

Phys 321: Lecture 3

The Interaction of Light and Matter

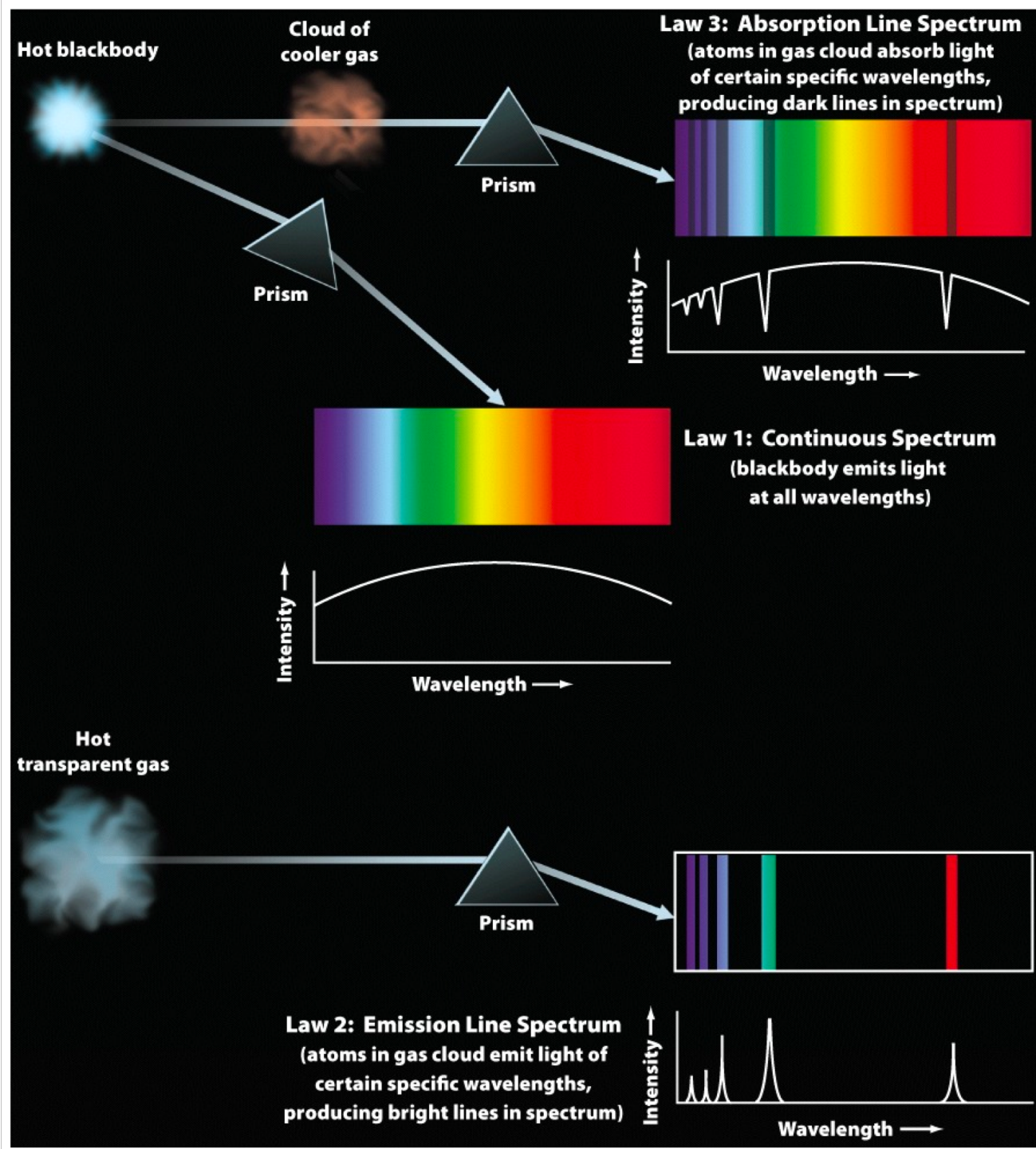
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Fraunhofer Lines

- By 1814, German optician Joseph von Fraunhofer cataloged 475 of dark lines in the solar spectrum, known today as the “**Fraunhofer Lines**”
- [A video story](#)

Kirchhoff's Laws

1. A **hot, dense** gas or hot solid object produces a **continuous spectrum**
2. A **hot, diffuse** gas produces bright spectral lines (**emission lines**)
3. A **cool, diffuse** gas in front of a source of a continuous spectrum produces dark spectral lines (**absorption lines**) in the continuous spectrum

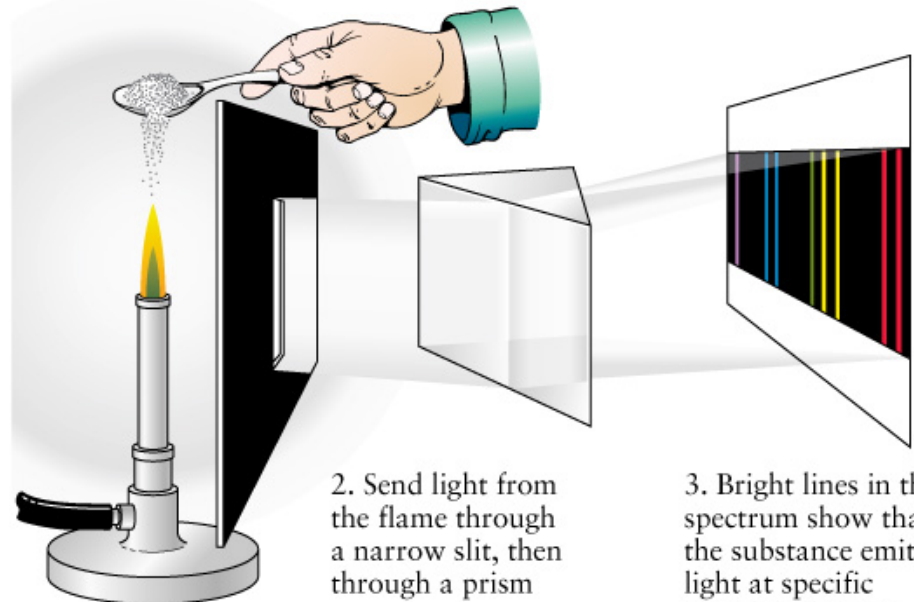


Measuring Spectra: Prism

Astronomers use *spectrographs* to separate the light in different wavelengths and measure the spectra of stars and galaxies



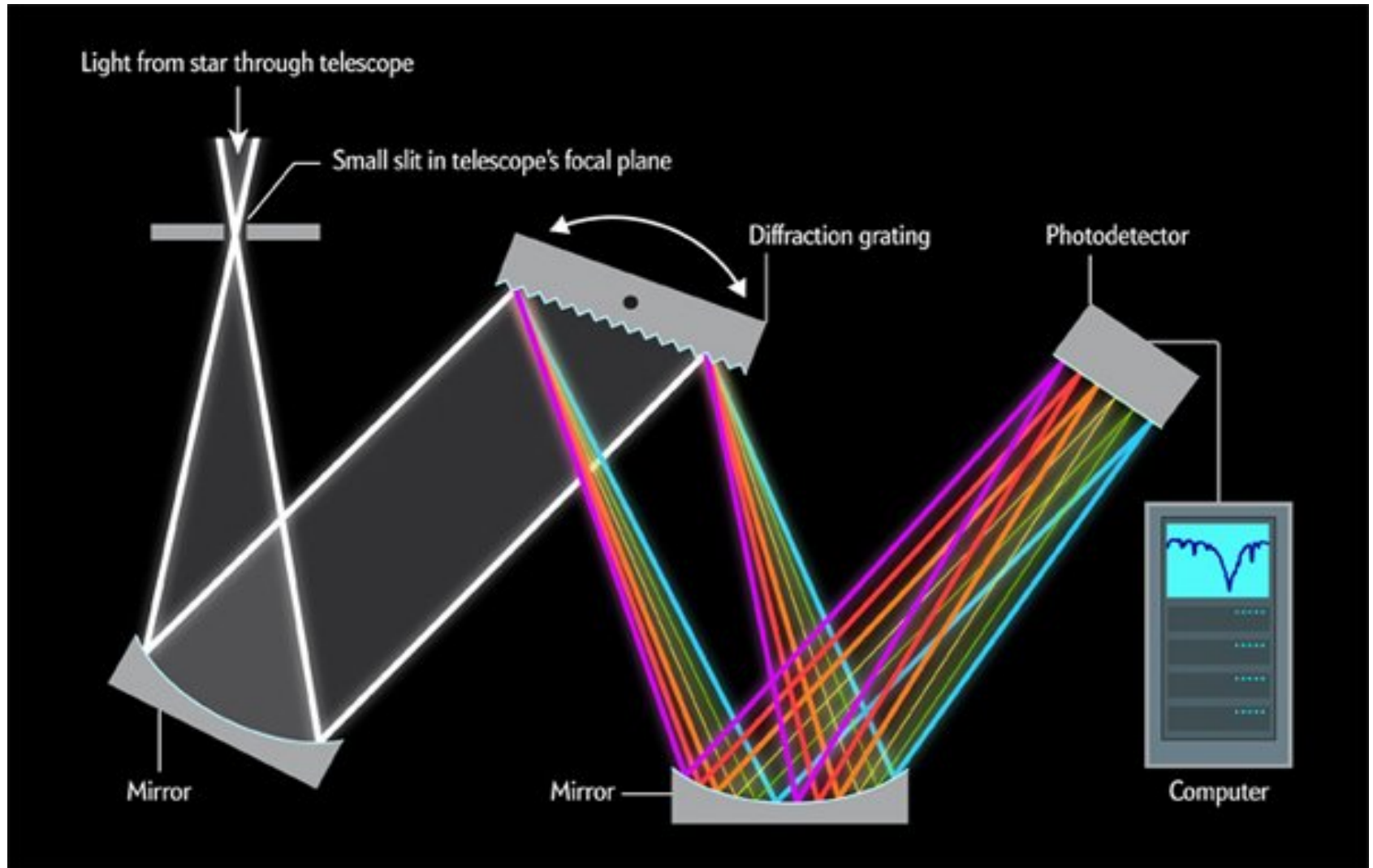
1. Add a chemical substance to a flame



2. Send light from the flame through a narrow slit, then through a prism

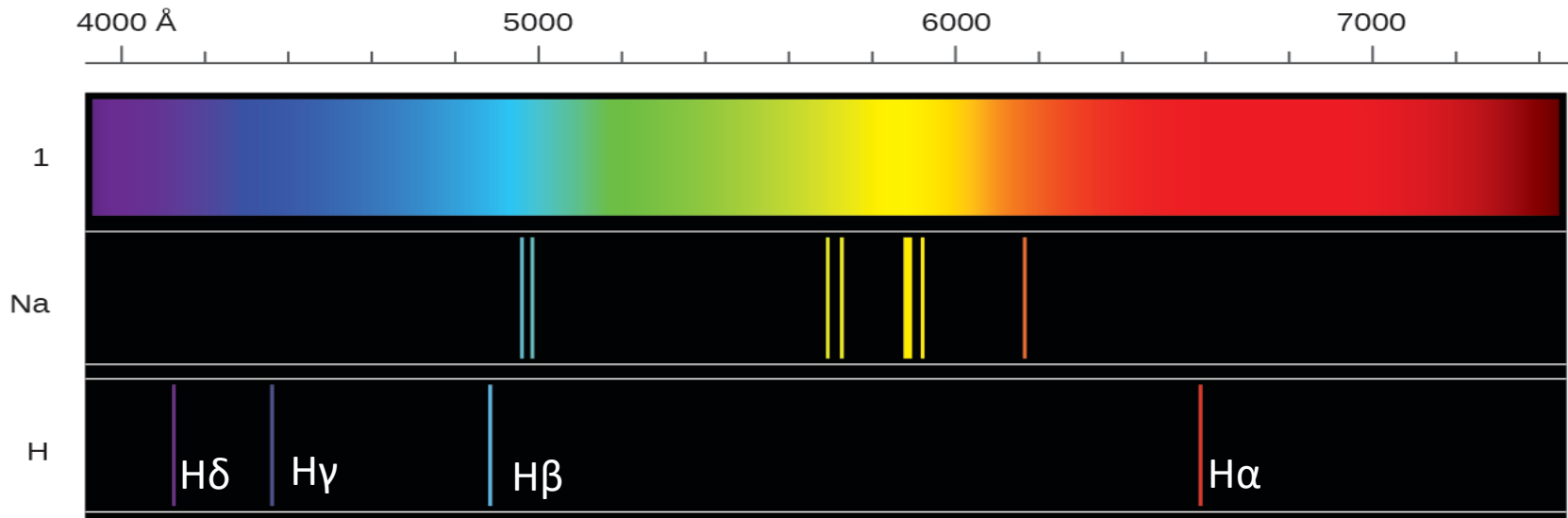
3. Bright lines in the spectrum show that the substance emits light at specific wavelengths only

Measuring Spectra: Diffraction Grating

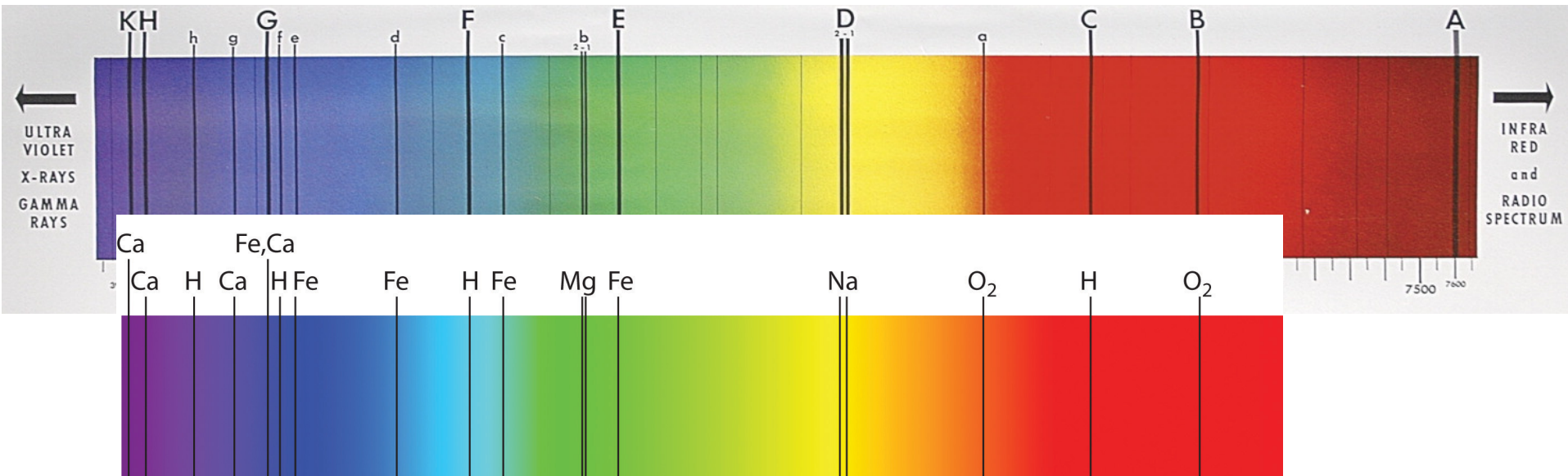


Spectra from different elements

Lab measurements



Fraunhofer lines in Solar Spectrum



Spectroscopy: Applications

- Identify elements on distant astronomical bodies:
Chemical Composition
- The element *Helium* is first discovered from the solar spectrum, not Earth.
- Measure physical properties: temperature, density, pressure, magnetic fields, high-energy particles, etc.
- Measure velocities of distant objects via *Doppler shift*

Doppler Shift

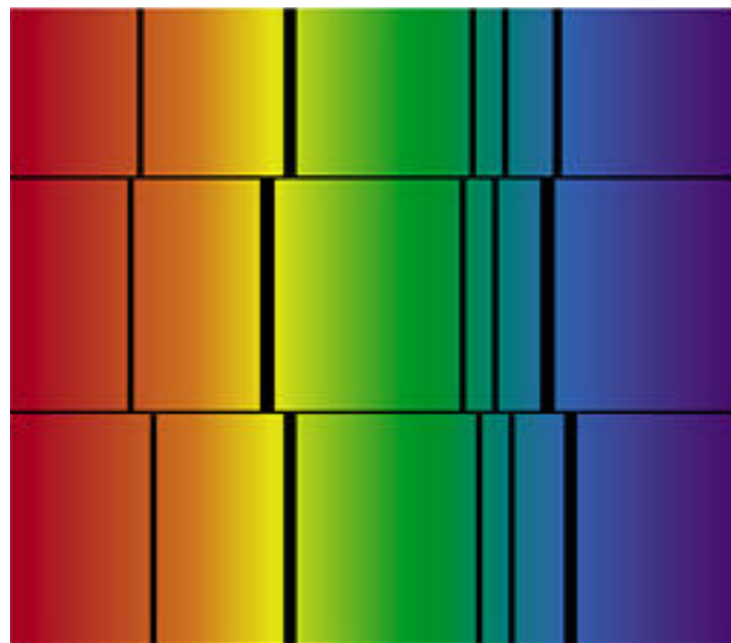
“It’s the apparent change in the frequency of a wave caused by relative motion between the source of the wave and the observer.” --- Sheldon Cooper

[A short animated explanation \(Youtube\)](#)

Reference spectrum in lab

Observed spectrum from Star A

Observed spectrum from Star B



Question: Which star is moving towards us?

Star A

Star B

Doppler Shift

$\lambda_{obs} > \lambda_{rest}$: **Redshifted** -> Object moving **away** from observer

$\lambda_{obs} < \lambda_{rest}$: **Blueshifted** -> Object moving **towards** observer

$$\frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}} = \frac{\Delta\lambda}{\lambda_{rest}} = \frac{v_r}{c}$$

v_r is the radial component of the objects velocity relative to the observer. **Positive** means the object is **moving away** and *vice versa*.

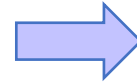
Radial Velocity of Vega

- The rest wavelength of H α is 656.281 nm as measured in the lab. From a ground-based telescope, the H α line from star Vega in constellation Lyra is measured at 656.251 nm. What is its radial velocity?

$$v_r = \frac{c (\lambda_{\text{obs}} - \lambda_{\text{rest}})}{\lambda_{\text{rest}}} = -13.9 \text{ km s}^{-1}$$

Back to nature of light discussion...

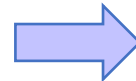
Young's double-slit experiment



Light = Waves

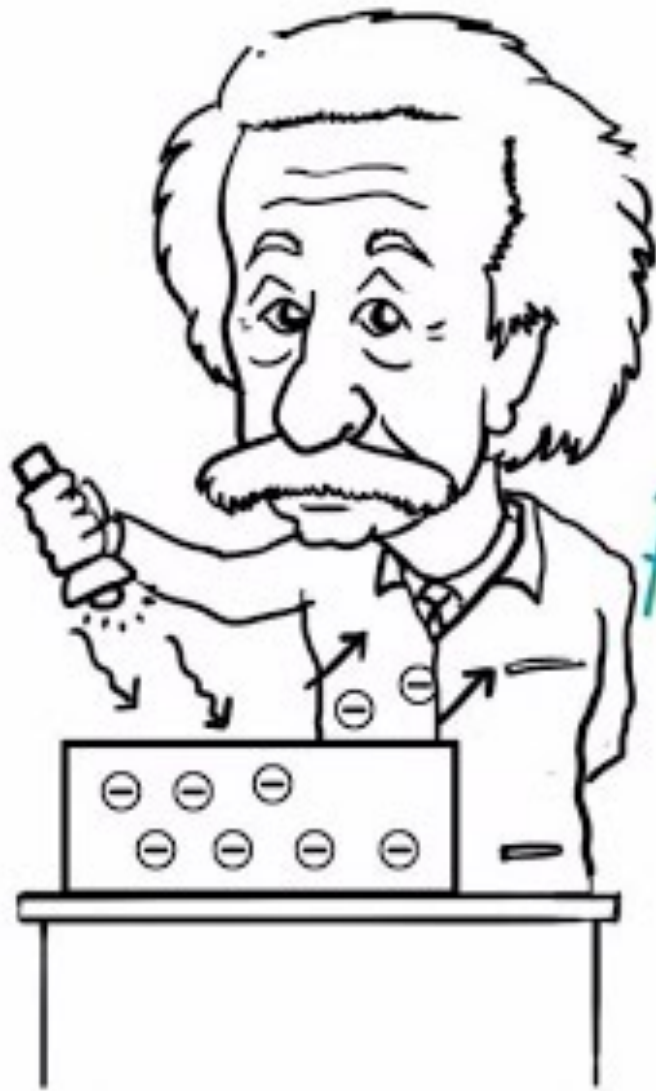
Maxwell's Equations

Ultraviolet Catastrophe of
blackbody radiation



Something is wrong!

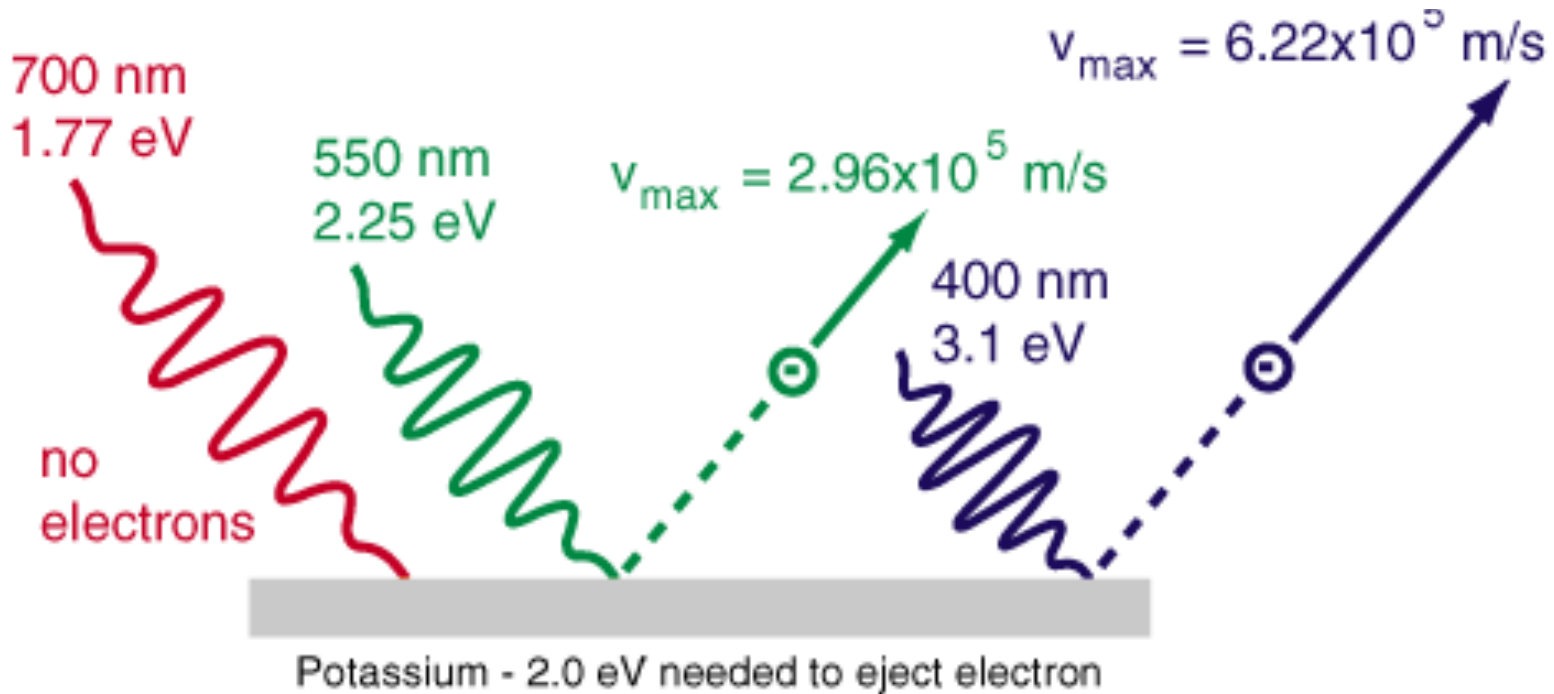
Discrete emission/absorption lines



*It's the
PhotoElectric Effect*

(Link to a Youtube video)

Photoelectric Effect



- Whether or not an electron is produced depends **only** on the **frequency** of the light, **not** the **flux** of the light
- Condition for producing an electron: $E_{\text{photon}} > \phi$
- A single photon has energy: $E_{\text{photon}} = h\nu$

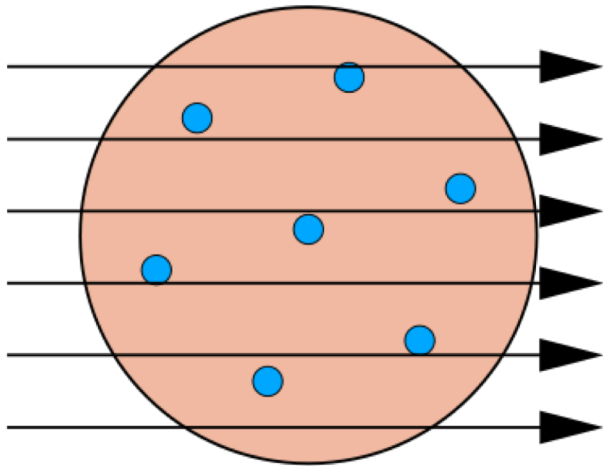
Planck's quantized energy of light waves!

LIGHT IS A

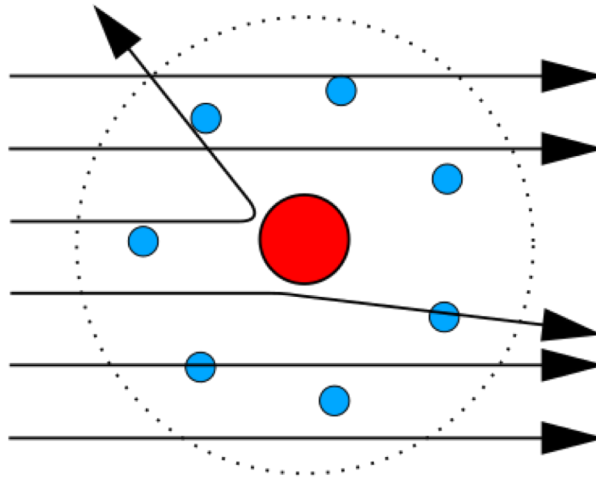
WAVE!

Atomic Structure

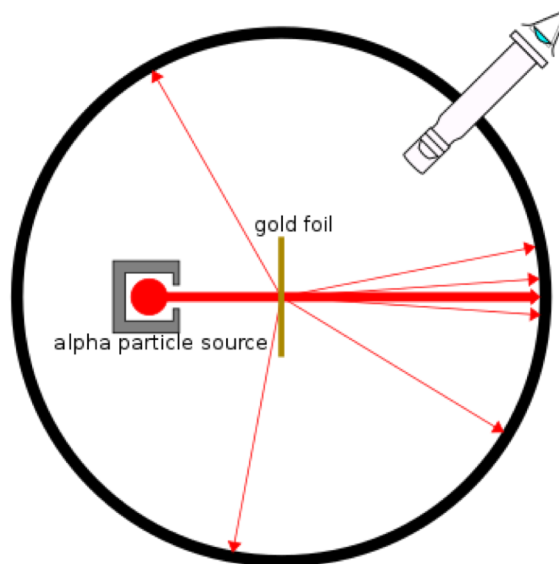
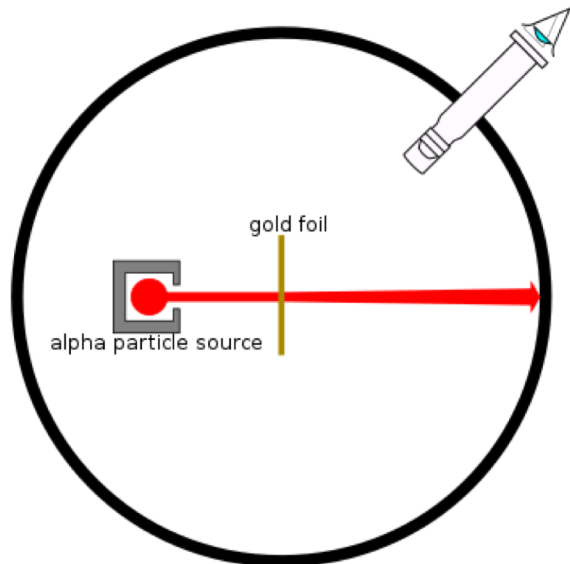
THOMSON MODEL



RUTHERFORD MODEL



**Most of the space
within an atom is
empty!**



The radius the of nucleus
was 10,000 times smaller
than the radius of the atom

OBSERVED RESULT

Hydrogen Spectral Lines

- In 1885, a Swiss teacher, Johann Balmer found an empirical formula to reproduce the wavelengths of hydrogen lines (in visible wavelengths), today known as the **Balmer series**

$$\frac{1}{\lambda} = R_H \left(\frac{1}{4} - \frac{1}{n^2} \right)$$

where $n = 3, 4, 5, \dots$, and $R_H = 1.09677583 \times 10^7 \pm 1.3 \text{ m}^{-1}$
is the experimentally determined Rydberg constant for hydrogen

Hydrogen Spectral Lines

More generally, $\frac{1}{\lambda} = R_H \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$

- **Lyman series:** $m = 1$
- **Balmer series:** $m = 2$
- **Paschen series:** $m = 3$

Series Name	Symbol	Transition	Wavelength (nm)
Lyman	Ly α	2 \leftrightarrow 1	121.567
	Ly β	3 \leftrightarrow 1	102.572
	Ly γ	4 \leftrightarrow 1	79.254
	Ly _{limit}	$\infty \leftrightarrow 1$	91.18
Balmer	H α	3 \leftrightarrow 2	656.281
	H β	4 \leftrightarrow 2	486.134
	H γ	5 \leftrightarrow 2	434.048
	H δ	6 \leftrightarrow 2	410.175
	H ϵ	7 \leftrightarrow 2	397.007
	H ζ	8 \leftrightarrow 2	388.905
	H _{limit}	$\infty \leftrightarrow 3$	364.6
Paschen	Pa α	4 \leftrightarrow 3	1875.10
	Pa β	5 \leftrightarrow 3	1281.81
	Pa γ	6 \leftrightarrow 3	1093.81
	Pa _{limit}	$\infty \leftrightarrow 3$	820.4

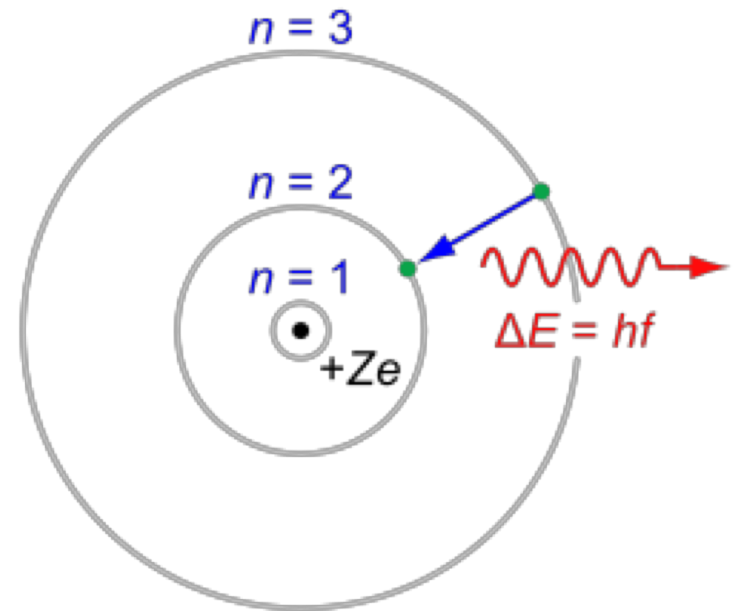
Bohr's Semiclassical Atomic Model

Model:

- Nucleus is a point mass sitting at the center of the atom
- Electrons move in circular orbits

Bohr's Postulations:

- Only a discrete number of orbits of the electrons (those with angular momentum that is an integral multiple of $\hbar = h/2\pi$) is allowed
- Radiation of a discrete quantum of energy is emitted/absorbed only when the electrons jumps from one orbit to another
- The radiated energy equals to the energy difference between the orbits



The unit of Planck's constant

Planck's constant has units of

$$\text{J s} = [\text{kg m}^2 \text{s}^{-2}] \text{s} = \text{kg} [\text{m s}^{-1}] \text{m}$$

which are the units of angular momentum (mvr).

Bohr's Model of Hydrogen Atom

Electric force provides centrifugal acceleration

$$\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{\mathbf{r}} = -\mu \frac{v^2}{r} \hat{\mathbf{r}},$$

Kinetic energy

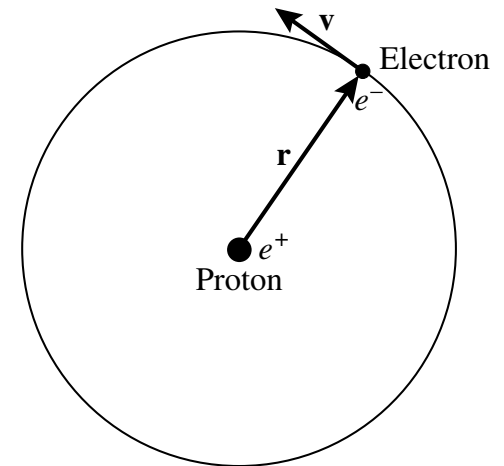
$$K = \frac{1}{2} \mu v^2 = \frac{1}{8\pi\epsilon_0} \frac{e^2}{r}$$

Electrical potential energy

$$U = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r} = -2K$$

Total energy

$$E = K + U = K - 2K = -K = -\frac{1}{8\pi\epsilon_0} \frac{e^2}{r}.$$



Bohr's Model of Hydrogen Atom

Now use Bohr's quantization of angular momentum

$$L = \mu v r = n \hbar$$

To rewrite kinetic energy

$$\frac{1}{8\pi\epsilon_0} \frac{e^2}{r} = \frac{1}{2} \mu v^2 = \frac{1}{2} \frac{(\mu v r)^2}{\mu r^2} = \frac{1}{2} \frac{(n \hbar)^2}{\mu r^2}$$

We have the radius of allowed the electron orbits

$$r_n = \frac{4\pi\epsilon_0 \hbar^2}{\mu e^2} n^2 = a_0 n^2,$$

Inserting this to the expression of total energy

$$E_n = -\frac{\mu e^4}{32\pi^2 \epsilon_0^2 \hbar^2} \frac{1}{n^2} = -13.6 \text{ eV} \frac{1}{n^2}.$$

n is the **principal quantum number**

Hydrogen spectral lines derived

Wavelength of the emitted photon

$$E_{\text{photon}} = E_{\text{high}} - E_{\text{low}}$$

Or

$$\frac{hc}{\lambda} = \left(-\frac{\mu e^4}{32\pi^2 \epsilon_0^2 \hbar^2} \frac{1}{n_{\text{high}}^2} \right) - \left(-\frac{\mu e^4}{32\pi^2 \epsilon_0^2 \hbar^2} \frac{1}{n_{\text{low}}^2} \right)$$

Which gives

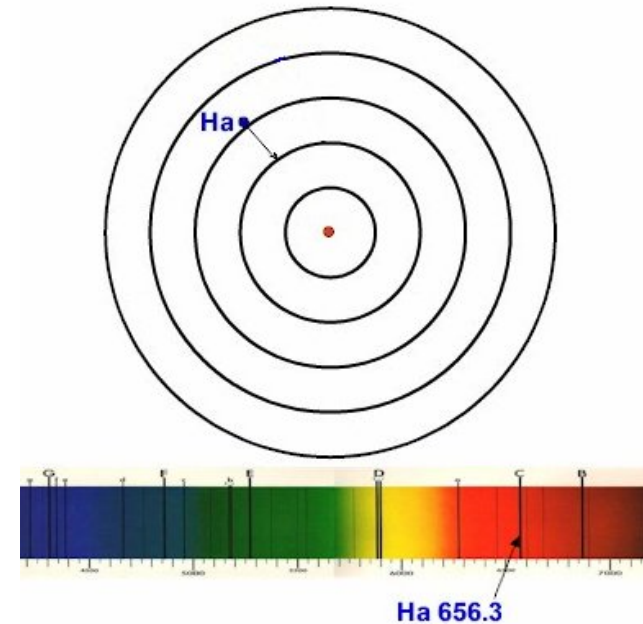
$$\frac{1}{\lambda} = \frac{\mu e^4}{64\pi^3 \epsilon_0^2 \hbar^3 c} \left(\frac{1}{n_{\text{low}}^2} - \frac{1}{n_{\text{high}}^2} \right) \quad R_H = \frac{\mu e^4}{64\pi^3 \epsilon_0^2 \hbar^3 c} = 10967758.3 \text{ m}^{-1}$$

Compare with Balmer's empirical expression

$$\frac{1}{\lambda} = R_H \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$

Example: H α line

- H α line is produced when the electron in the hydrogen atom makes the transition from the $n = 3$ to the $n = 2$ orbit. What is the wavelength?



$$\frac{1}{\lambda} = \frac{\mu e^4}{64\pi^3 \epsilon_0^2 \hbar^3 c} \left(\frac{1}{n_{\text{low}}^2} - \frac{1}{n_{\text{high}}^2} \right) \quad R_H = \frac{\mu e^4}{64\pi^3 \epsilon_0^2 \hbar^3 c} = 10967758.3 \text{ m}^{-1}$$

$$E_{\text{photon}} = E_{\text{high}} - E_{\text{low}}$$

$$\frac{hc}{\lambda} = -13.6 \text{ eV} \frac{1}{n_{\text{high}}^2} - \left(-13.6 \text{ eV} \frac{1}{n_{\text{low}}^2} \right) \quad \Rightarrow \quad \lambda = 656.469 \text{ nm}$$

$$= -13.6 \text{ eV} \left(\frac{1}{3^2} - \frac{1}{2^2} \right).$$

Comparing to the measured wavelength in the air:

$$\lambda_{\text{air}} = 656.275 \text{ nm}$$

Why?

Which of the following transitions of the hydrogen atom has the **shortest** wavelength?

A. $n = 3 \rightarrow n = 2$ ($H\alpha$)

B. $n = 4 \rightarrow n = 2$ ($H\beta$)

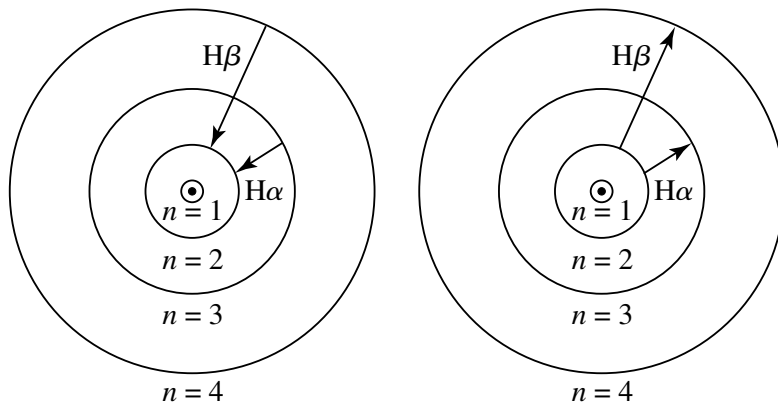
☒ C. $n = 2 \rightarrow n = 1$ (Lyman α)

D. $n = 5 \rightarrow n = 3$ (Paschen γ)

E. $n = 4 \rightarrow n = 3$ (Paschen α)

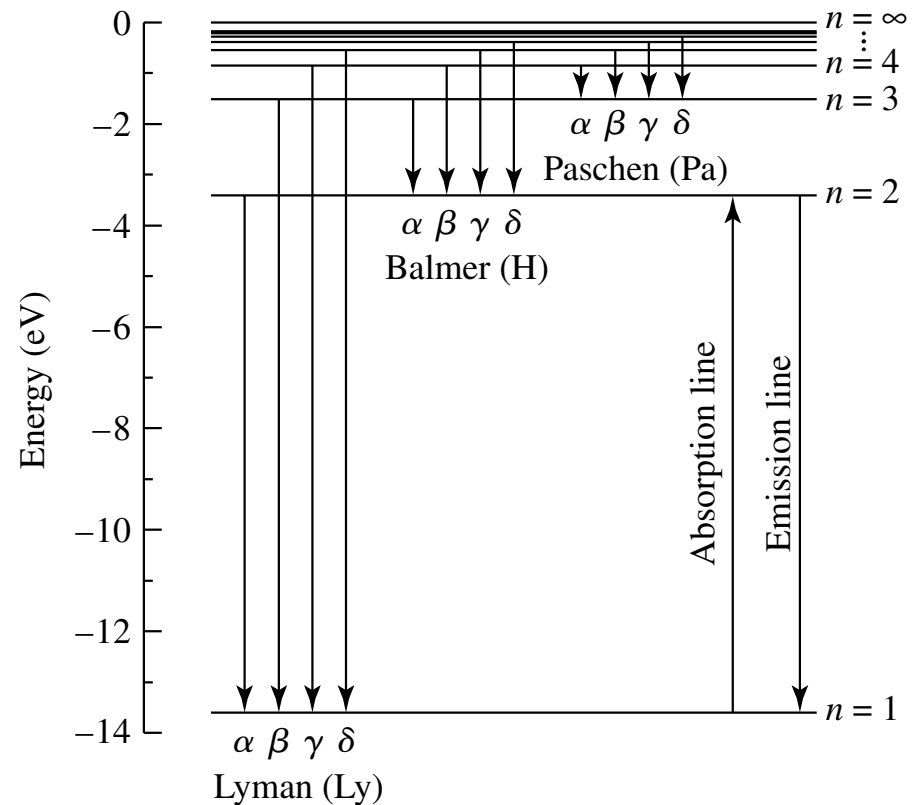
Hydrogen spectral lines and Kirchhoff's Law

1. A **hot, dense** gas or hot solid object produces a **continuous spectrum**
2. A **hot, diffuse** gas produces bright spectral lines (**emission lines**)
3. A **cool, diffuse** gas in front of a source of a continuous spectrum produces dark spectral lines (**absorption lines**) in the continuous spectrum



Emission Lines

Absorption Lines



More quantum physics...

Useful for later lectures

Wave-particle duality of *all* particles

- In his 1927 Ph.D. thesis, de Broglie's extended the wave-particle duality to *all* particles
- de Broglie wavelength

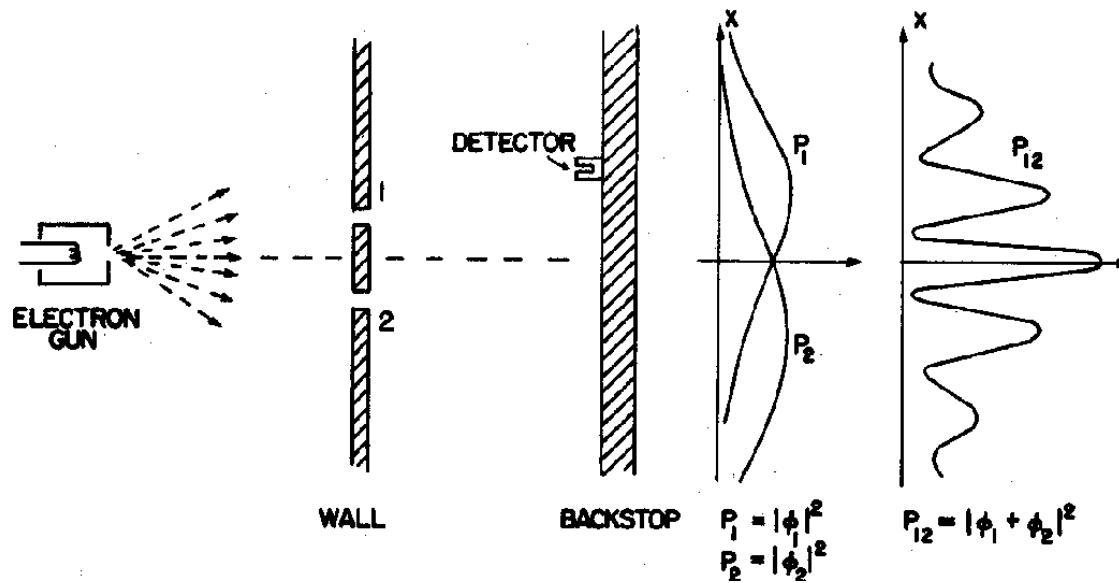
$$\lambda = \frac{h}{p}$$

- ***Everything*** exhibits wave properties!

If a light-wave could also act like a particle, why shouldn't matter-particles also act like waves?

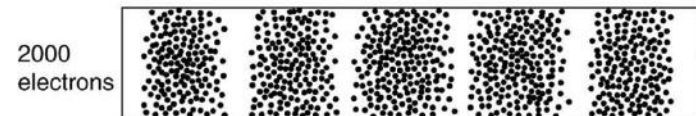
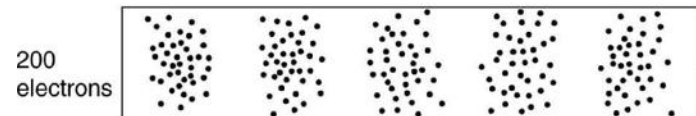
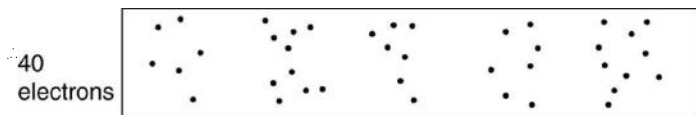


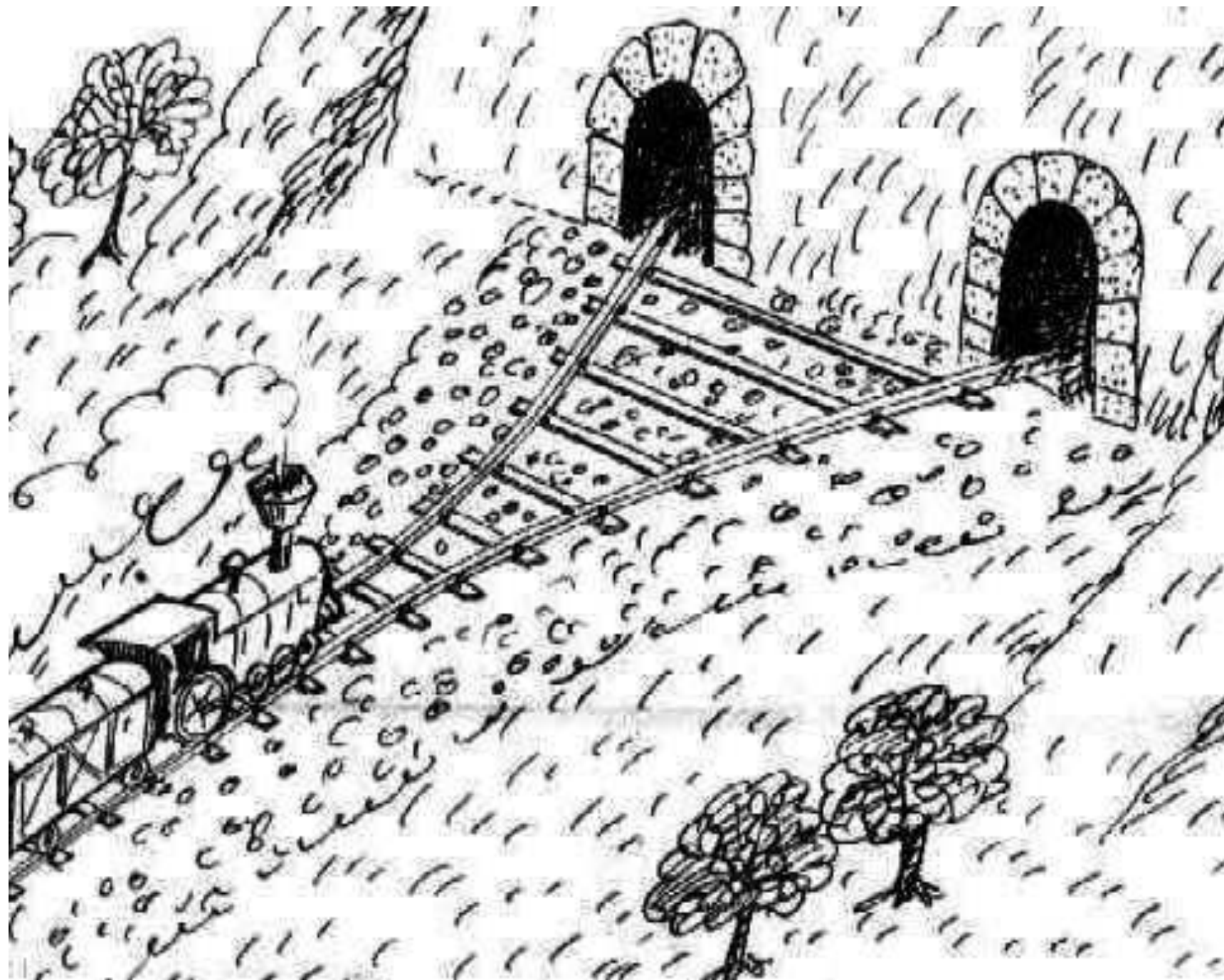
Electron double-slit experiment



Electrons exhibit wave-particle duality

Each electron passes ***both*** slit



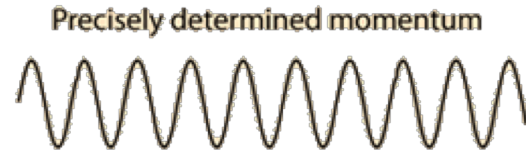


Example: Wavelength of a jogger

- What is the wavelength of a 70-kg man jogging at 3 m/s?

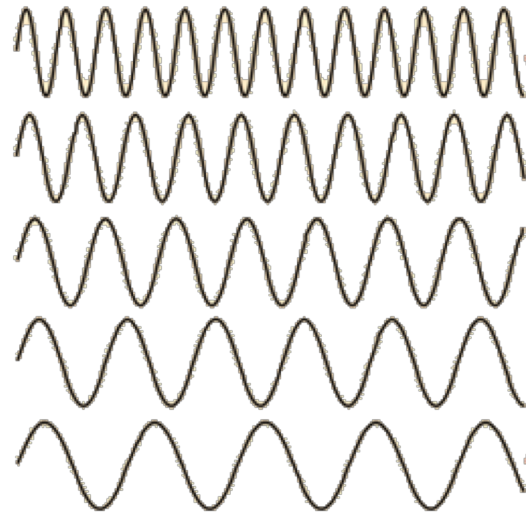
$$\lambda = \frac{h}{p} = \frac{h}{m_{\text{man}} v} = 3.16 \times 10^{-36} \text{ m}$$

Heisenberg's Uncertainty Principle

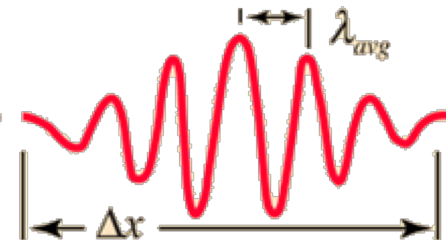


A sine wave of wavelength λ implies that the momentum is precisely known. But the wavefunction and the probability of finding the particle $\Psi^*\Psi$ is spread over all of space! p -precise
x-unknown

$$p = \frac{h}{\lambda}$$



Adding several waves of different wavelength together will produce an interference pattern which begins to localize the wave.



But that process spreads the momentum values and makes it more uncertain. This is an inherent and inescapable increase in the uncertainty Δp when Δx is decreased.

$$\Delta x \Delta p > \frac{\hbar}{2}$$

Forms often used:

$$\Delta x \Delta p \approx \hbar.$$

$$\Delta E \Delta t \approx \hbar.$$

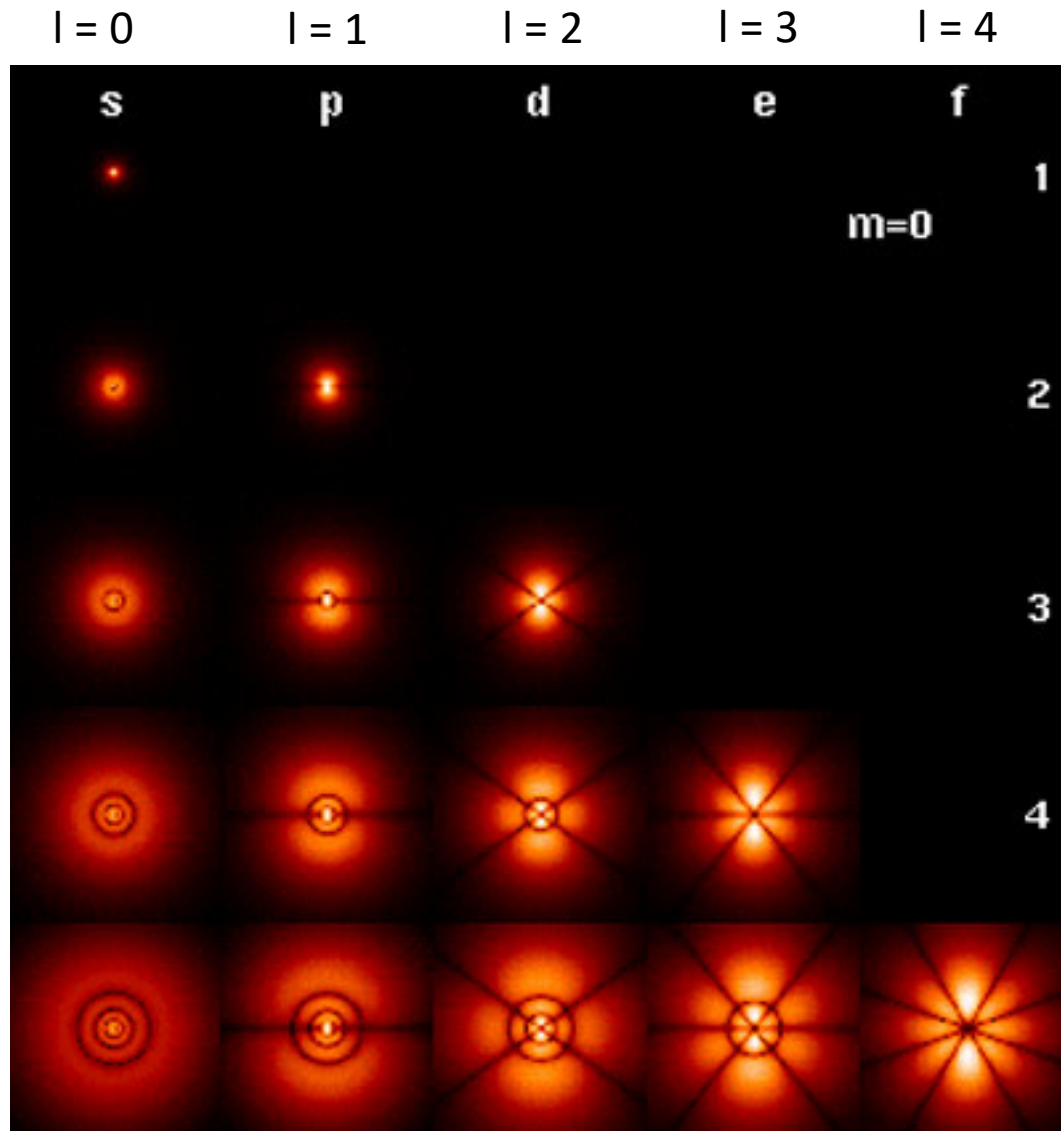
More quantum numbers

- Heisenberg's uncertainty principle implies that electrons cannot have well-defined circular orbits and angular momentum at the same time

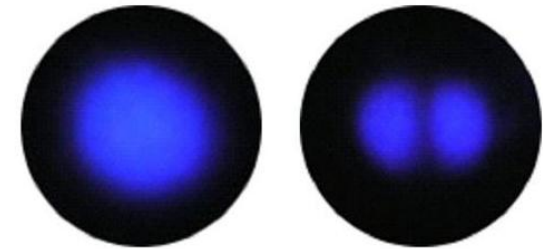
➡ Bohr's model must be updated!

- Electron orbits are “fuzzy”
- More than one quantum number
 - Principle quantum number ***n***: $n = 1, 2, 3 \dots$
 - ***Energy*** and ***Size*** of the orbital
 - Angular momentum quantum number ***l***: $l = 0, \dots, n-1$
 - ***Shape*** of the orbital
 - Magnetic quantum number ***m_l***: $m_l = -l, \dots, 0, \dots, +l$
 - ***Orientation*** of the orbital
 - Spin quantum number ***m_s***: $m_s = +\frac{1}{2}$ or $-\frac{1}{2}$
 - ***Orientation of the spin axis*** of the electron

Atomic Orbitals: Probability Map

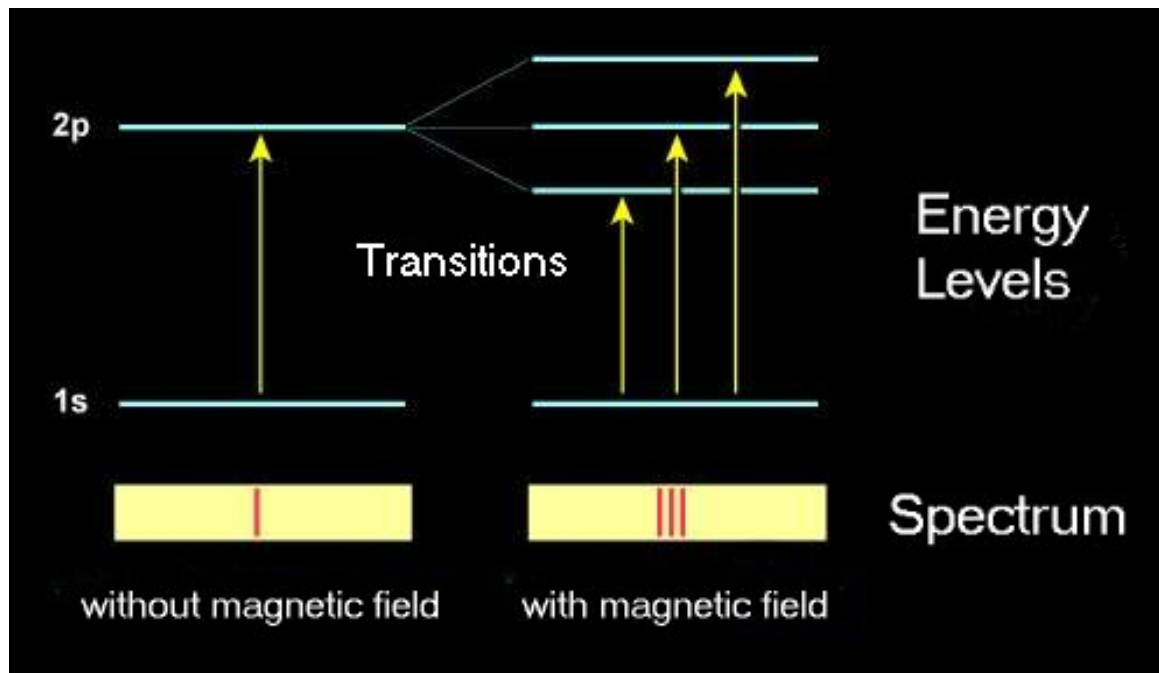


[Comparing with experimental results](#)



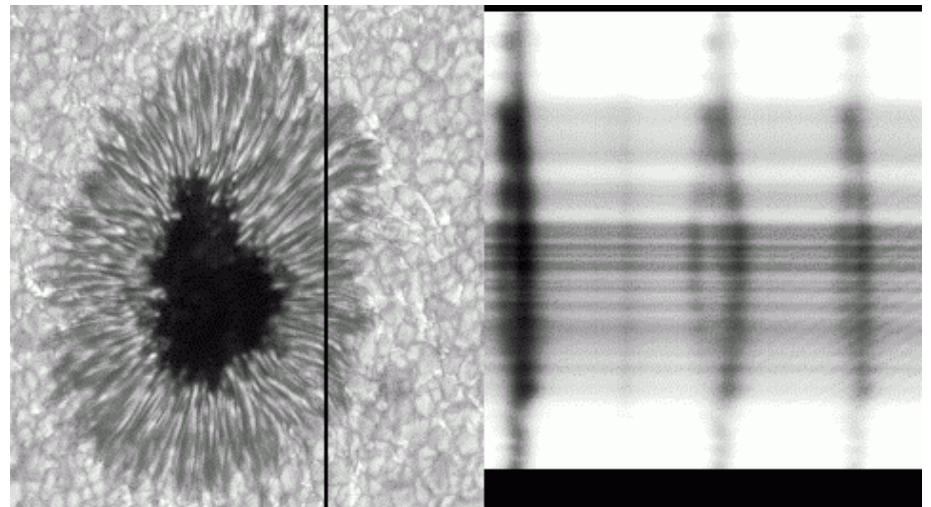
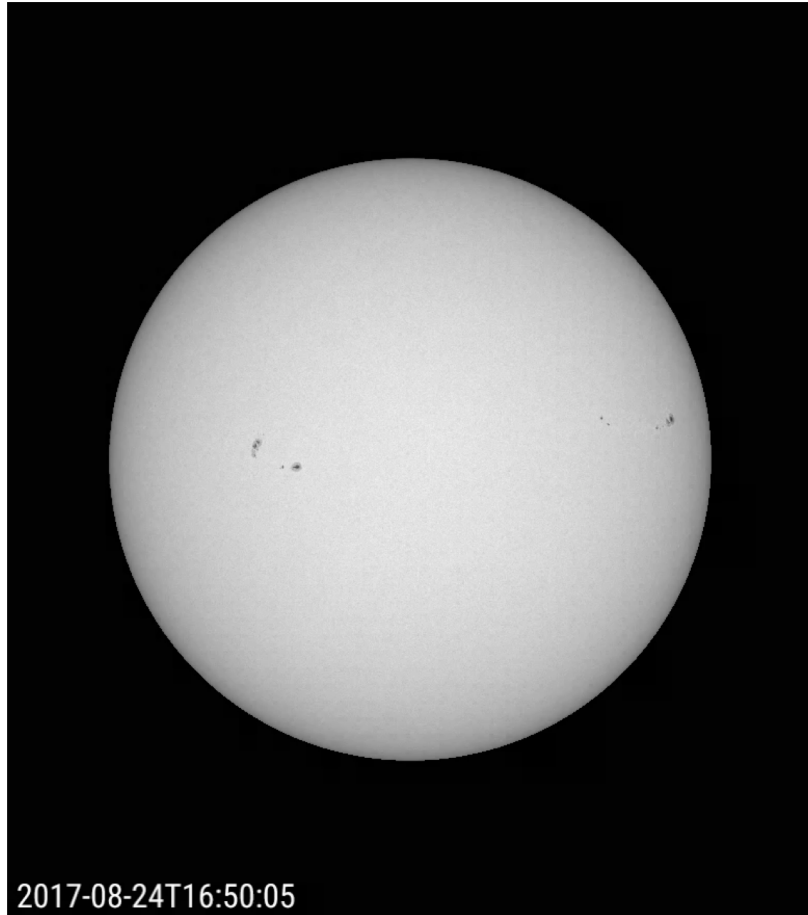
Zeeman Effect

- Energy states correspond to the magnetic quantum number m_l (which describes the orientation of the electron's motion) are **separated** under the presence of **external magnetic field**

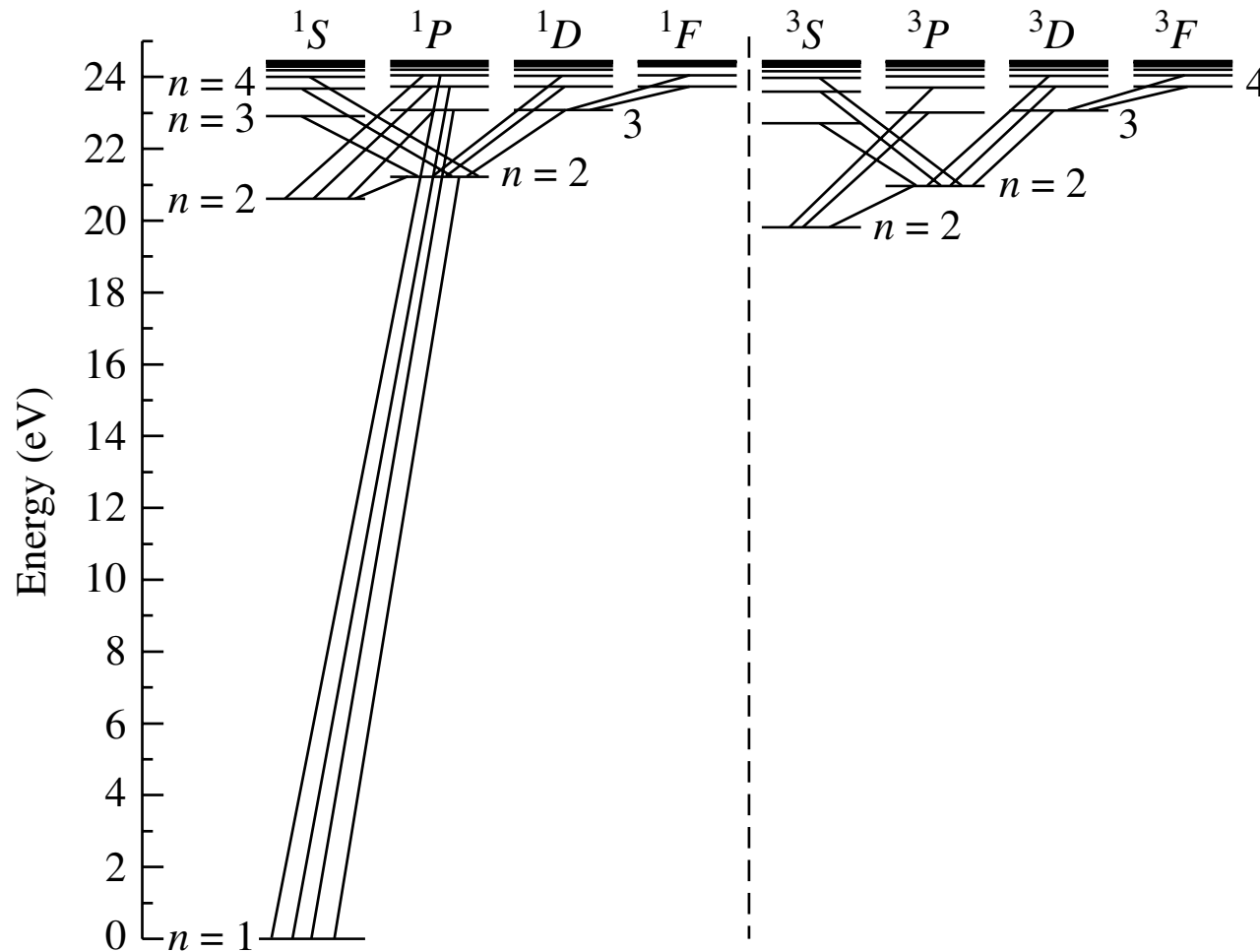


$$\nu = \nu_0 \quad \text{and} \quad \nu_0 \pm \frac{eB}{4\pi\mu},$$

Zeeman Effect and the Sun's magnetic field



The Complex Spectra of Atoms



Some of the electronic energy levels of the helium atom and possible allowed transitions