Hale COLLAGE 2017 Lecture 21

Radiative processes from energetic particles II: Gyromagnetic radiation

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Previous lectures

1) Magnetic reconnection and energy release
2) Particle acceleration and heating
3) Chromospheric evaporation, loop heating and cooling

Following lectures:
How to diagnose the accelerated particles and the environment?
• What?
• Where?
• When? → How?
Outline

• Radiation from energetic particles
  • Bremsstrahlung → Previous lecture
  • Gyromagnetic radiation (“magnetobremsstrahlung”) → This lecture
• Other radiative processes → Briefly in the next lecture
  • Coherent radiation, inverse Compton, nuclear processes

• Suggested reading:
  • Synchrotron radiation: Chapter 5 of “Essential Radio Astronomy” by Condon & Ransom 2016
  • Gyroresonance radiation: Chapter 5 of Gary & Keller 2004
  • Gyrosynchrotron radiation: Dulk & Marsh 1982

• Next two lectures: Diagnosing flare energetic particles using radio and hard X-ray imaging spectroscopy
Radiation from an accelerated charge

Larmor formula:
\[
\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} a^2 \sin^2 \theta \quad P = \frac{2q^2}{3c^3} a^2
\]

Relativistic Larmor formula:
\[
\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} \frac{(a_\perp^2 + \gamma^2 a_\parallel^2)}{(1 - \beta \cos \theta)^4} \sin^2 \theta
\]
\[
P = \frac{2q^2}{3c^3} \gamma^4 (a_\perp^2 + \gamma^2 a_\parallel^2)
\]

Radio and HXR/gamma-ray emission in flares:
• Acceleration experienced in the Coulomb field: bremsstrahlung
• Acceleration experienced in a magnetic field: gyromagnetic radiation
Gyromagnetic radiation

• Gyromagnetic radiation (sometimes called “gyroemission”) is due to the acceleration experienced by an electron as it gyrates in a B field due to the Lorentz force.

• Acceleration is perpendicular to $v_e$
Gyroemission from a single electron

- Let’s start from Larmor’s formula:

\[
\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} a^2 \sin^2 \theta \quad P = \frac{2q^2}{3c^3} a^2
\]

- Perpendicular acceleration: \( a_\perp = \omega_{ce} v_\perp \), where \( \omega_{ce} \) is the (angular) electron gyrofrequency

\[
\omega_{ce} = 2\pi v_{ce} = \frac{eB}{m_e c} \approx 2\pi \cdot 2.8B \text{ MHz}
\]

- (Direction integrated) Larmor’s equation becomes:

\[
P = \frac{2e^2}{3c^3} \omega_{ce}^2 v_\perp^2
\]

- Relativistic case:

\[
P = \frac{2e^2}{3c^3} \gamma^4 \omega_B^2 v_\perp^2, \text{ with } \omega_B = \frac{eB}{\gamma m_e c} = \frac{\omega_{ce}}{\gamma}
\]
Radiation pattern: non-relativistic

• Larmor’s Equation

\[ \frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3} a^2 \sin^2 \theta \]

Dipole pattern

Null at \( \theta = 0 \)

Observer
Radiation pattern: relativistic

- Relativistic case ($\gamma \gg 1$)
  - In the rest frame of the electron
    
    $$\frac{dP'}{d\Omega'} = \frac{q^2}{4\pi c^3} a^2 \sin^2 \theta'$$

- In the observer’s frame, radiation pattern found from Lorentz transform from the electron rest frame
  
  Null occurs at $\theta = \pm \arccos(1/\gamma)$

Strongly beamed forward along the direction of the electron!
Relativistic gyroemission: sharply pulsed radiation

\[ \Delta t_p \propto \frac{1}{\gamma^3 \omega_B} = \frac{1}{\gamma^2 \omega_{ce}} \]

\[ \Delta t_p = t \text{ (end of pulse)} - t \text{ (start of pulse)} \]

\[ = \frac{\Delta x}{v} + \frac{x - \Delta x}{c} - \frac{x}{c} = \frac{\Delta x}{v} \left( 1 - \frac{v}{c} \right) \ll \frac{\Delta x}{v} = \Delta t \]
Power spectrum \( P(\nu) \)

- For a nonrelativistic electron, radiation field \( E(t) \) is a sinusoid with frequency \( \omega_{ce} \)
- Power spectrum is a single tone at the electron gyrofrequency
Power spectrum $P(\nu)$

- As the electron speed picks up, mild beaming effect takes place, $E(t)$ is non-sinusoidal
- Low harmonics of electron gyrofrequency show up in the power spectrum

Can you identify two effects in the $E(t)$ plot?
Power spectrum $P(\nu)$

- When the electron is relativistic $E(t)$ is highly pulsed
- The power spectrum shows contribution from many harmonics
Types of gyromagnetic radiation

- Gyromagnetic radiation behaves very differently with different electron distributions
- A precise general expression valid for all electron energies is *not* available. Instead, we use approximate expressions for various electron energy regimes

- Non-relativistic or thermal ($\gamma - 1 \ll 1$):
  Gyroresonance or cyclotron radiation
- Mildly relativistic ($\gamma - 1 \sim 1 - 5$):
  Gyrosynchrotron radiation
- Ultra-relativistic ($\gamma - 1 \gg 1$):
  Synchrotron radiation
Thermal gyroresonance radiation

• At a given B, thermal gyroresonance radiation is essentially a “spectral line” centered at $s\nu_{ce}$, where $s = 1, 2, 3 \ldots$ is the harmonic number

• Particularly relevant above active regions at microwave frequencies – Why?

• Spectral width of a given resonance line

  $$\Delta \nu / s\nu_{ce} \approx \sqrt{\frac{k_B T}{m_e c^2}}$$

  Very narrow in the corona (~1/3000)

• High opacity only at these “resonance layers”
Thermal gyroresonance opacity

- Two different wave modes: ordinary (o mode) and extraordinary (x mode, gyrates with the same sense of rotation as an electron)

\[ \tau_{x,o}(s, \nu, \theta) = 0.0133 \frac{n_e L_B(\theta)}{\nu} \frac{s^2}{s!} \left( \frac{s^2 \sin^2 \theta}{2 \mu} \right)^{s-1} F_{x,o}(\theta) \]

Where \( F_{x,o}(\theta) \approx (1 - \sigma \cos \theta)^2 \) and \( \mu = m_e c^2 / k_B T \)

\( \sigma = -1 \) for x mode and 1 for o mode, \( L_B \) is the scale length of B

- Opacity for two different wave modes

Why?

Which mode has a larger opacity? Why?
Thermal gyroresonance opacity

Figure 5.1. The (integrated) optical depth of the $s = 2$, 3, 4 gyroresonance layers at 5 GHz (left, middle and right panels, respectively) as a function of the angle $\mu$ between the line of sight and the magnetic field direction. The temperature in the source is $3 \times 10^{6}$ K, and the magnetic scale height $L_B$ is $10^9$ cm. In each panel the solid line is the optical depth of the layer in the $x$ mode, and the dashed line is the optical depth in the $o$ mode. The dotted lines show the optical depth obtained using the circularly-polarized mode approximation (2). The density used for this calculation was decreased as $s$ increases to simulate the decrease of $n_e$ with height: the values are shown in each panel.

- The opacity drops sharply towards small $\mu$ in both modes. At angles very close to $90 \pm \theta$, the $o$ mode opacity dips sharply since it must be a factor of $\theta$ smaller than the $x$ mode opacity exactly at $\mu = 90^\circ$ (e.g., Bornatici et al. 1983, Robinson 1991). By (1), the opacity is zero at $\mu = 0^\circ$ for $s > 1$.

- For each increase of $s$ by 1, the opacity in a given mode at a given angle drops by slightly more than 2 orders of magnitude. This is largely due to the $\theta^s$ dependence of (1). The importance of this large change in opacity from one layer to the next is that a given harmonic layer is likely to be either optically thick over a wide range of angles $\mu$, or else optically thin everywhere.

Typically, optically thick at $S=2$ (o-mode) and 3 (x-mode)  

From White 2004
Gyroresonance emission of a sunspot

Figure 5.2. Plots of the gyroresonance layers of a dipole sunspot model (upper panel) and the predicted brightness temperatures resulting from an observation of such a spot (lower panel), viewed nearly vertically (actually $\pm 10^\circ$ off vertical). In the upper panel the thin solid lines are magnetic lines of force and the dotted lines are the $s=1, 2, 3, 4$ gyroresonance layers, with $s=4$ the highest and $s=1$ the lowest layer. Where the gyroresonance layers are optically thick (i.e., $\mu \approx 1$) in the $o$ mode, they have been overplotted with a thick solid line. Except in the $s=1$ layer where the $x$ mode does not propagate, a layer which is optically-thick in the $o$ mode is also thick in the $x$ mode. If a gyroresonance layer is optically thick in the $x$ mode but not in the $o$ mode, it is overplotted with a thick dashed line. In the lower panel, the $x$-mode brightness temperature is shown by a solid line and the $o$ mode brightness temperature by a dashed line. The frequency is 5.0 GHz, the dipole is buried at a depth of $1.2 \times 10^9$ cm, and the maximum field strength at the surface is 2500 G. In the model temperature increases with radial height from $1.0 \times 10^6$ K at the base of the corona (zero height in this case) to $3.0 \times 10^6$ K at about 15000 km.

Temperature of order $10^5$ K, which provides the outer boundary of the radio source.

From White 2004
Actual observation from the VLA

Q: Which polarization is the x-mode?

Made by B. Chen for AR 12158 (unpublished)
Nonthermal synchrotron radiation

- Ultra-relativistic \((\gamma - 1 \gg 1)\)
- From a single electron, adjacent "spikes" are separated in frequency by only \(\Delta \nu = \frac{\nu_{ce}}{\gamma}\)
- Fluctuations in electron energy, B strength, or pitch angle cause "broadening" of the spikes
- Spectrum is virtually continuous
Synchrotron spectrum $P(\nu)$ from a single electron

Most of the energy is emitted at $\nu \approx \nu_c$, where

$$\nu_c = \frac{3}{2} \gamma^2 \nu_{ce} \sin \alpha$$

is the critical frequency ($\alpha$ is the pitch angle)
Synchrotron spectrum of an optically thin source

• One electron of electron $E$ nearly emits all energy at a single frequency $\nu \approx \gamma^2 \nu_{ce}$

• Optically thin source $\rightarrow$ to get emissivity $j_\nu$ in $(\nu, \nu + d\nu)$, just add $P(\nu) = -dE/dt$ up from all electrons within $(E, E + dE)$:

$$j_\nu d\nu = -\frac{dE}{dt} f(E) dE$$

• Assume a power law electron energy distribution:

$$f(E) = C n_e E^{-\delta}$$

• The emissivity $j_\nu \propto \nu^{-(\delta - 1)/2}$
Synchrotron spectrum: optically thick regime

- Synchrotron brightness cannot be arbitrarily high → self-absorption becomes important at low frequencies
- The spectrum has a power law of slope $5/2$ for optically thick source
Gyrosynchrotron radiation

• From mildly relativistic electrons (~1 to several MeV)

• Expressions for the emission and absorption coefficient are much more complicated than the nonrelativistic (thermal gyroresonance) and ultra-relativistic (synchrotron) case

  “exact”           approximate
  Ramaty 1969       Petrosian 1981
  Klein 1987
Spectrum is also more complicated

Klein (1987)
Variation with $B$

$\nu_{pk} \sim B^{3/4}$

$B=1000 \text{ G}$
$B=500 \text{ G}$
$B=200 \text{ G}$
$B=100 \text{ G}$

Arbitrary

Frequency (GHz)
$n_{\text{rel}} = 1 \times 10^7 \text{ cm}^{-3}$

$n_{\text{rel}} = 5 \times 10^6 \text{ cm}^{-3}$

$n_{\text{rel}} = 2 \times 10^6 \text{ cm}^{-3}$

$n_{\text{rel}} = 1 \times 10^6 \text{ cm}^{-3}$

$\nu_{\text{pk}} \sim n_{\text{rel}}^{1/4}$
\[ q = 20 \]
\[ q = 40 \]
\[ q = 60 \]
\[ q = 80 \]
\[ n_{pk} \sim \theta^{1/2} \]

Variation with LOS

\( \theta = 80^\circ \)
\( \theta = 60^\circ \)
\( \theta = 40^\circ \)
\( \theta = 20^\circ \)
(Gyro)synchrotron spectrum

Schematic diagram from Dulk & Marsh 1982
Gyrosynchrotron in flares

Flare observed by SOHO, GOES, and Nobeyama Radioheliograph at 17 and 34 GHz
- Microwave: gyrosynchotron
- EUV/SXR: hot thermal plasma
A schematic model of a flare loop
Summary

- **Gyromagnetic radiation** results from electrons accelerated in the magnetic field.
- Three different regimes based on energy of the source electrons: gyroresonance, gyrosynchrotron, and synchrotron.
- **Gyroresonance** can be used to diagnose B fields in active regions.
- **Gyrosynchrotron** can be used to probe flare-accelerated electrons and diagnose B field in flare loops.
- **Synchrotron** is more relevant to cosmic sources, but still possible on the Sun (e.g., the mysterious sub-THz flare component).