during solar eclipse (experiment led by Arthur Eddington in 1919)

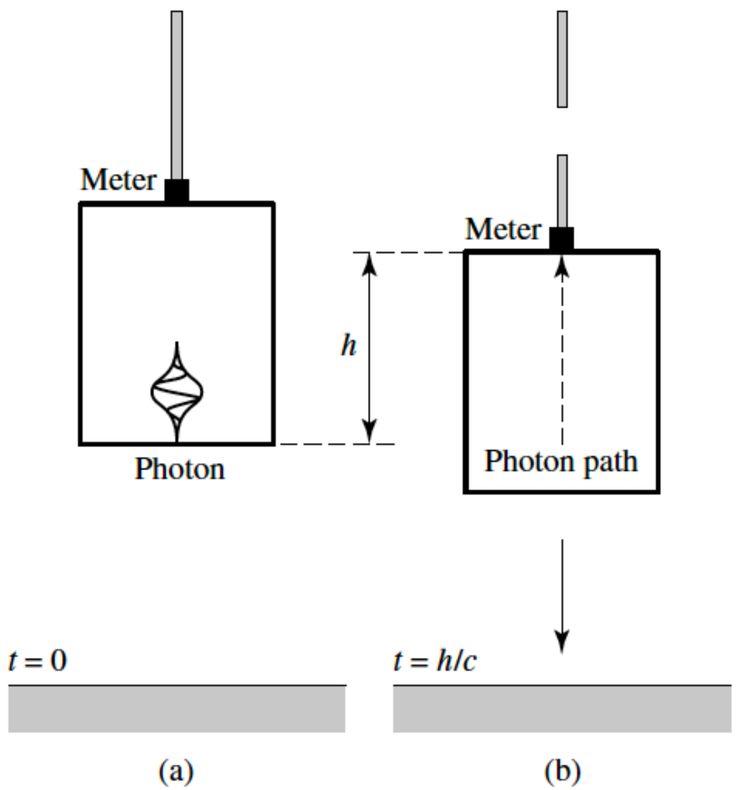


Gravitational Lensing

Time is also “messed up”!



Gravitational redshift and time dilation



An elevator at rest   A free-falling elevator

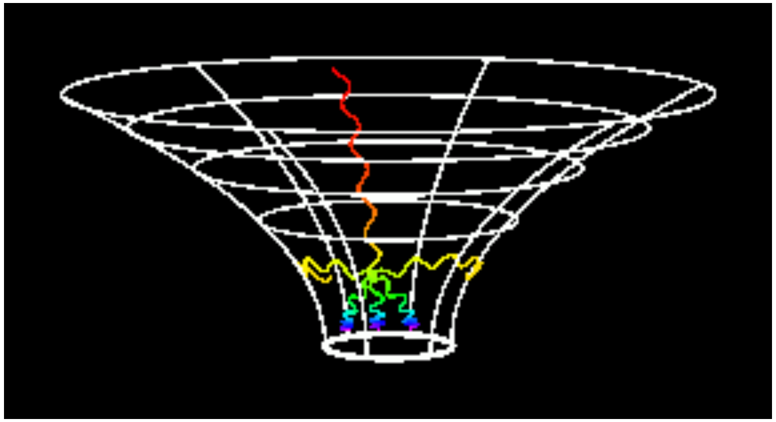
- The observer on the ground thinks the meter should measure a blueshifted light
- Yet the equivalence principle says the meter should measure exactly the light with the same frequency
- So from the point of view of the ground-based observer, the light should be redshifted on its journey to the meter
- This is the gravitational redshift: any photon escaping from the gravitational potential well would be redshifted and lose its energy

$$\frac{\Delta \nu}{\nu_0} = -\frac{v}{c} = -\frac{gh}{c^2}$$

Very small for small gh, but huge near the holes

Gravitational redshift and time dilation

If a light beam escapes from an initial position of r_0 outside a massive object to infinity, the light gets redshifted continuously



The light observed by a distant observer would have a frequency

$$\frac{\nu_\infty}{\nu_0} = \left(1 - \frac{2GM}{r_0 c^2}\right)^{1/2}.$$

Note time is just an inversion of frequency, so clock also slows down

$$\frac{\Delta t_0}{\Delta t_\infty} = \frac{\nu_\infty}{\nu_0} = \left(1 - \frac{2GM}{r_0 c^2}\right)^{1/2}.$$

At the Schwarzschild radius

$$\boxed{\frac{\Delta t_0}{\Delta t_\infty} = \frac{\nu_\infty}{\nu_0} = \left(1 - \frac{2GM}{r_0 c^2}\right)^{1/2}.}$$

$R_S = 2GM / c^2$

$$\frac{\Delta t_0}{\Delta t_\infty} = 0$$

Time is frozen!

$$\frac{\nu_\infty}{\nu_0} = 0$$

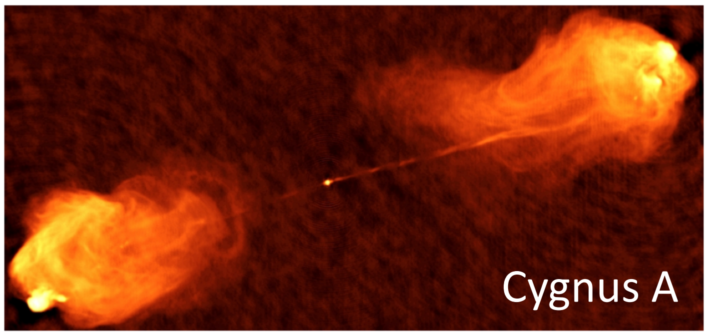
Light is infinitely redshifted

Nothing escapes from within the event horizon

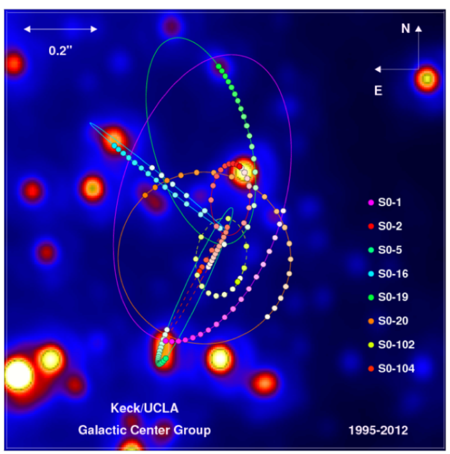
Falling into a black hole

Black holes: Observational evidence

Supermassive black holes

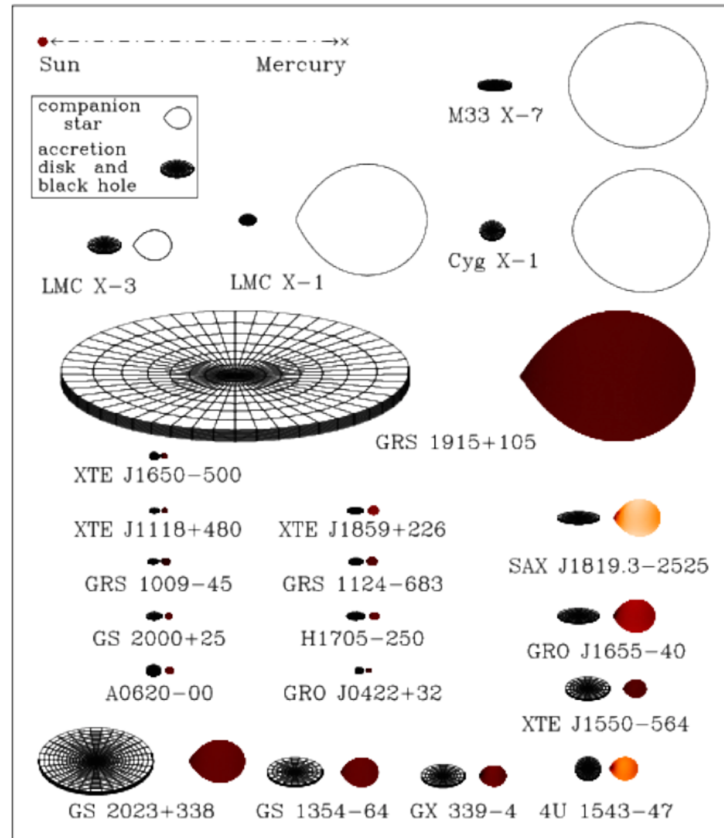


The Galactic Center



Mass and size estimates

Stellar-mass black holes



Dynamic mass in binary systems