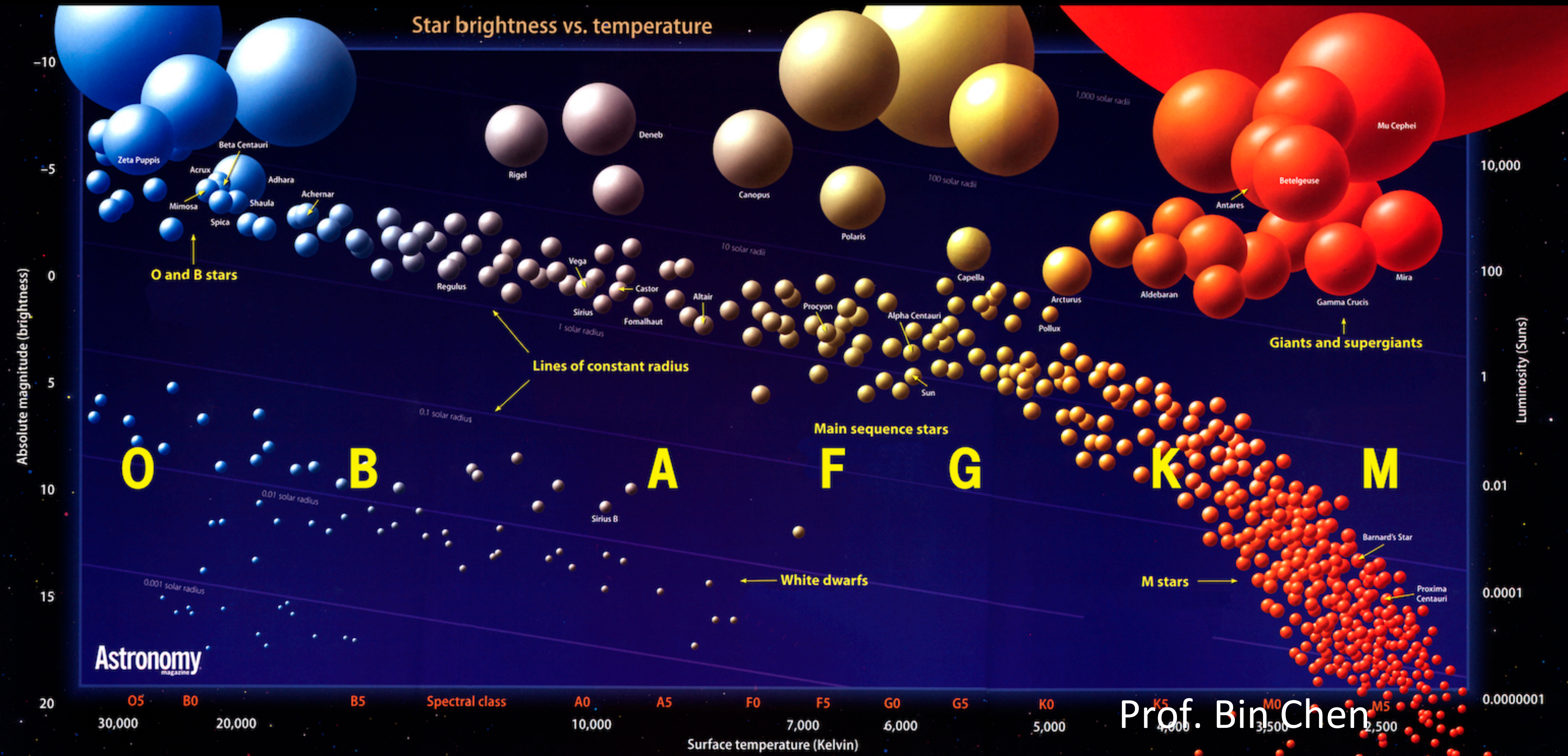
Phys 321: Lecture 3

Stellar Spectra and HR Diagram



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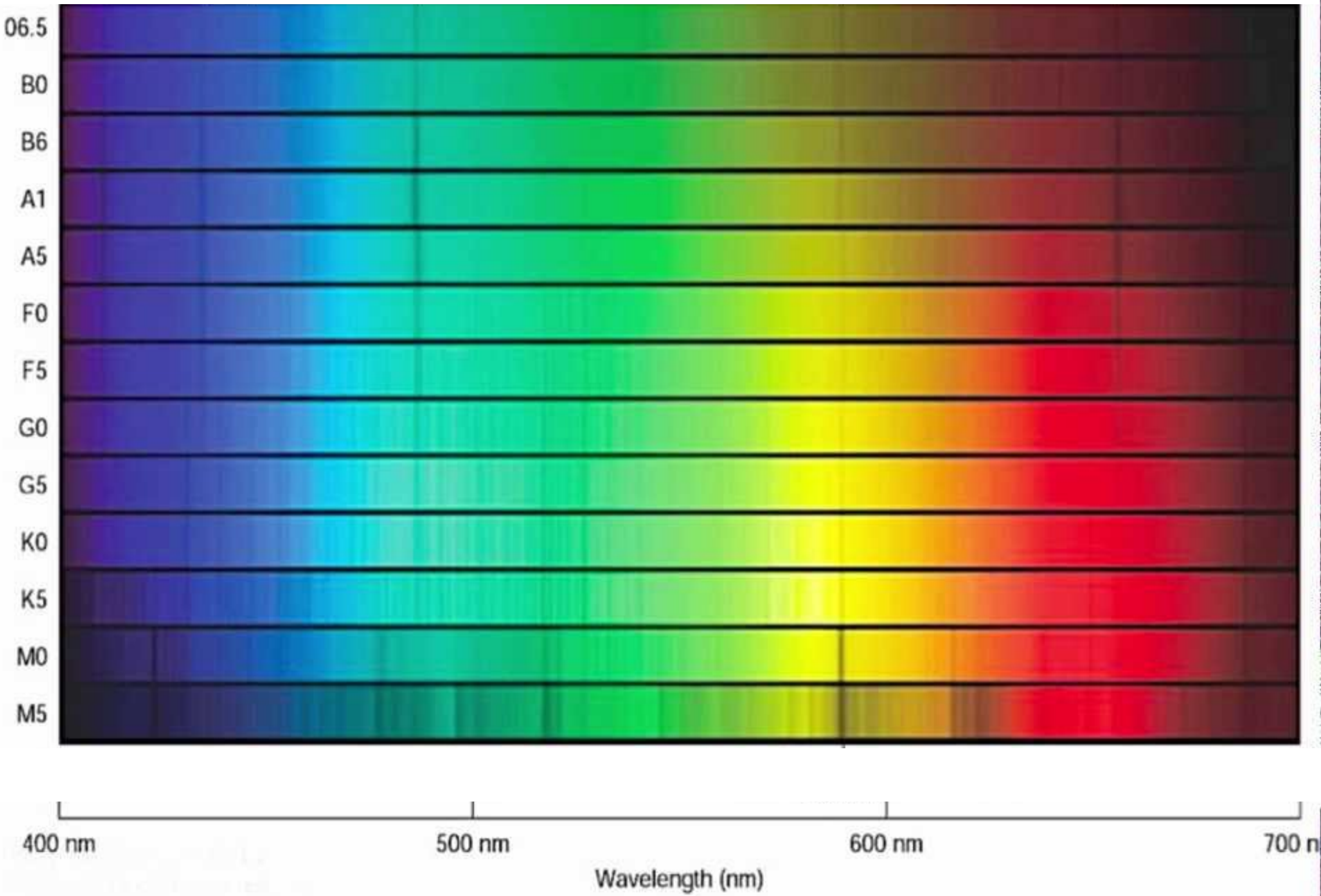
Stellar Spectra



Spectral Type of Stars

- Earlier the spectra were classified based on the Balmer lines (A, B)
- Later the spectra are re-ordered in **surface temperature** using the continuum as the guide (Annie J. Cannon)

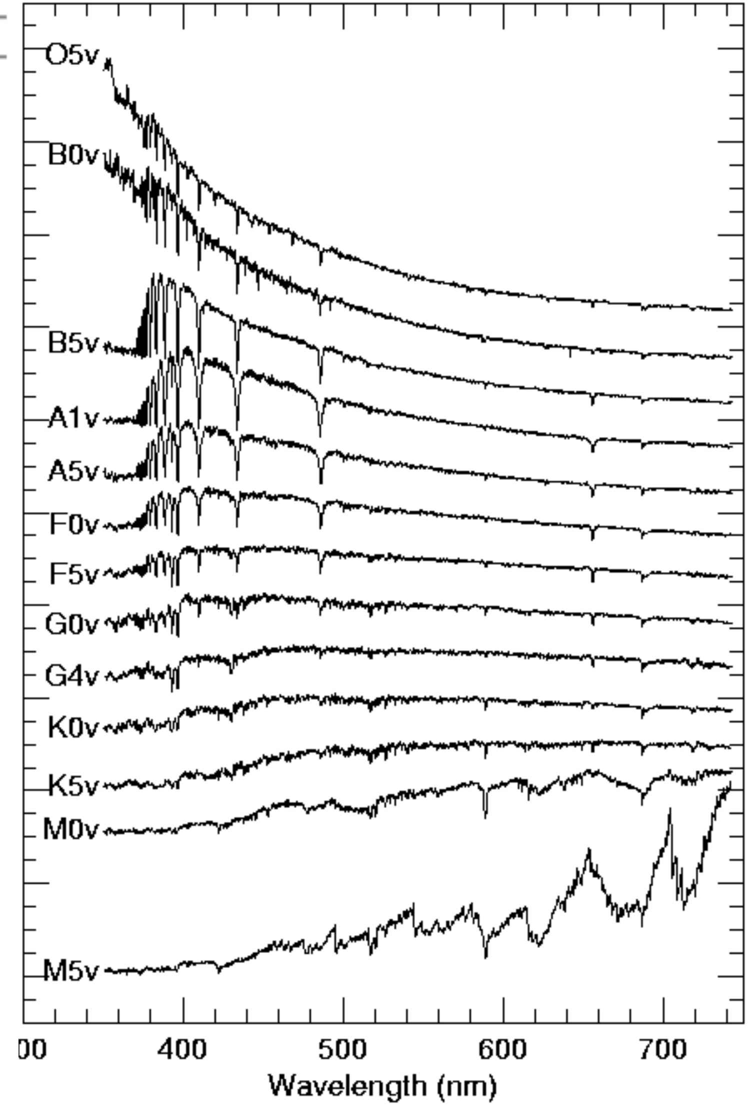
Annie Jump Cannon

Who classified over 200,000 stellar spectra included in the Henry Draper Catalogue

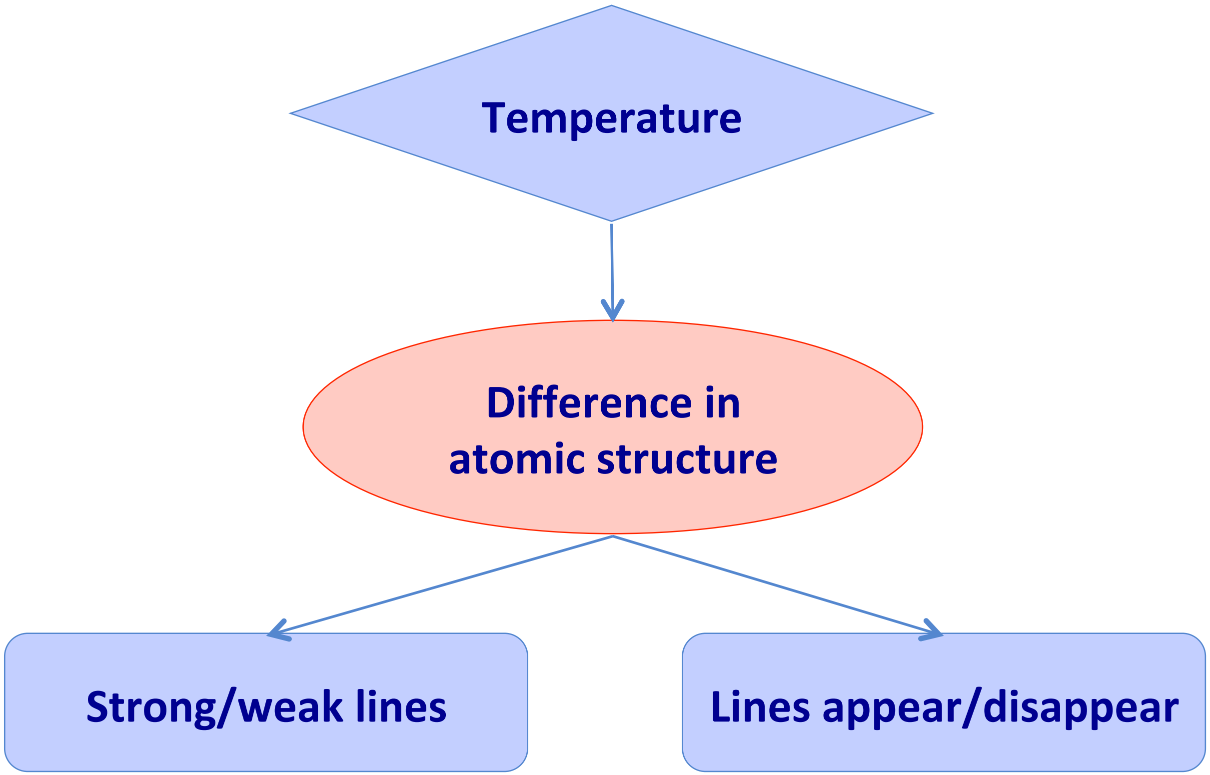
O, B, A, F, G, K, M (**O**h **B**e **A** **F**ine **G**irl/**G**uy **K**iss **M**e)
(early) (late)

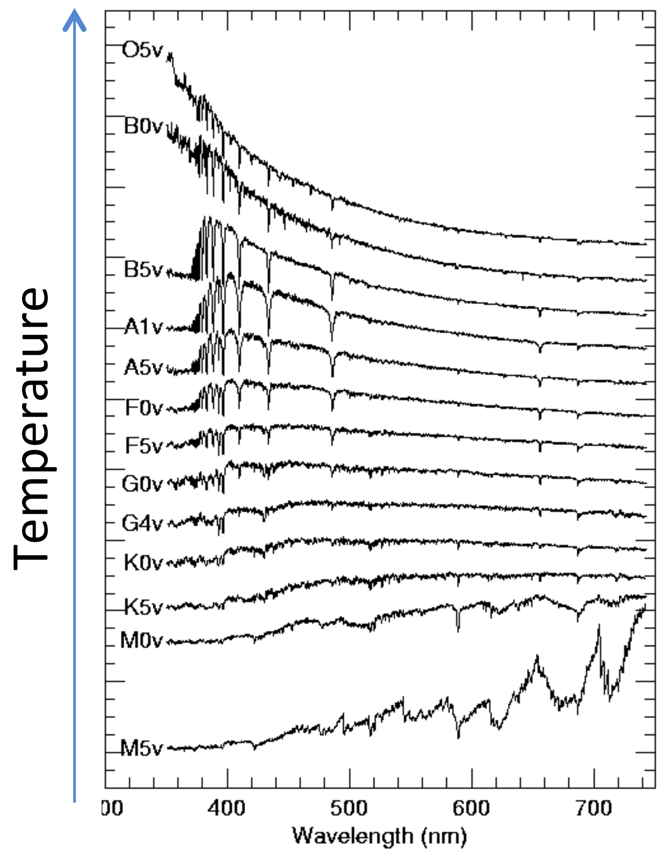
...B7, B8, B9, A0, A1, A2, A3, ..., A7, A8, A9, F0, F1... etc.
(early A) (late A)

Spectral Type	Characteristics
O	Hottest blue-white stars with few lines Strong He II absorption (sometimes emission) lines. He I absorption lines becoming stronger.
B	Hot blue-white He I absorption lines strongest at B2. H I (Balmer) absorption lines becoming stronger.
A	White Balmer absorption lines strongest at A0, becoming weaker later. Ca II absorption lines becoming stronger.
F	Yellow-white Ca II lines continue to strengthen as Balmer lines continue to weaken. Neutral metal absorption lines (Fe I, Cr I).
G	Yellow Solar-type spectra. Ca II lines continue becoming stronger. Fe I, other neutral metal lines becoming stronger.
K	Cool orange Ca II H and K lines strongest at K0, becoming weaker later. Spectra dominated by metal absorption lines.
M	Cool red Spectra dominated by molecular absorption bands, especially titanium oxide (TiO) and vanadium oxide (VO). Neutral metal absorption lines remain strong.
L	Very cool, dark red Stronger in infrared than visible. Strong molecular absorption bands of metal hydrides (CrH, FeH), water (H_2O), carbon monoxide (CO), and alkali metals (Na, K, Rb, Cs). TiO and VO are weakening.
T	Coolest, Infrared Strong methane (CH_4) bands but weakening CO bands.



Physics behind different spectral types





Questions

As a function of temperature

- In **what orbitals** are electrons most likely to be found?
- What are the relative numbers of atoms in **various stages of ionization**?

Statistical Mechanics

Maxwell-Boltzmann velocity distribution

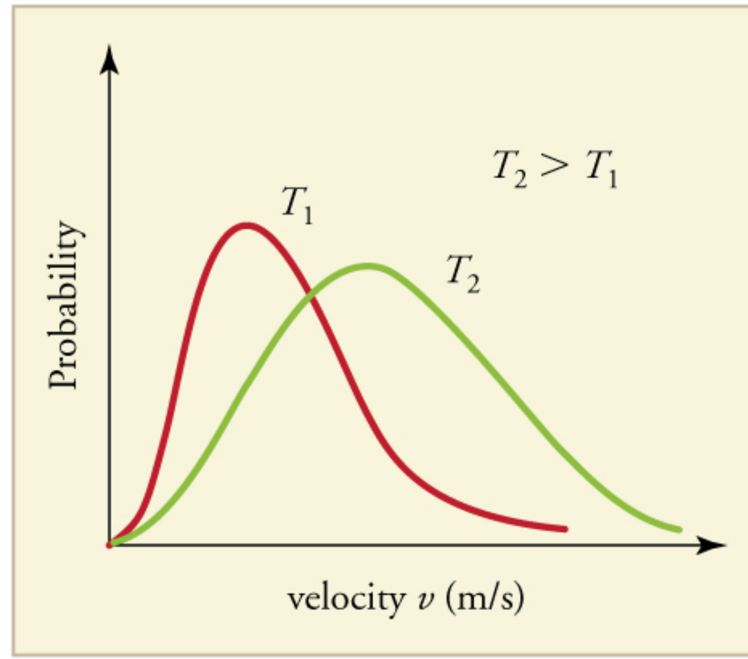
Particle mass

$$n_v\,dv = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT}\,4\pi v^2\,dv,$$

Total number density (# per m^3)

Temperature

Velocity



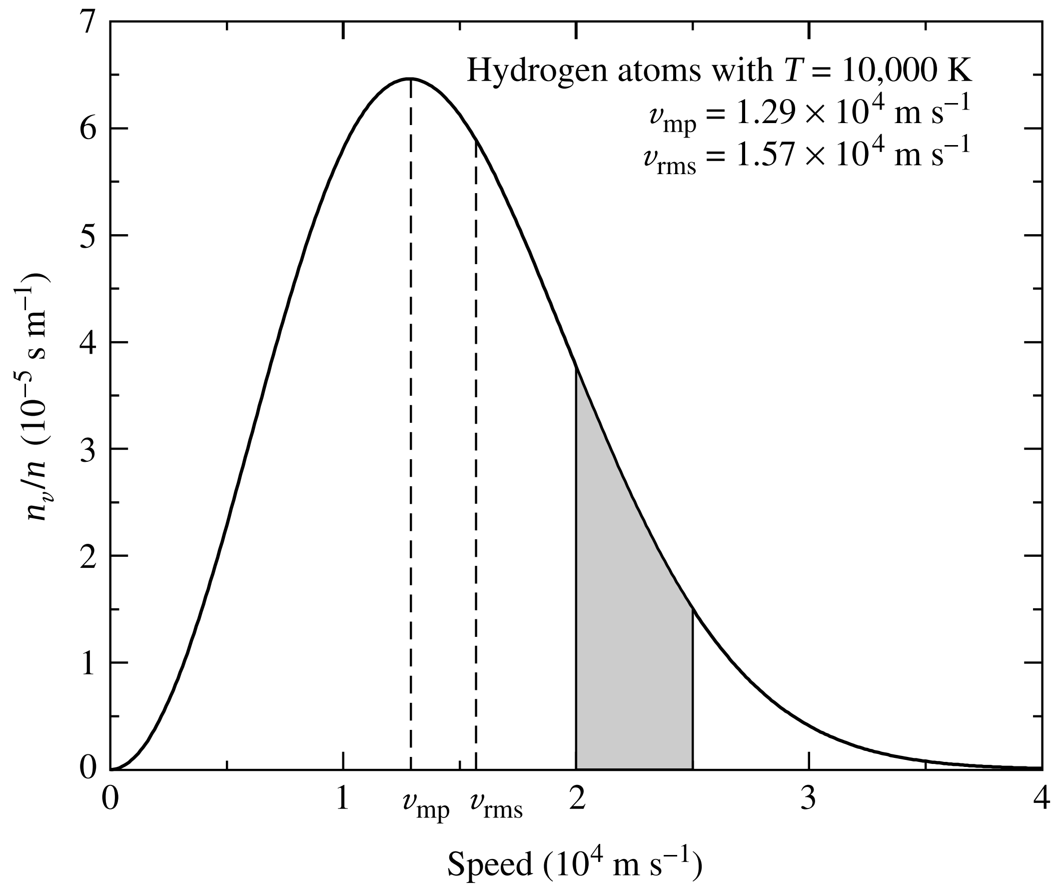
Maxwell-Boltzmann velocity distribution

Most probable speed

$$v_{\text{mp}} = \sqrt{\frac{2kT}{m}}.$$

Root-mean-square (average) speed

$$\boxed{v_{\text{rms}} = \sqrt{\frac{3kT}{m}}.}$$



The Boltzmann Equation

- Orbitals of higher energy are less likely to be occupied by electrons

Probability of the system in state S_b

Energy in state S_b

Temperature

$$\frac{P(s_b)}{P(s_a)} = \frac{e^{-E_b/kT}}{e^{-E_a/kT}} = e^{-(E_b - E_a)/kT}$$

Probability of the system in state S_a

Energy in state S_a

The Boltzmann Equation

- Often more than one state can have the same energy – the energy levels may be degenerate
- E.g., in a hydrogen atom
    - n = 1 (-13.6 eV) state is two fold degenerate -> g = 2
    - n = 2 (-3.40 eV) state is eight fold degenerate -> g = 8

Probability of the system in **Energy E_b**

Statistical weight of E_b state(s)

Temperature

$$\frac{P(E_b)}{P(E_a)} = \frac{g_b\, e^{-E_b/kT}}{g_a\, e^{-E_a/kT}} = \frac{g_b}{g_a}\, e^{-(E_b - E_a)/kT}$$

Probability of the system in **energy E_a**

Statistical weight of E_a state(s)

The Boltzmann Equation

- For a large number of atoms (as in stellar atmosphere), the ratio of probabilities is indistinguishable from the ratio of numbers of atoms:

$$\frac{N_b}{N_a} = \frac{g_b\, e^{-E_b/kT}}{g_a\, e^{-E_a/kT}} = \frac{g_b}{g_a}\, e^{-(E_b-E_a)/kT}.$$

Example

- Degeneracy of hydrogen atoms of energy level n is $g=2n^2$
- At what temperature (in K) will equal numbers of H I atoms have electrons in the ground state (n = 1, g = 2, E = -13.6 eV) and in the first excited state (n = 2, g = 8, E = -3.40 eV)?

$$\frac{N_b}{N_a} = \frac{g_b\, e^{-E_b/kT}}{g_a\, e^{-E_a/kT}} = \frac{g_b}{g_a}\, e^{-(E_b-E_a)/kT}.$$

k = 1.38 x 10^{-23} J/K,
1 eV = 1.602 x 10^{-19} J

$$1 = \frac{2(2)^2}{2(1)^2}\, e^{-[(-13.6\text{ eV}/2^2)-(-13.6\text{ eV}/1^2)]/kT},$$

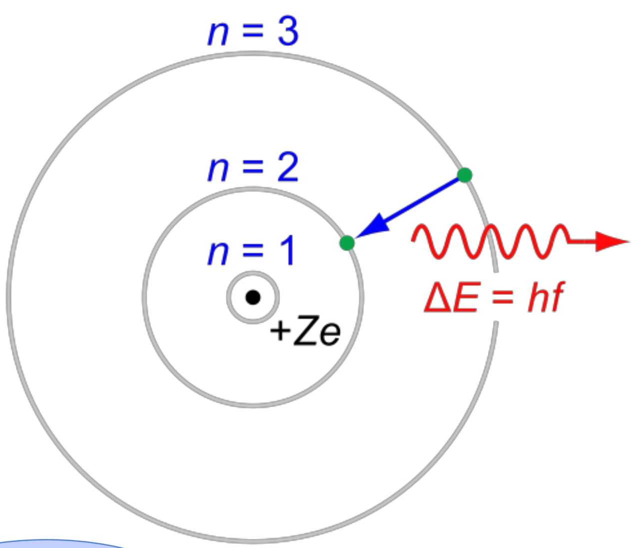
or

$$\frac{10.2\text{ eV}}{kT} = \ln(4).$$

$$T = \frac{10.2\text{ eV}}{k\ln(4)} = 8.54\times10^4\text{ K}$$

Strength of Hydrogen Balmer lines

Balmer series: n = 2 -> n = 2, 3, 4, …



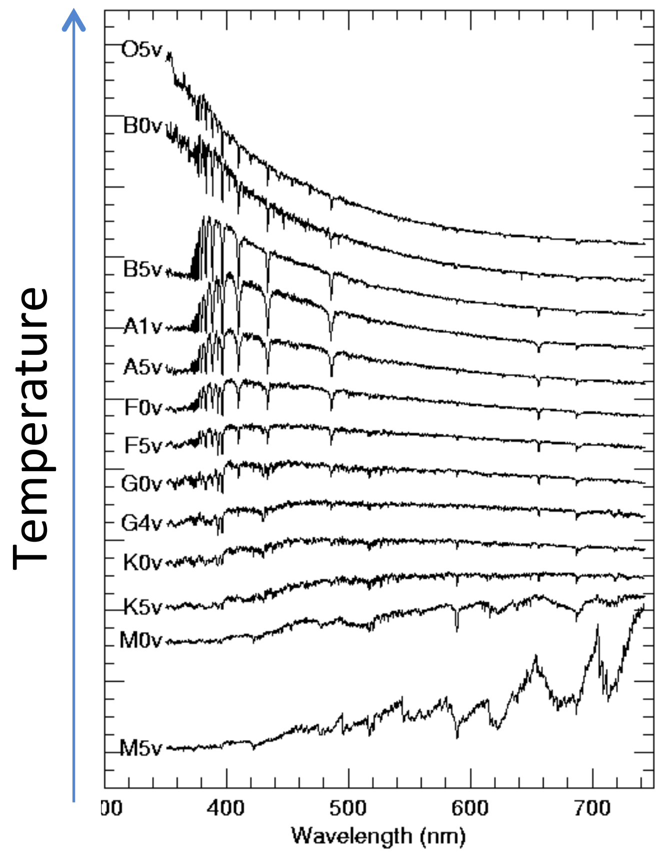
Higher T

-> more electrons populate n = 2 level

-> more transitions from n = 2 level

-> stronger Balmer lines?

Why?



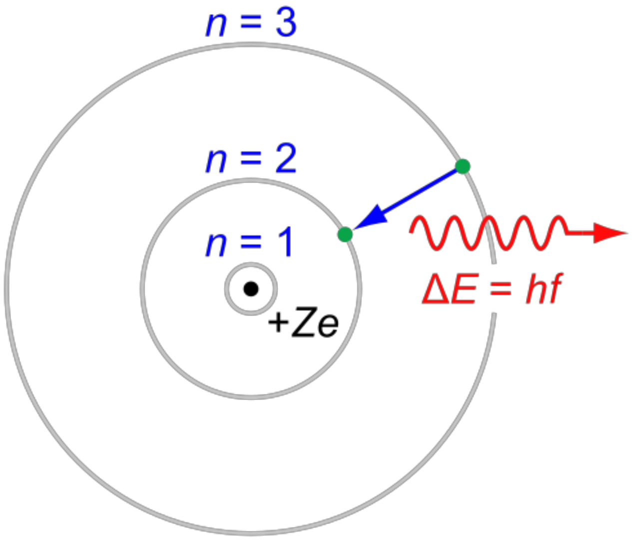
Balmer lines strongest at A0 (~9520 K). **Weaker** for later **and earlier** spectral types.

Strength of Hydrogen Balmer lines

- Higher T
  - -> more electrons populate n = 2 level
  - -> stronger Balmer lines
- Even higher T, atoms become ionized!
  - -> less atoms have any electrons at all
  - -> weaker Balmer lines

Ionization Stages:

H I = neutral hydrogen
H II = ionized hydrogen
He I = neutral helium
He II = singly-ionized helium
He III = doubly-ionized helium



Ionization of hydrogen from the ground n=1 state requires photons with wavelength less than 91 nm. What is the minimum wavelength to ionize from the n=2 level?

A. 45.5 nm
B. 125 nm
C. 182 nm
D. 364 nm

The Saha Equation

- To get a hydrogen atom's electron from n = 1 to n = 2, it needs 10.2 eV of energy
- To get the same electron to n = ∞ (and becomes a H II atom), it only needs 3.4 eV more energy!
- Let χ_i be the energy needed to remove an electron from an atom (or ion) in the ground state, taking it from ionization stage i to stage i + 1. For hydrogen atoms, it is 13.6 eV.

$$\frac{N_{i+1}}{N_i} \sim \frac{Z_{i+1}}{Z_i} \exp(-\chi_i / kT)$$

N_i: Number of atoms in ionization stage i

Z_i: **partition function** for ionization stage i

$$Z = \sum_{j=1}^{\infty} g_j \, e^{-(E_j - E_1)/kT} .$$

The Saha Equation

- Extra contribution from free electrons:

  -> more free electrons, more chance for the N_{i+1} stage to recombine to N_i stage

- Full Saha Equation:

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_i/kT}.$$

Electron number density

Example: H between 5,000 and 25,000 K

- Partition function for H II is Z_{II} = 1, since there is no degeneracy for a bare proton
- Partition function for H I:

$$Z = \sum_{j=1}^{\infty} g_j \, e^{-(E_j - E_1)/kT}.$$

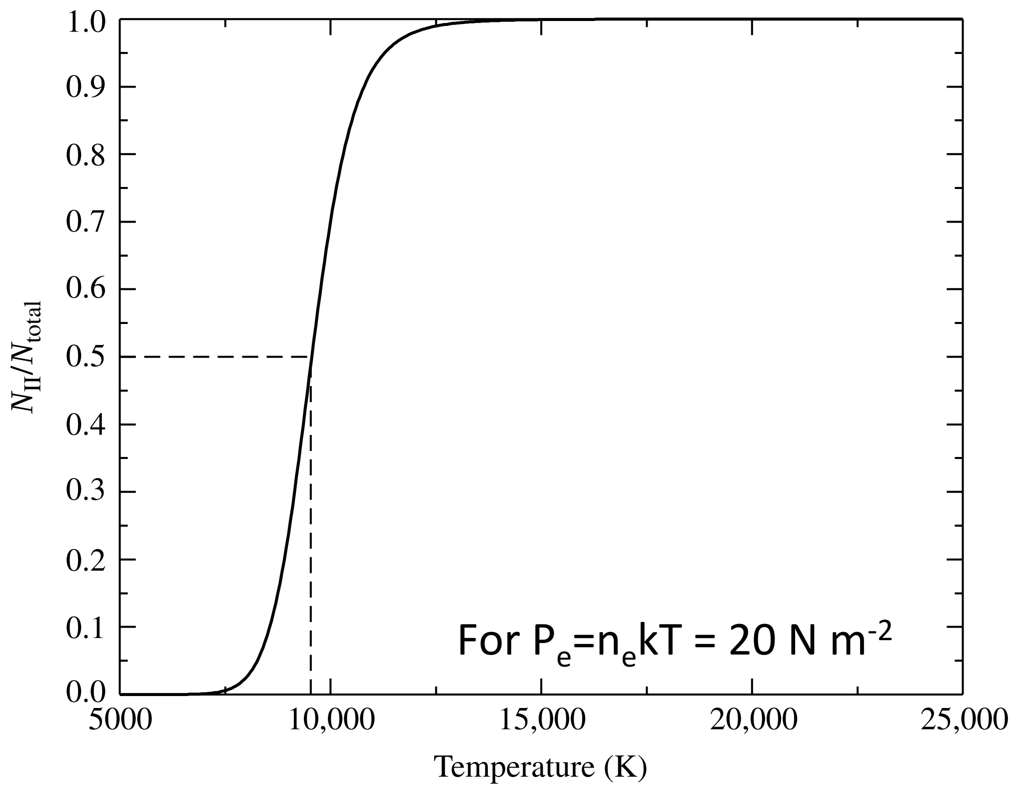
Let's just take the first two terms (**Why?**)

$$Z_I \approx 2 + 8\exp(-[(-3.4eV) - (-13.6eV)]/kT) = 2 + 8\exp(-10.2eV/KT)$$

The thermal energy kT for 5,000 to 25,000 K is from ~0.5 eV to 2.5 eV, so $Z_I \approx 2$

Example: H between 5,000 and 25,000 K

- Inserting the values we get N_{II}/N_I
- Sometimes we want $$\frac{N_{II}}{N_{total}} = \frac{N_{II}}{N_I + N_{II}} = \frac{N_{II}/N_I}{1 + N_{II}/N_I}$$



Above T ~ 9,600 K, more than 50% of hydrogen atoms are ionized!

Combining Boltzmann and Saha Equations

- Let's evaluate what fraction of atoms are in the HI n = 2 state responsible for Balmer lines N_2/N_{total}
- Nearly all the neutral hydrogen atoms are either in n = 1 or n = 2 state $N_1 + N_2 \simeq N_\mathrm{I}$

$$\frac{N_2}{N_\mathrm{total}} = \left(\frac{N_2}{N_1 + N_2}\right)\left(\frac{N_\mathrm{I}}{N_\mathrm{total}}\right)$$

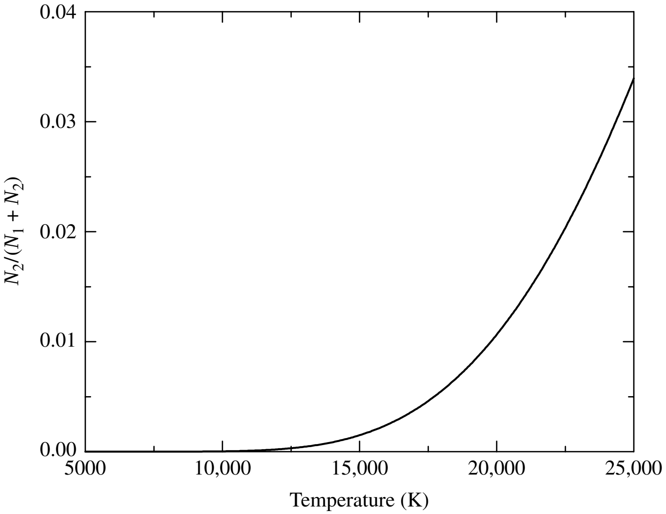
$$= \left(\frac{N_2/N_1}{1 + N_2/N_1}\right)\left(\frac{1}{1 + N_\mathrm{II}/N_\mathrm{I}}\right)$$

Boltzmann Equation

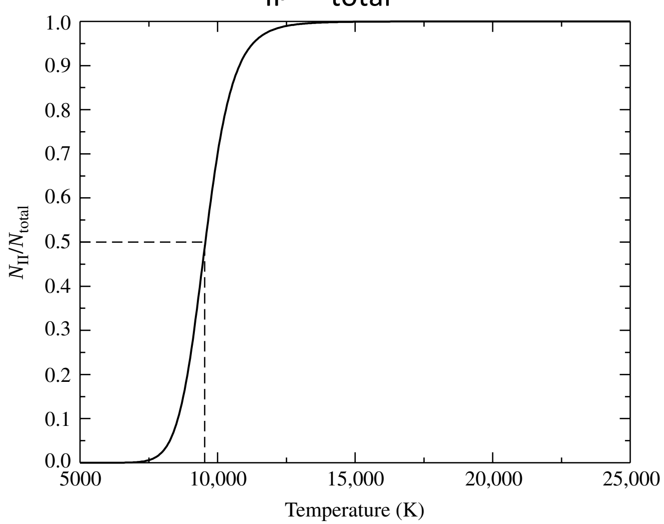
Saha Equation

Combining Boltzmann and Saha Equations

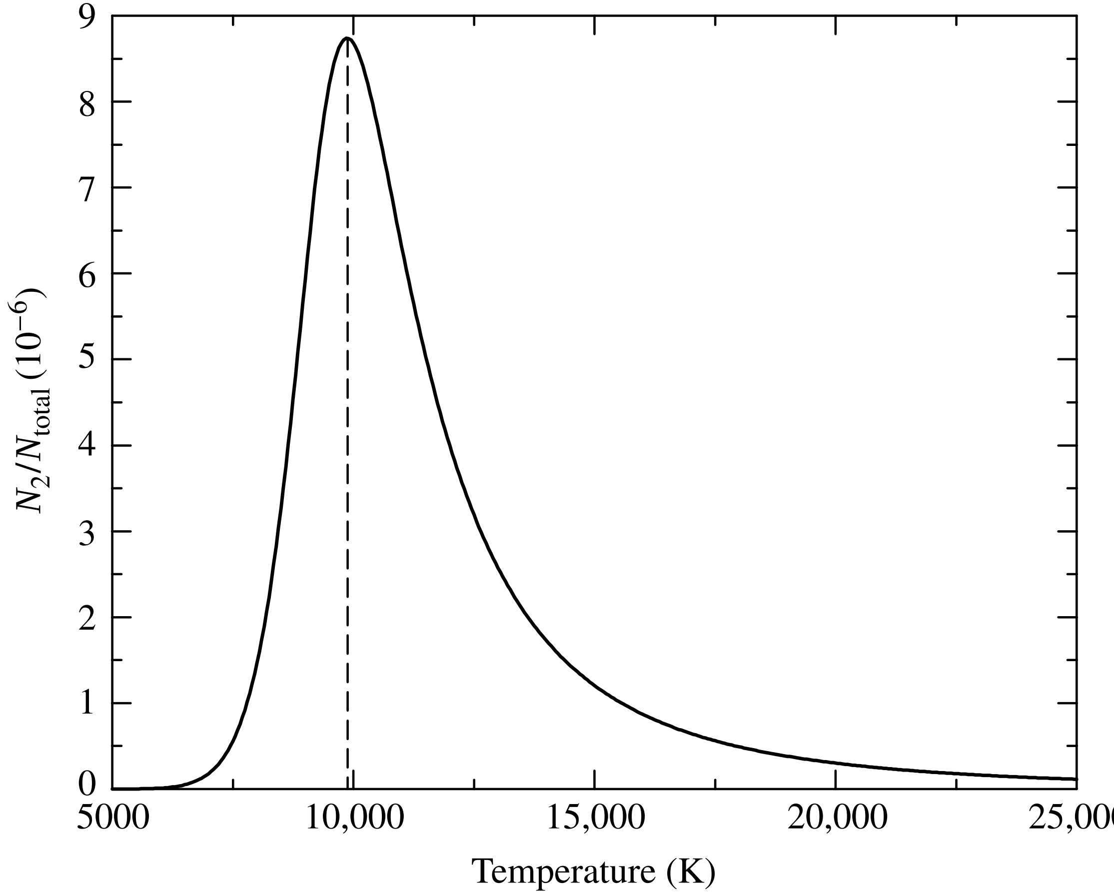
$N_2/(N_1+N_2)$



N_{II}/N_{total}

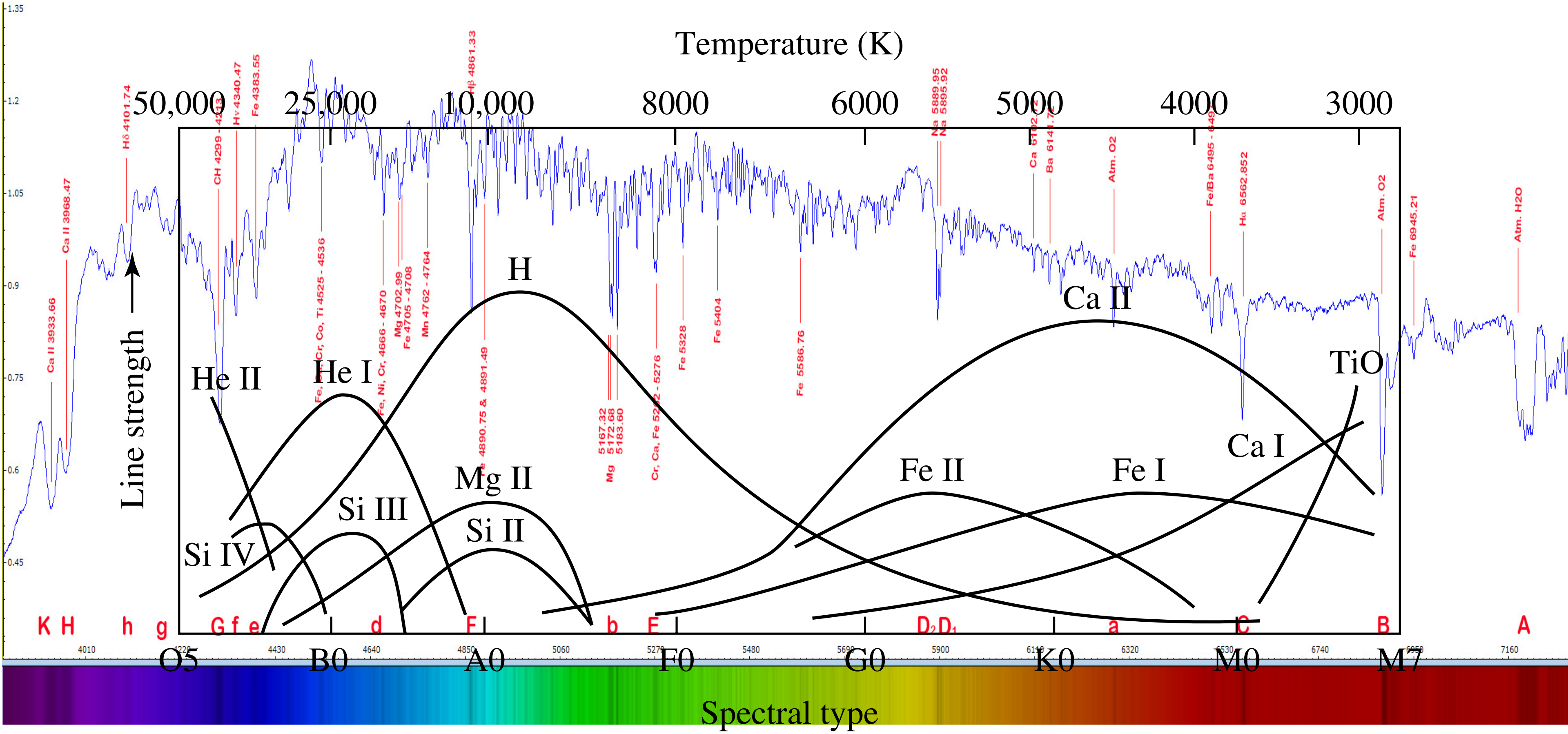


$$\frac{N_2}{N_{\text{total}}} = \left(\frac{N_2}{N_1 + N_2}\right)\left(\frac{N_{\text{I}}}{N_{\text{total}}}\right)$$



Strength of Different Spectral Lines

Question: For each calcium atom, there are 500,000 hydrogen atoms! But why in the solar spectrum, the Ca II (H & K) lines are more profound than the Balmer lines?



Temperature

Wien's Law

$$T \approx 6000K \frac{500nm}{\lambda_{peak}}$$

Color index

B - V

Spectral type

O B A F G K M

← **T**

Luminosity

Stefan-Boltzmann Law

$$L = 4\pi R^2 \sigma T^4$$

Flux-distance relation

$$F = \frac{L}{4\pi d^2}$$

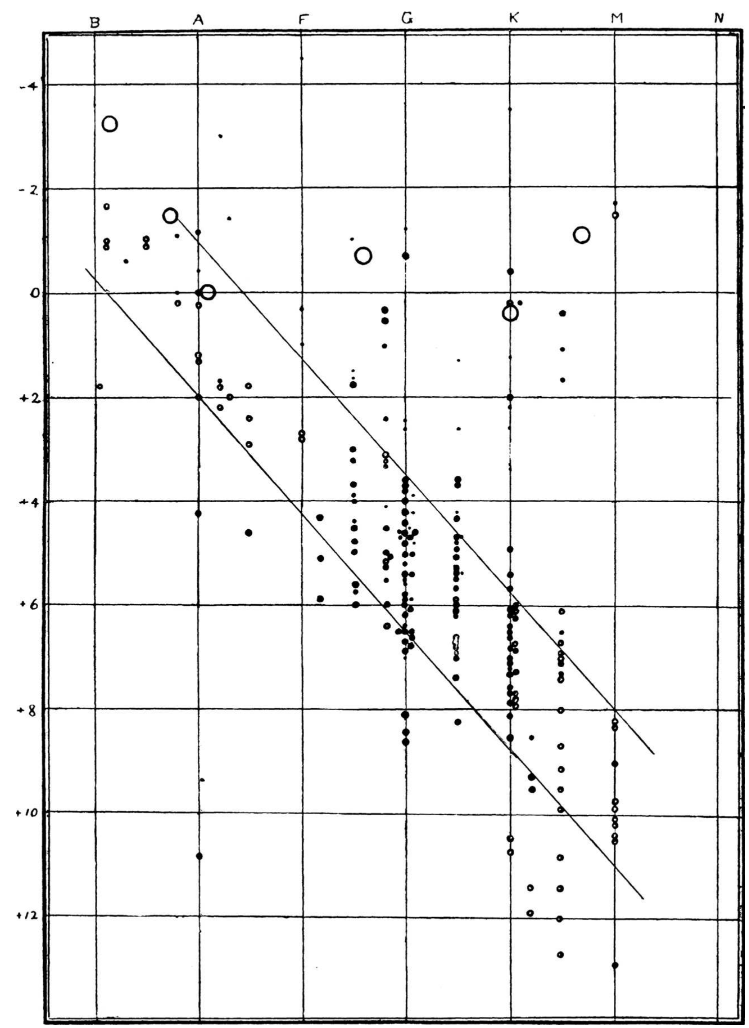
Absolute magnitude

$$m - M = 5\log_{10}\left(\frac{d}{10pc}\right)$$

Hertzprung-Russell Diagram

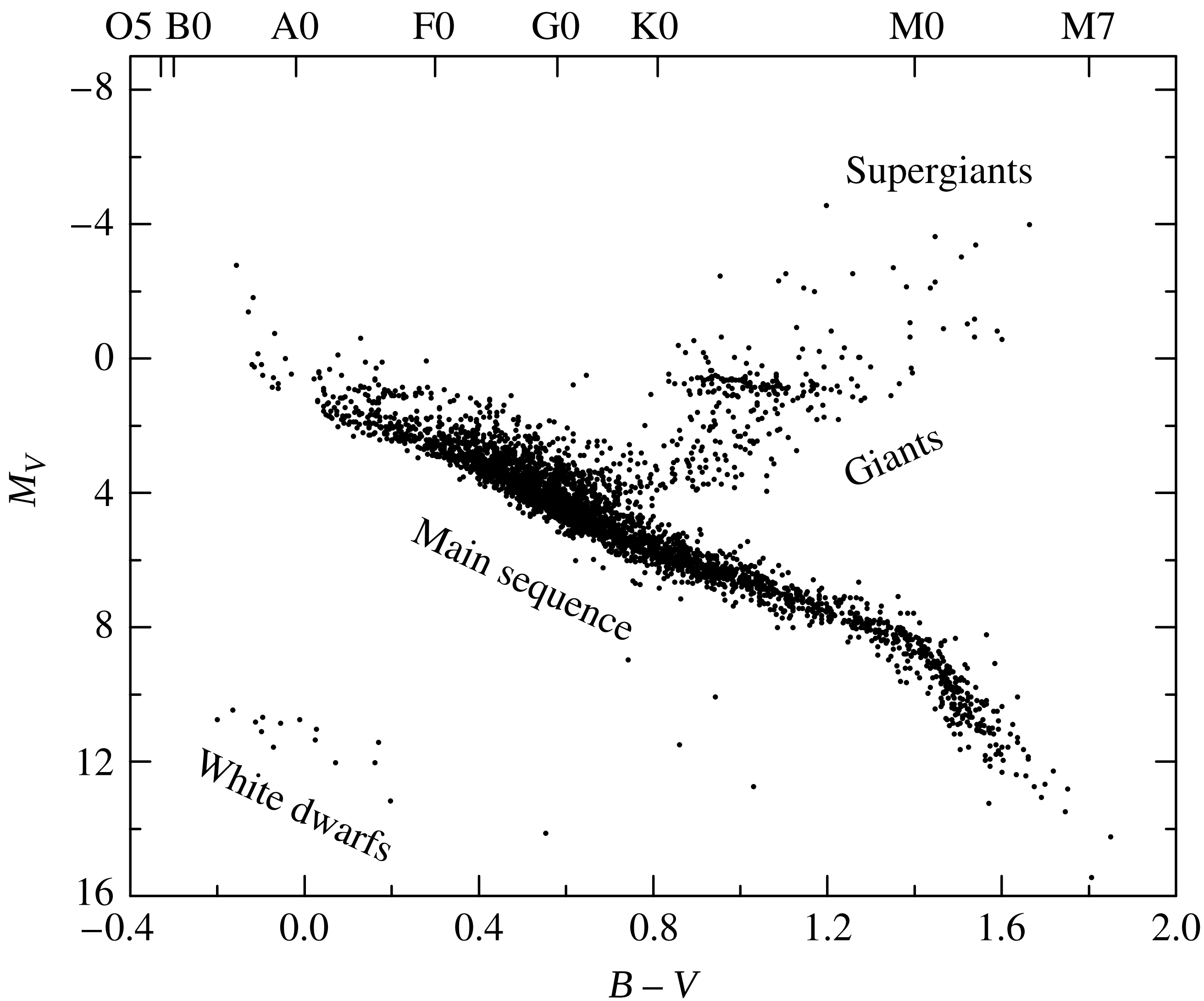
Hertzprung-Russell Diagram

- X axis: B-V color index or spectral type
- Y axis: absolute magnitude

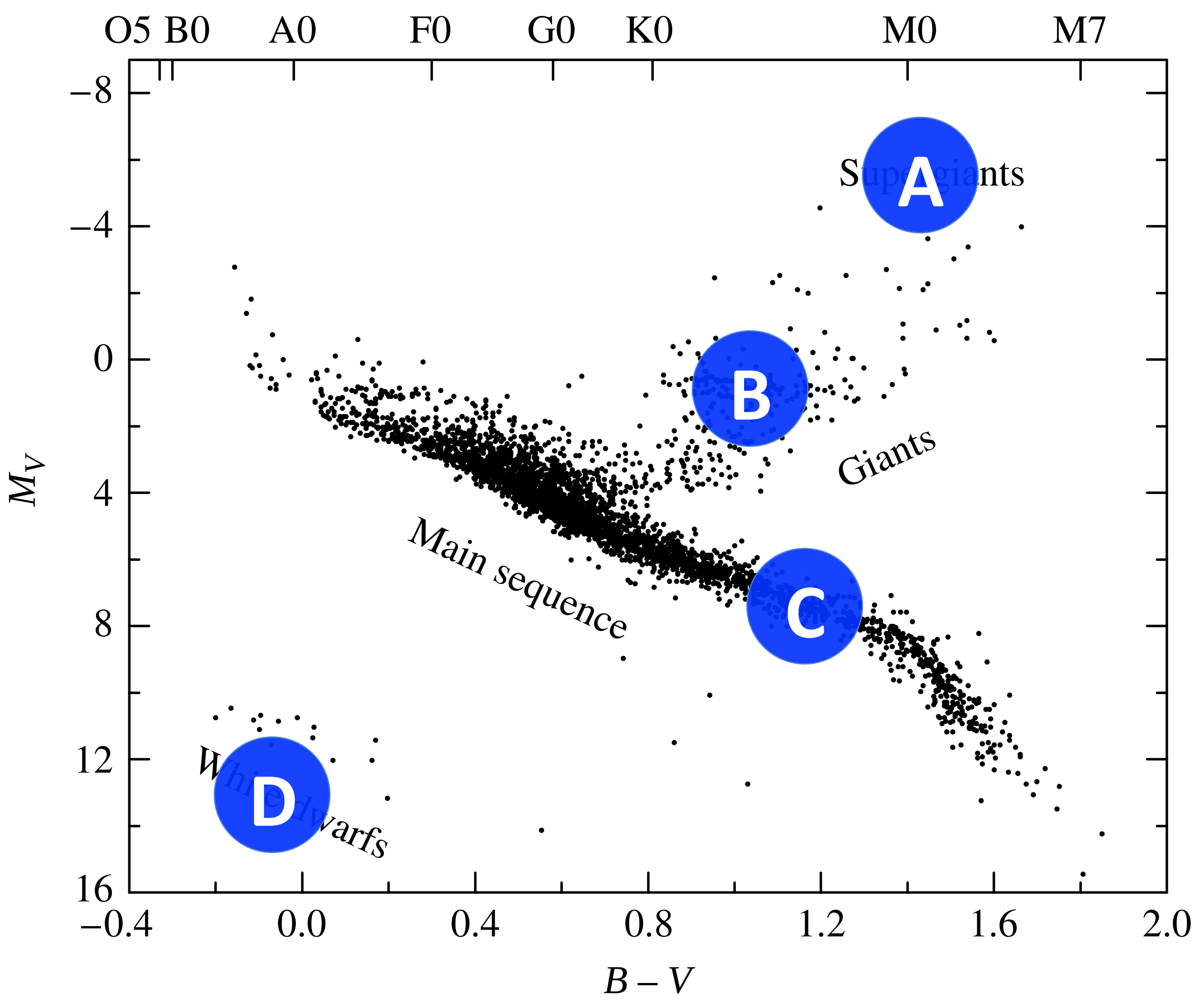


Henry N. Russell’s first diagram (1914)

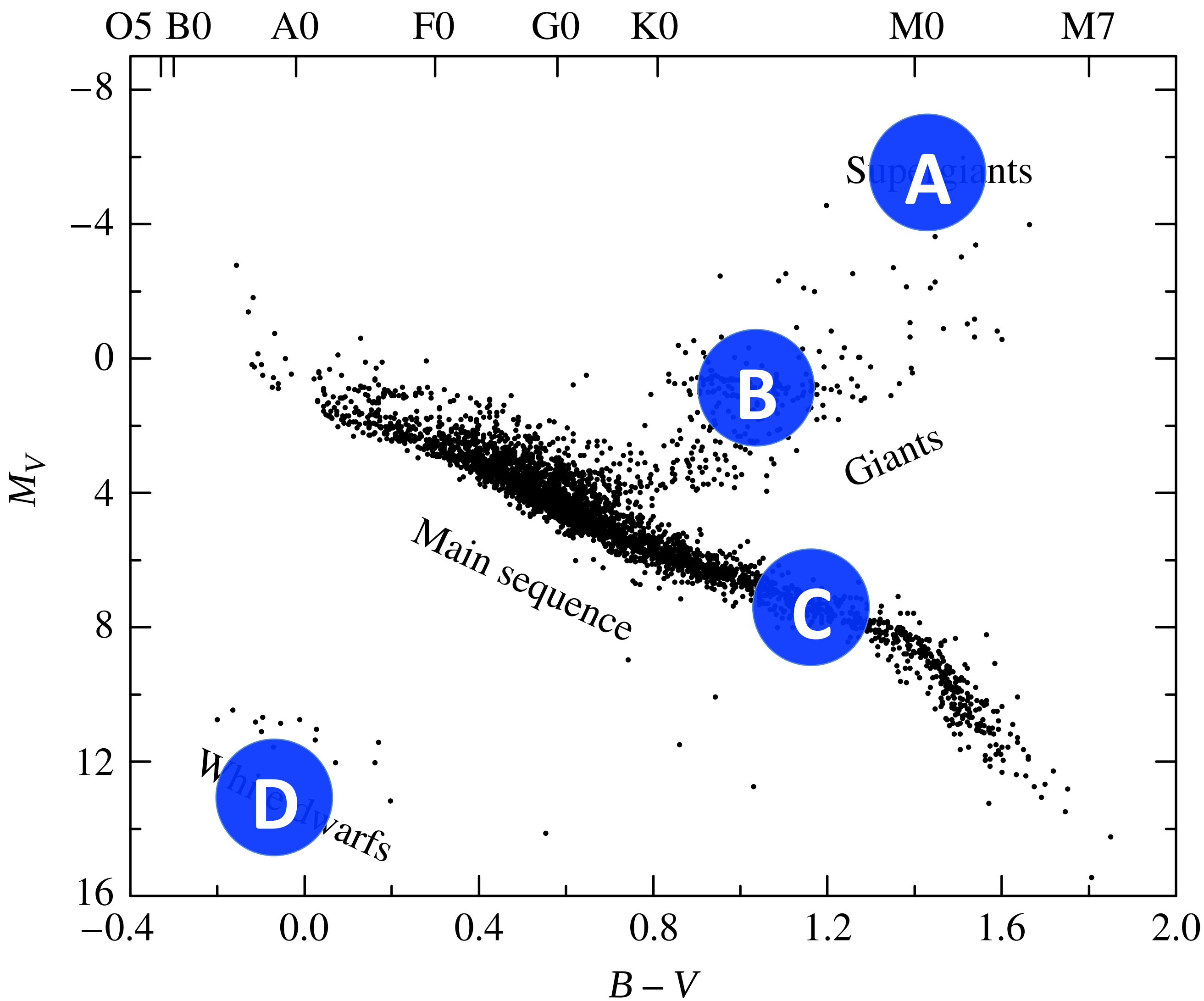
Hertzprung-Russell Diagram



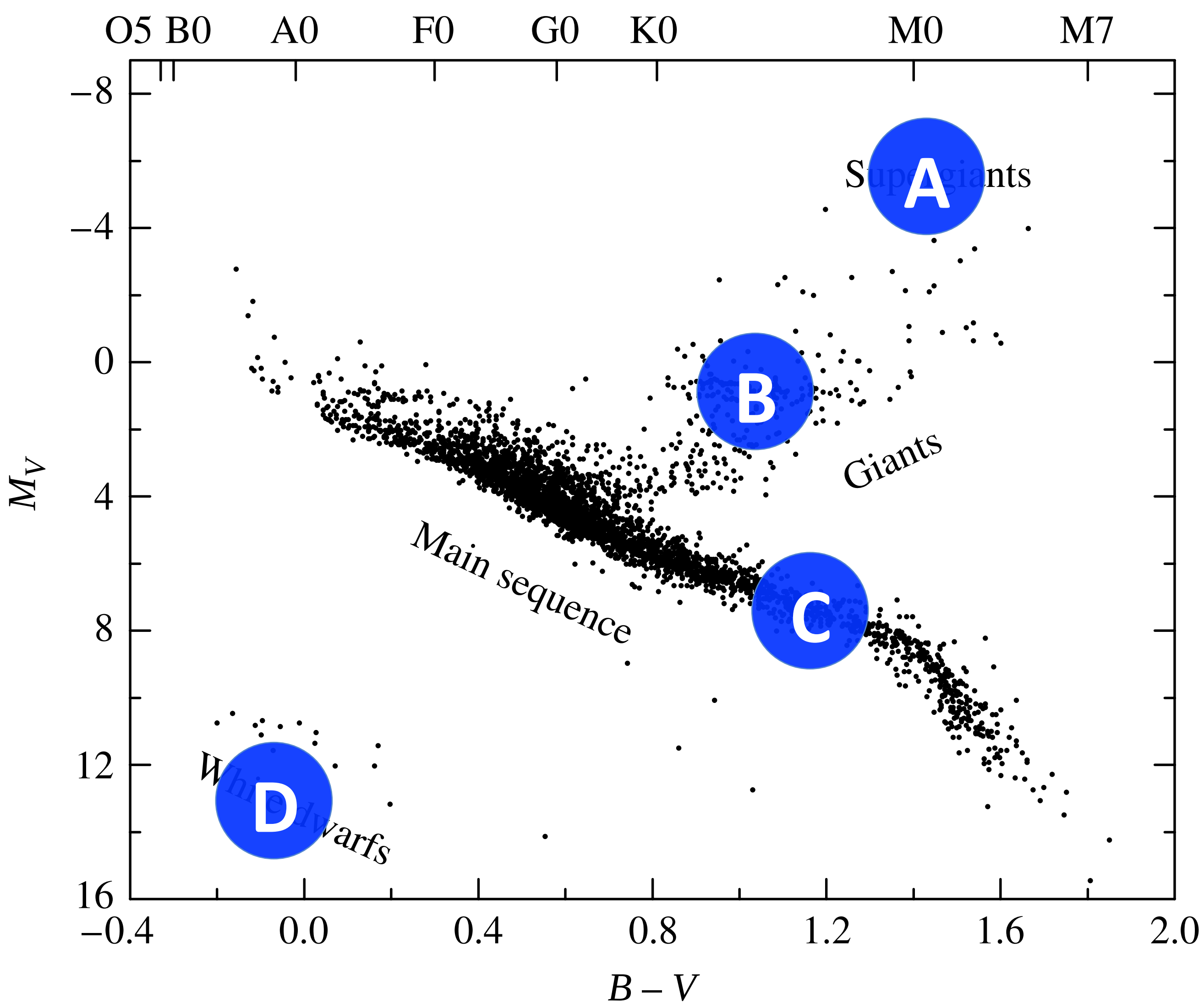
Which star is the hottest?



Which star is the most luminous?



Which star has the largest radius?

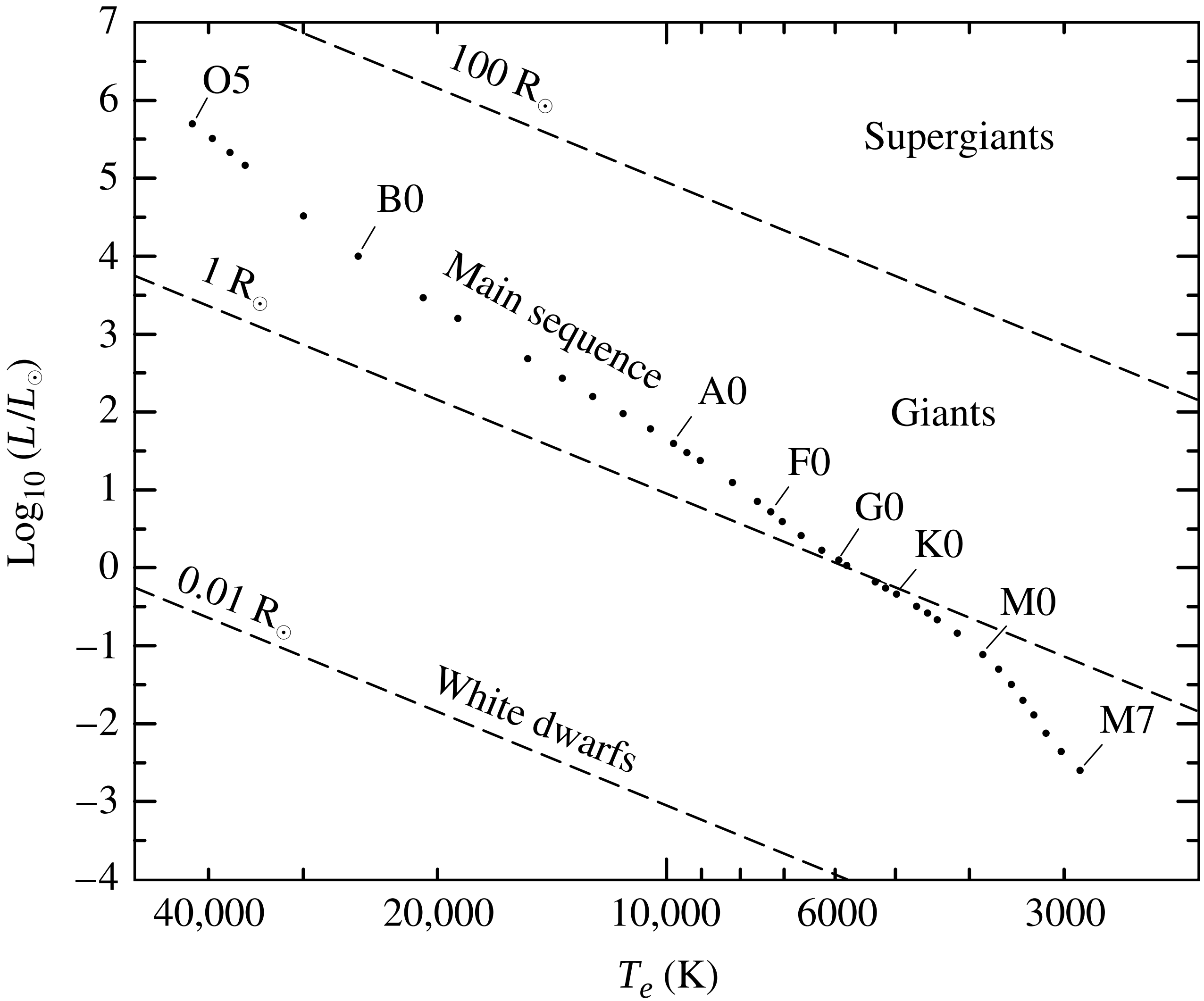


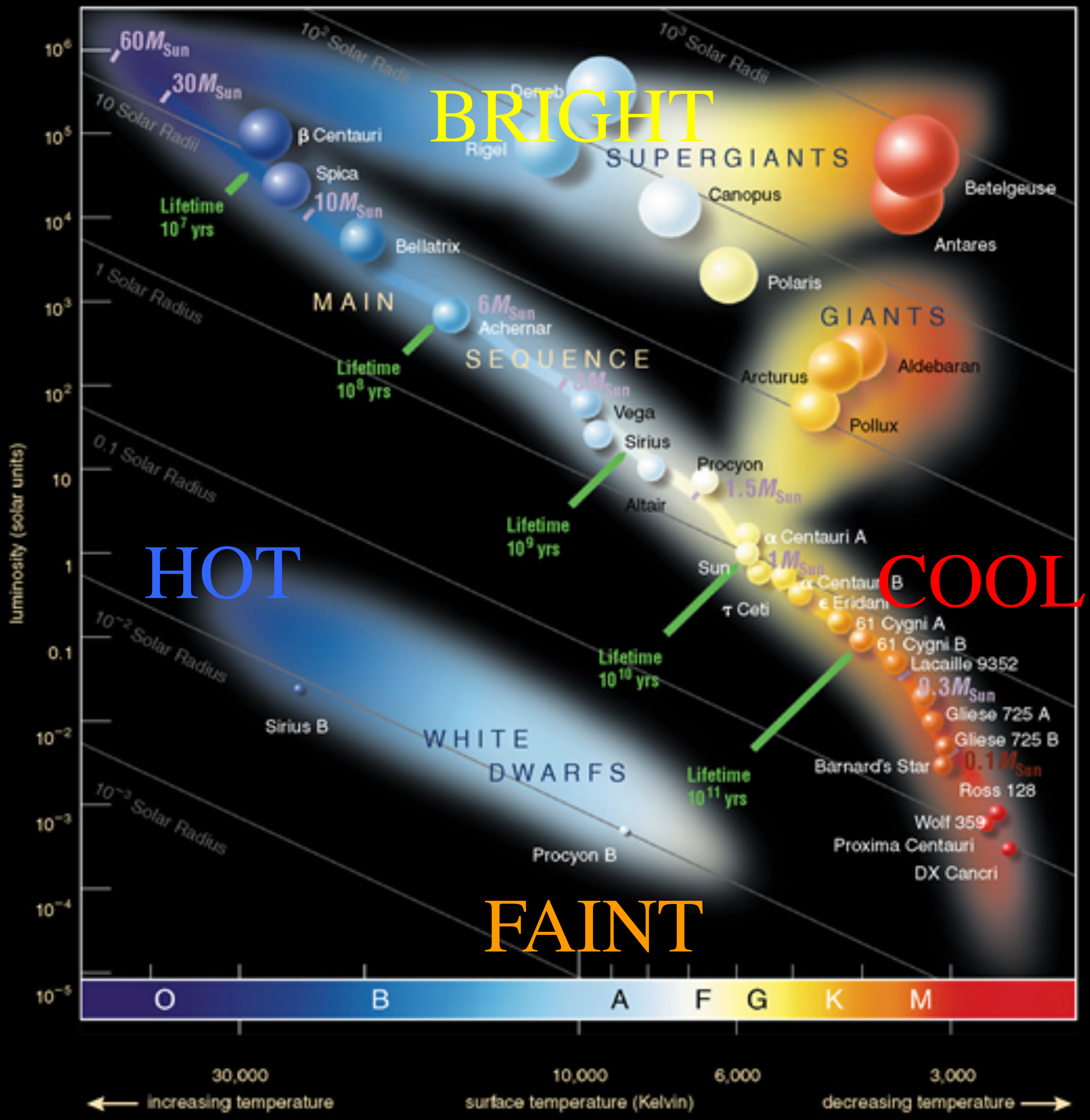
Theorist’s HR Diagram

A theorist’s HR diagram:

- X axis: Temperature
- Y axis: Luminosity

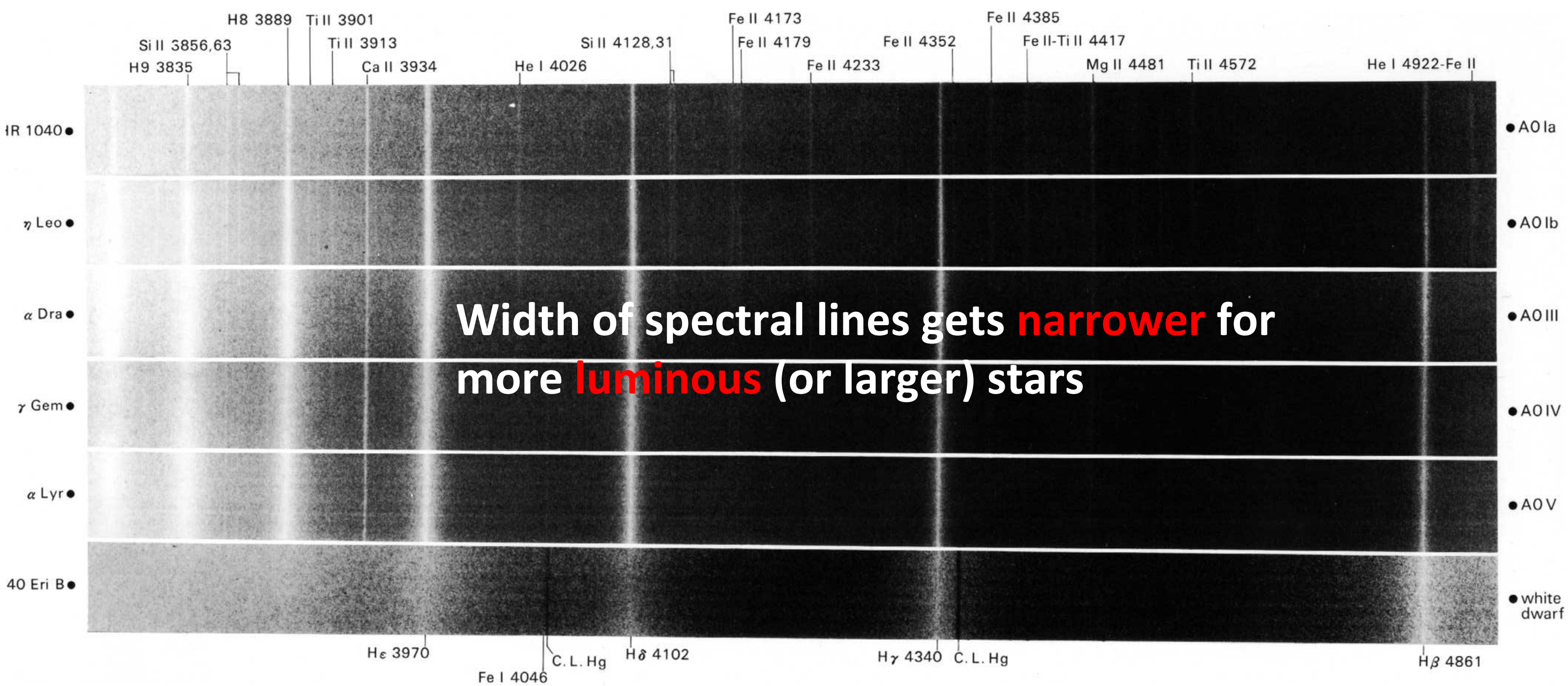
$$L = 4\pi R^2 \sigma T^4$$




BRIGHT
HOT
COOL
FAINT
SUPERGIANTS
GIANTS
MAIN SEQUENCE
WHITE DWARFS
luminosity (solar units)
surface temperature (Kelvin)
increasing temperature
decreasing temperature
30,000
10,000
6,000
3,000
O B A F G K M
Deneb
Rigel
Betelgeuse
Antares
Canopus
Polaris
Aldebaran
Arcturus
Pollux
β Centauri
Spica
Bellatrix
Achernar
Vega
Sirius
Altair
Procyon
α Centauri A
Sun
α Centauri B
τ Ceti
ε Eridani
61 Cygni A
61 Cygni B
Lacaille 9352
Gliese 725 A
Gliese 725 B
Barnard's Star
Ross 128
Wolf 359
Proxima Centauri
DX Cancri
Sirius B
Procyon B
Lifetime 10^7 yrs
Lifetime 10^8 yrs
Lifetime 10^9 yrs
Lifetime 10^{10} yrs
Lifetime 10^{11} yrs
$60 M_{Sun}$
$30 M_{Sun}$
$10 M_{Sun}$
$6 M_{Sun}$
$3 M_{Sun}$
$1.5 M_{Sun}$
$1 M_{Sun}$
$0.3 M_{Sun}$
$0.1 M_{Sun}$
10^3 Solar Radii
10^2 Solar Radii
10 Solar Radii
1 Solar Radius
0.1 Solar Radius
10^{-2} Solar Radius
10^{-3} Solar Radius
10^6
10^5
10^4
10^3
10^2
10
1
0.1
10^{-2}
10^{-3}
10^{-4}
10^{-5}

Luminosity Class

- For a given spectral type (or color index/temperature), there are stars with different luminosities and hence, sizes – dwarf, giant, supergiant
- Correspond to the Luminosity Class, categorized by subtle differences in the spectra with the same spectral class



Luminosity Class

Class	Type of Star
Ia-O	Extreme, luminous supergiants
Ia	Luminous supergiants
Ib	Less luminous supergiants
II	Bright giants
III	Normal giants
IV	Subgiants
V	Main-sequence (dwarf) stars
VI, sd	Subdwarfs
D	White dwarfs