PHYS 321: PROBLEM SET 4

Due Feb 26 Tue @ 02:30 pm

Solve the problems listed below, and ***write up your answers clearly and completely****. Do not turn in rough work – instead, make a clean copy after checking your calculations. Use English sentences and phrases to explain your solution and describe your answers step by step. Even if you did not get the correct answer, you may get partial credits for these steps!*

1. (4 credits) Consider a box of electrically neutral hydrogen gas that is maintained at a constant volume V. In this simple situation, the number of free electrons must equal the number of H II ions: $n_e V = N_{II}$. Also, the total number of hydrogen atoms (both neutral and ionized), N_t, is related to the density of the gas by $N_t = \rho V/(m_p + m_e) \approx \rho V/m_p$, where m_p is the mass of the proton (The tiny mass of electron may be safely ignored). Let the density of the gas be 10^{-6} kg m^{-3}, typical of the photosphere of an A0 star.
   (a) Make these substitutions into the Saha Equation (Eq. 8 in your textbook) to derive a quadratic equation for the fraction of ionized atoms in the following form:
   $$\left(\frac{N_{II}}{N_t}\right)^2 + \left(\frac{N_{II}}{N_t}\right)\left(\frac{m_p}{\rho}\right)\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_I/kT} - \left(\frac{m_p}{\rho}\right)\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_I/kT} = 0$$
   (b) Solve the quadratic equation in part (a) for the fraction of ionized hydrogen, N_{II}/N_t, for a range of temperatures between 5000 K and 25,000 K. Use your favorite programming language to make a graph of your results of N_{II}/N_t vs. T (compare your results to Fig. 8 of your textbook).

2. (2 credits) Using the root-mean-square speed $v_{\rm rms}$, estimate the mean free path of the nitrogen molecules in your classroom at room temperature (300 K). What is the average time between collisions? Take the radius of a nitrogen molecule to be 0.1 nm and the density of air to be 1.2 kg m^{-3}. A nitrogen molecule contains 28 nucleons (protons and neutrons).

3. (2 credits) If the temperature of a star's atmosphere is increasing outward, what type of spectral lines would you expect to find in the star's spectrum at those wavelengths where the opacity is greatest?

4. (2 credits) Consider a large hollow spherical shell of hot gas surrounding a star. Under what circumstances would you see the shell as a glowing ring around the star? What can you say about the optical thickness of the shell?