Radiative Processes in Flares I: Bremsstrahlung

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1) Magnetic reconnection and energy release
2) Particle acceleration and heating
3) Chromospheric evaporation and loop heating

Previous lectures

Following lectures: How to diagnose the accelerated particles?
- What?
- Where?
- When? How?
Outline

• Introduction

• Radiation from energetic particles
  • Bremsstrahlung → this lecture
  • Gyrosynchrotron
  • Other radiative processes (time permitting)
    • Coherent emission
    • Inverse Compton
    • Nuclear processes

• Suggested reading: Ch. 5 of Tandberg-Hanssen & Emslie
Thermal and non-thermal radiation

• Refer to the distribution function of source particles $f(E)$ (# of particles per unit energy per unit volume)

• Radiation from a Maxwellian particle distribution is referred to as thermal radiation

• Radiation from a non-Maxwellian particle distribution is referred to as nonthermal radiation
  • In flare physics, the nonthermal population we consider usually has much larger energies than that of the thermal “background”

• Example of a nonthermal distribution function:

  $$f(E)dE = Cn_e E^{-\delta} dE,$$

  where $C$ is the normalization factor to make $\int_0^\infty f(E) dE = n_e \rightarrow \text{“power law”}$

*See Lecture 17 (by Prof. Longcope) for details on the distribution function
Radiation from an accelerated charge

Larmor formula: \[ \frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} a^2 \sin^2 \theta \quad P = \frac{2q^2}{3c^3} a^2 \]

Relativistic Larmor formula: \[
\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} \left( a_\perp^2 + \gamma^2 a_\parallel^2 \right) \frac{\sin^2 \theta}{(1 - \beta \cos \theta)^4}
\]
\[ P = \frac{2q^2}{3c^3} \gamma^4 \left( a_\perp^2 + \gamma^2 a_\parallel^2 \right) \]

Radio and HXR/gamma-ray emission in flares:
- Acceleration experienced in the Coulomb field: bremsstrahlung
- Acceleration experienced in a magnetic field: gyromagnetic radiation
Electron-ion bremsstrahlung

- e-i bremsstrahlung is relevant to quiet Sun, flares, and CMEs
Bremsstrahlung

- Each electron-ion interaction generates a single pulse of radiation
Power spectrum of a single interaction

Single pulse duration $\tau \sim b/\nu$

Emitted energy

$$W_\nu \approx \frac{\pi^2}{2} \frac{Z^2 e^6}{c^3 m_e^2} \left( \frac{1}{b^2 \nu^2} \right)$$
A note on the impact parameter

- **Maximum value** $b_{max}$
  - Debye Length $\lambda_D = \left(\frac{kT}{4\pi n_e e^2}\right)^{1/2} = v_{th}/\omega_{pe}$, Where $v_{th}$ is the thermal speed and $\omega_{pe} = \sqrt{\frac{4\pi n_e e^2}{m_e}}$ is the plasma frequency
  - Why this length scale sets $b_{max}$?
  - $b_{max} \approx \frac{v}{\omega}$, where $\omega$ is the observing frequency

- **Minimum value** $b_{min}$
  - $b_{min} \approx \frac{Ze^2}{m_e v^2}$, given by maximum possible momentum change of the electron $\Delta p_e = 2p_{e0}$
  - $b_{min} = \frac{\hbar}{m_e v}$, from uncertainty principle
Bremsstrahlung emissivity

- # of electrons passing by any ion within \((b, b + db)\) and \((\nu, \nu + d\nu)\)

\[
n_e \ (2\pi b \ db) \ \nu f (\nu) \ d\nu
\]

- # of collisions per unit volume per unit time

\[
\dot{n}_c (\nu, b) = (2\pi b) \nu f (\nu) \ n_e n_i
\]

- Spectral power emitted at \(\nu\)

\[
4\pi j_\nu = \int_{b=0}^{\infty} \int_{v=0}^{\infty} W_\nu (\nu, b) \ \dot{n}_c (\nu, b) \ d\nu \ db.
\]

\[
4\pi j_\nu = \frac{\pi^3 Z^2 e^6 n_e n_i}{c^3 m_e^2} \int_{v=0}^{\infty} \frac{f (\nu)}{v} d\nu \int_{b_{\text{min}}}^{b_{\text{max}}} db b
\]
Thermal bremsstrahlung

- Distribution function of the electron $f(v)$ is Maxwellian

\[
f(v) = \frac{4v^2}{\sqrt{\pi}} \left( \frac{m_e}{2kT} \right)^{3/2} \exp \left( -\frac{m_e v^2}{2kT} \right).
\]

Emission coefficient:

\[
j_{\nu} = \frac{\pi^2 Z^2 e^6 n_e n_i}{4c^3 m_e^2} \left( \frac{2m_e}{\pi kT} \right)^{1/2} \ln \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right).
\]
Thermal bremsstrahlung

• Absorption coefficient:

\[ \kappa = \frac{j_\nu}{B_\nu(T)} \]

• Let’s first consider radio wavelengths. In the Rayleigh-Jeans regime \( B_\nu(T) = \frac{2kT\nu^2}{c^2} \), so

\[
\kappa = \frac{1}{\nu^2 T^{3/2}} \left[ \frac{Z^2 e^6}{c} \frac{n_e n_i}{\sqrt{2\pi (m_e k)^3}} \right] \frac{\pi^2}{4} \ln \left( \frac{b_{max}}{b_{min}} \right).
\]

\[ \kappa(\nu) \propto \nu^{-2.1} \]
Thermal bremsstrahlung opacity

• Considering a spherical cow...

\[ \tau = - \int_{\text{los}} \kappa \, ds \propto \int \frac{n_e n_i}{\nu^2 T^{3/2}} \, ds \approx \int \frac{n_e^2}{\nu^2 T^{3/2}} \, ds. \]

• At low frequencies, \( \tau \gg 1 \), optically thick:

\[ S \approx B_\nu(T) = \frac{2kT \nu^2}{c^2} \propto \nu^2 \]

• At high frequencies, \( \tau \ll 1 \), optically thin:

\[ S \propto \frac{2kT \nu^2}{c^2} \tau(\nu) \propto \nu^{-0.1} \]
Consider a spherical cow….

At frequencies low enough that \( T_{\text{HII}} \) becomes opaque, its spectrum approaches that of a blackbody with brightness temperature approaching the electron temperature \( T_e \), and its flux density obeys the Rayleigh–Jeans approximation.

On a log-log plot, the overall spectrum of a uniform H\(_{\text{II}}\) region looks like Figure 4.8, with the spectral break corresponding to the frequency at which the optical depth, and at much higher frequencies the spectral slope.

\[ \text{slope} \approx -0.1 \]

\[ \text{slope} \leq 2 \]

\[ \tau = 1 \]
Emission measure

- Optical depth is essential in thermal bremsstrahlung

\[ \tau = - \int_{\text{los}} \kappa \, ds \propto \int \frac{n_e n_i}{\nu^{2.1} T^{3/2}} \, ds \approx \int \frac{n_e^2}{\nu^{2.1} T^{3/2}} \, ds. \]

- Assuming constant \( T \)

\[ \tau \approx \nu^{-2.1} T^{-1.5} (n_e^2 L) \]

Column emission measure \( EM_c \)
Recap from Lecture 7: Radiative Transfer Equation

• Brightness temperature

\[ I_{\nu} = B_{\nu}(T_B) = \frac{2\nu^2}{c^2} kT_B \]

• Effective temperature

\[ S_{\nu} = \frac{2\nu^2}{c^2} kT_{eff} \]

• Using our definitions of brightness temperature and effective temperature, the transfer equation can be rewritten

\[ \frac{dT_B}{d\tau_{\nu}} = -T_B + T_{eff} \]

• Optically thick source, \( \tau_{\nu} \gg 1, T_B \approx T_{eff} \)
• Optically thin source, \( \tau_{\nu} \ll 1, T_B \approx \tau_{\nu} T_{eff} \)
$T_b$ spectrum

- **Optically thin regime**

$$T_b = T(1 - e^{-\tau}) \approx T\tau$$

$$\approx \nu^{-2.1} T^{-0.5} EM_c$$

- **Optically thick**

$$T_b = T(1 - e^{-\tau}) \approx T$$
Bremsstrahlung for X-rays

Single pulse duration $\tau \sim b/\nu$

- Higher electron energy $\rightarrow$ larger $\nu$
- Closer encounters $\rightarrow$ smaller $b$
Introducing the Cross Section

- The **cross section** $\sigma_B$ is defined as if the radiation all comes from the impact within an area around a target ion.

- # of electrons that encounter a single target in $dt$ (assume all $e$ have the same speed and emit a photon at the same wavelength):
  $$n_e \sigma_B v dt$$

- # of photons produced per unit volume per unit time:
  $$n_e n_i \sigma_B v$$

- Photon flux at Earth ($\text{cm}^{-2} \text{s}^{-1}$ per unit energy) if the incident electron population remains roughly unchanged, or a "thin target" scenario:
  $$n_e n_i \sigma_B v V_s / (4\pi R^2) \quad R = 1 \text{ AU}$$

\[
\approx \frac{\sigma_B v S_s \int n_e n_i dl}{4\pi R^2} \\
\approx \sigma_B v S_s / (4\pi R^2) EM_c
\]
Differential Cross Section

In fact, $\sigma_B$ depends on:

- Incident electron energy $E$
- Outgoing photon energy $\epsilon$,
- Outgoing photon direction $\Omega$

We need a **differential cross section**: $d^2\sigma_B/dEd\Omega$, written as $\sigma_B(\epsilon, E, \Omega)$
Bremsstrahlung cross section

• Bremsstrahlung from weak interactions

\[
4\pi j_L = \frac{\pi^3 Z^2 e^6 n_e n_i}{c^3 m_e^2} \int_{v=0}^{\infty} \frac{f(v)}{v} dv \int_{b_{\text{min}}}^{b_{\text{max}}} \frac{db}{b}
\]

• For close encounters, \( \sigma_B(\epsilon, E, \Omega) \) is much more complicated (quantum physics). In the non-relativistic case, a direction-integrated cross section

\[
\sigma_{\text{NRBH}}(\epsilon, E) = \frac{\sigma_0 Z^2}{\epsilon E} \ln \frac{1 + (1 - \epsilon/E)^{1/2}}{1 - (1 - \epsilon/E)^{1/2}} \quad \text{cm}^2 \text{ keV}^{-1}
\]

where \( \sigma_0 = 7.90 \times 10^{-25} \) cm\(^2\) keV \( (\sigma_{\text{NRBH}} = 0 \text{ for } \epsilon > E) \)

Known as the Bethe-Heitler cross section

*See Koch & Motz 1959 for full relativistic, angle and polarization dependent cross section
Thin target bremsstrahlung

- With a differential cross section $\sigma_B(\varepsilon, E)$
- Taking into account electron distribution $f(E)$
- The directional integrated **thin-target bremsstrahlung flux** $F(\varepsilon)$ (photons cm$^{-2}$ s$^{-1}$ per unit energy) becomes

$$F(\varepsilon) = \frac{S_s N_i}{4\pi R^2} \int_{E=\varepsilon}^{\infty} f(E)\nu(E)\sigma_B(\varepsilon, E)dE$$

Where $N_i = \int n_i dl$ is the column density of the target
Thick target bremsstrahlung

• Incident electrons are completely stopped, or thermalized in the source → requires high density.
  ❖ Usually occurs when energetic electrons precipitating onto the chromosphere

• Much quicker energy loss from electrons → lots of X-ray photons emitted → Produces intense X-ray emission

• Usually dominates the hard X-ray (~10 keV – 300 keV) spectrum
Thick target bremsstrahlung

• Electrons change their energy in time (quickly)

\[ F(\epsilon) = \frac{S_s N_i}{4\pi R^2} \int_{E_0=\epsilon}^{\infty} f(E) \nu(E) \sigma_B(\epsilon, E) dE \]

Time and space dependent

• Instead we need to have

\[ F(\epsilon) = \frac{S_s}{4\pi R^2} \int_{E_0=\epsilon}^{\infty} f(E_0) \nu(E_0) m(\epsilon, E_0) dE_0 \]

where

\[ m(\epsilon, E_0) = \int_{t_1(E=E_0)}^{t_2(E=\epsilon)} n_i(l(t)) \sigma_B(\epsilon, E(t)) \nu(E(t)) dt \]

is the number of photons at energy \( \epsilon \) emitted per unit energy by an electron of initial energy \( E_0 \)
Thick target bremsstrahlung

• We need something to describe $E(t)$
• Q: what is the main mechanism for electron energy loss?
• Energy loss mainly due to e-e Coulomb collisions. We need another cross section to describe $dE/dt$

→ the Rutherford cross section:

$$\sigma_e = \frac{C}{E^2} \approx 10^{-17} \text{ cm}^2 \times \left(\frac{E}{\text{keV}}\right)^{-2}, \text{ where } C = 2\pi e^4 \ln \Lambda$$

So

$$\frac{dE}{dt} = -\sigma_e(E) n_i v(E) E$$
Thick target bremsstrahlung

- Photon flux

\[ F(\epsilon) = \frac{S_s}{4\pi R^2 C} \int_{E_0=\epsilon}^{\infty} f(E_0)\nu(E_0) \left( \int_{\epsilon}^{E_0} E\sigma_B(\epsilon, E)dE \right) dE_0 \]

- Comparing to the thin-target case

\[ F(\epsilon) = \frac{S_s N_i}{4\pi R^2} \int_{E=\epsilon}^{\infty} f(E)\nu(E)\sigma_B(\epsilon, E)dE \]

- Effective column density

\[ N_{eff}(\epsilon, E_0) = \frac{1}{C\sigma_B(\epsilon, E_0)} \int_{\epsilon}^{E_0} E\sigma_B(\epsilon, E)dE \]

\[ \propto E^2 \]

Similar to the stopping column \( N_c \)!

\[ N(s) = \int n_e(s') ds' \quad [\text{cm}^{-2}] \]

Stopping column: \( N \sigma_e = 1 \)

\[ N_c = \frac{E_c^2}{6\pi e^4 \Lambda} = 1.4 \times 10^{17} \text{ cm}^{-2} E_{c,\text{keV}}^2 \]

Stop cut-off

From Lecture 10 by Prof. Longcope
Thin target vs. Thick target

• The effective column depth $N_{\text{eff}}(\epsilon, E_0)$ is independent of the target

• $N_i \ll N_{\text{eff}}(\epsilon, E_0)$: incident electron distribution is nearly unchanged $\rightarrow$ thin target

• $N_i \geq N_{\text{eff}}(\epsilon, E_0)$: substantial change in incident electron distribution $\rightarrow$ we must use the thick target expression
From X-ray spectrum to electron distribution

• $F(\epsilon)$ is what we observe (after taking out instrument response)

Thin target

$$F(\epsilon) = \frac{S_S N_i}{4\pi R^2} \int_{E=\epsilon}^{\infty} f(E) \nu(E) \sigma_B(\epsilon, E) dE$$

Thick target

$$F(\epsilon) = \frac{S_S}{4\pi R^2 C} \int_{E_0=\epsilon}^{\infty} f(E_0) \nu(E_0) \left( \int_{\epsilon}^{E_0} E \sigma_B(\epsilon, E) dE \right) dE_0$$

• Obtaining $f(E)$ becomes an inversion problem

• Many approaches (Brown 1971 and after), but difficult to obtain an accurate $f(E)$ due to the “smoothing” effect of the integral (e.g., Craig & Brown 1985)
If we pretend to know the form of $f(E)$...

- **Thermal:**
  
  Maxwellian: $f(E) = \frac{2n_e}{\pi^{1/2}(KT)^{3/2}}E^{1/2} \exp(-E/KT)$

- **Nonthermal:**
  
  Power law: $f(E) = Cn_e E^{-\delta}$

  Kappa: $f(E) \propto \left[1 + \frac{E}{kT(\kappa-3/2)}\right]^{-(\kappa+1)}$
Thermal bremsstrahlung

• Resulted from $f(E)$ with a Maxwellian distribution
  • Radio thermal bremsstrahlung do not require high speed electrons
  • X-ray thermal bremsstrahlung does require high speed electrons $E > \epsilon$. For 3 keV X-ray, $T_e \sim 3.5 \times 10^7 K$

→ from flaring loops

Question:
• Why is thermal bremsstrahlung not so important in optical and UV?

From Krucker & Battaglia 2014
Nonthermal thin target bremsstrahlung

- Assume electron distribution is a power law:
  \[ f(E) = C n_e E^{-\delta} \]
- Plug in \( F(\epsilon) \) for thin target, we have
  \[
  F(\epsilon) = 2\beta K \epsilon^{-(\delta+1/2)} \int_1^\infty u^{-(\delta+1/2)} \log \left( \sqrt{u} + \sqrt{1-u} \right) du
  \]
  \[ F(\epsilon) \propto \epsilon^{-(\delta+1/2)} \]
  \[ u = E/\epsilon \]
- \( F(\epsilon) \) usually have a power-law shape in HXRs:
  \[ F(\epsilon) = K \epsilon^{-\gamma} \]
  If thin-target, the electron energy spectrum is
  \[ f(E) \propto E^{-(\gamma-1/2)} \]
  * See J. Brown 1971 for details
Nonthermal thick target bremsstrahlung

- Assume electron distribution is a power law:
  \[ f(E) = C n_e E^{-\delta} \]
- Plug in \( F(\epsilon) \) for thick target, we have
  \[ F(\epsilon) \propto \epsilon^{-(\delta-1)} \]
  Much flatter than thin-target!
- Inverting from a power-law photon spectrum
  \[ f(E) \propto E^{-(\gamma+3/2)} \]
- Inferred spectral index is **steeper by 2** for thick target than thin target
X-ray emission in flares

In solar corona:
- low density $\rightarrow$ very few collisions
- energy loss small ($dE \ll E$)
- faint X-ray emission

$\text{n} \sim 10^8 - 10^{10} \text{ cm}^{-3}$

Below transition region:
- high density $\rightarrow$ many collisions
- energy loss very fast
- strong X-ray emission

$n > 10^{12} \text{ cm}^{-3}$

acceleration site

Thick target

Thermal

Thin target

Thick target

acceleration site

intense HXR emission

THICK target

Thin target

intense HXR emission

Thin target
Other bremsstrahlung contributions

- At higher (relativistic) energies, corrections for the e-i cross section must be included. Moreover, the radiation pattern becomes highly beamed.

- Additional contributions need to be included
  - e-e bremsstrahlung $\Rightarrow$ important at $\epsilon > 300$ keV with a flatter spectrum (Haug 1975; Kontar 2007)
  - e$^+$/e bremsstrahlung (Haug 1985)
  - i-e bremsstrahlung is usually insignificant (Emslie & Brown 1985; Haug 2003)
Summary

• Bremsstrahlung emission is one of the most important diagnostics for energetic electrons in flares
• Thermal bremsstrahlung: radio, X-ray
• Nonthermal bremsstrahlung: X-ray
  • Thin target $\rightarrow$ corona
  • Thick target $\rightarrow$ chromosphere (sometimes corona)
• To obtain $f(E)$, we need
  • Observation of X-ray spectrum $F(\epsilon)$ with high resolution
  • Application of the correct emission mechanism(s)
  • Appropriate inversion