INTRODUCTION TO RADIATIVE-HYDRODYNAMIC MODELING

Adam Kowalski (CU/NSO/LASP)

With Contributions from Joel Allred (NASA/GSFC) and Mats Carlsson (Univ. of Oslo)



RADIATIVE-HYDRODYNAMIC (RHD) MODELING

- Suggested reading (on Prof Longcope's website):
 - Allred et al. 2015 ApJ
 - Carlsson 1998

RADIATIVE-HYDRODYNAMIC (RHD) MODELING

- Goals for today's lecture:
 - Motivate modeling the chromosphere
 - Introduce the essentials of radiative-hydrodynamics
 - On Monday: will do an IDL-based lab with RADYN (Carlsson & Stein 1997 ApJ) model output
 - analyze the physics of chromospheric evaporation and condensation resulting from electron beam heating

OBSERVATIONAL MOTIVATION FOR MODELING THE RADIATIVE TRANSFER

- White-light continuum radiation
 - chromospheric? photospheric? A combination?
- T ~ 9000 10,000 K blackbody(-like) in optical and NUV





Hawley & Fisher (1992)

OBSERVATIONAL MOTIVATION FOR MODELING THE DYNAMICS

Redshifts in Hα in solar flares: signatures of chromospheric condensation (heated, downflowing compressions)





Ichimoto & Kurokawa 1984: 40-140 km/s redshifts in Ha

...Sometimes blueshifts observed too

OBSERVATIONAL MOTIVATION FOR MODELING THE DYNAMICS

Redshifts in Mg II (and other singly ionized chromospheric lines from IRIS data) in solar flares



INGREDIENTS FOR RADIATIVE-HYDRODYNAMIC FLARE MODELING

- ID model atmosphere with photosphere, chromosphere, transition region, and corona.
- Solve the radiative-transfer, rate, and charge conservation equations simultaneously with the equations of mass, momentum, and energy conservation.
- Flare heating prescription from solving the Fokker-Planck equation, as appropriate for partially ionized chromosphere.

1D PLANE-PARALLEL MODELING

- All motion confined to vertical (z-direction), so what are we assuming?
 - The magnetic pressure is much greater than the gas pressure
 - The center of a strong magnetic flux tube



***For this lecture z=0 is at top of atmosphere; for lab z=0 is at photosphere



TERMINOLOGY: LTE VS. NON-LTE

- LTE: local thermodynamic equilibrium
 - Ievel populations determined by temperature (collisions)
 - When collisional rates are high (density is high), can make use of Saha-Boltzmann
 - The ratio of emissivity to opacity is the Planck function
- non-LTE:
 - level populations affected by the radiation field (which can originate from somewhere else in the atmosphere)
 - scattering of radiation changes levels from their Saha-Boltzmann values

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INITIAL MODEL ATMOSPHERE (1D)

Adaptive grid (Dorfi & Drury 1987) to resolve transition region and steep pressure gradients / shocks during flare.



 $\frac{\partial \rho v}{\partial t} \\ \frac{\partial \rho e}{\partial t}$

JATIONS OF RHD

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial z} = 0, \quad (1) \text{ mass}$$
(b) Charge conservation
$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial z} = 0, \quad (1) \text{ mass}$$
(c) Charge conservation
$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial z} + \frac{\partial (p + q_v)}{\partial z} + \rho g - A_{\text{beam}} = 0, \quad (2) \text{ momentum}$$

$$\frac{e}{t} + \frac{\partial \rho ve}{\partial z} + (p + q_v) \frac{\partial v}{\partial z} + \frac{\partial \rho ve}{\partial z} + (p + q_v) \frac{\partial v}{\partial z} + \frac{\partial (F_c + F_r)}{\partial z} - Q_{\text{cor}} - Q_{\text{beam}} - Q_{\text{rc}} = 0, \quad (3) \text{ energy}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial n_i v}{\partial z} - \left(\sum_{j \neq i}^{N'} n_j P_{ji} - n_i \sum_{j \neq i}^{N'} P_{ij}\right) = 0, \quad (4) \text{ frate or level population}$$

$$\mu \frac{\partial I_{\nu \mu}}{\partial z} = \eta_{\nu \mu} - \chi_{\nu \mu} I_{\nu \mu} \text{ opacity} \quad (5) \text{ rodiate transfer.}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial z} = 0, \qquad (1)$$

$$\frac{\partial \rho v}{\partial t} + \frac{\partial \rho v^2}{\partial z} + \frac{\partial (p+q_v)}{\partial z} + \rho g - A_{beam} = 0, \qquad (2)$$

$$\frac{\partial \rho e}{\partial t} + \frac{\partial \rho v e}{\partial z} + (p+q_v) \frac{\partial v}{\partial z} \quad \text{erg}[s] \text{ cm}^3 \qquad \text{erg}[s] \quad \text{erg}[s$$

m .

"beam" = nonthermal

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial z} = 0, \qquad (1)$$

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$$\mu \frac{\partial I_{\nu\mu}}{\partial z} = \eta_{\nu\mu} - \chi_{\nu\mu}I_{\nu\mu} \qquad (5)$$
Solve for
$$I_{v,\mu} (Y_{v}) \dots$$

. ..

OPTICALLY THIN RADIATIVE LOSS FUNCTION





$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial z} = 0, \qquad (1)$$

$$\frac{\partial \rho v}{\partial t} + \frac{\partial \rho v^{2}}{\partial z} + \frac{\partial (p + q_{v})}{\partial z} + \rho g - A_{beam} = 0, \qquad (2)$$

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Solve for
$$I_{v} \mu (Y_{v}) \dots$$

EQUATION OF RADIATIVE TRANSFER AND PLANE-PARALLEL GEOMETRY

$$I_{\nu\mu}(\tau_{\nu}) = \begin{cases} \frac{1}{\mu} \int_{\tau_{\nu}}^{\infty} S_{\nu}(t) e^{-(t-\tau_{\nu})/\mu} dt & \text{if } \mu > 0\\ \frac{1}{-\mu} \int_{0}^{\tau_{\nu}} S_{\nu}(t) e^{-(\tau_{\nu}-t)/(-\mu)} dt & \text{if } \mu < 0 \end{cases}$$

$$I_{\mu,\nu}(2\nu=0) = \text{emergent intensity}$$

= $\int_{0}^{\infty} \int_{0}^{\infty} (t) e^{t/\mu} dt$

* NOT GIT) from Lecture II

THE CONTRIBUTION FUNCTION*

Emergent Intensity =
$$Iv_{j\mu}(\tau_{v=0})$$

= $\int_{\mu}^{\infty} S_{v}(\tau_{v}) e^{-\tau_{v}/\mu} d\tau$
 $S_{v} = \frac{\pi_{v}}{\chi_{v}}; \quad \tau_{v} = \int \chi_{v} dz$
 $\exists v_{j\mu}(\tau_{z=0}) = \int_{\pi}^{\infty} \frac{1}{\pi} \tau_{v} e^{-\int \lambda v dz} dz$
The contribution function to
the emergent intensity.
 $(I(z))$ tells you where the emergent intensity.
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THE CONTRIBUTION FUNCTION

- Integrating the contribution function over height gives emergent intensity
- Integrating (via Gaussian quadrature sum) the emergent intensity over µ gives the emergent radiative flux:

$$F_{\nu} = 2\pi \sum_{\mu} I_{\nu,\mu} \mu w_{\mu}$$

$$w_{\mu} = [0.118, 0.239, 0.284, 0.239, 0.118]$$

$$\mu = [0.0469, 0.231, 0.5, 0.769, 0.953]$$

THE CONTRIBUTION FUNCTION: WHY DO WE CARE?

- Compare emergent flux from a flare atmosphere to observations.
- Use contribution function to determine the atmospheric parameters.

OPTICALLY THICK VS. OPTICALLY THIN

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Q: What happens in the energy equation if one assumes that all losses are optically thin?

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OPTICALLY THICK RADIATIVE TRANSFER: OPACITY SOURCES

OPTICALLY THICK RADIATIVE TRANSFER: OPACITY SOURCES

free-free

bound-bound

bound-free

using the *actual* ion density. Then summing over all levels and processes we have the *non-LTE opacity*

$$\chi_{\nu} = \sum_{i} \sum_{j>i} \left[n_{i} - (g_{i}/g_{j})n_{j} \right] \alpha_{ij}(\nu) + \sum_{i} (n_{i} - n_{i}^{*}e^{-h\nu/kT})\alpha_{i\kappa}(\nu) + \sum_{\kappa} n_{e}n_{\kappa}\alpha_{\kappa\kappa}(\nu, T)(1 - e^{-h\nu/kT}) + n_{e}\sigma_{e}$$
(7-1)

where the four terms represent, respectively, the contributions of boundbound, bound-free, and free-free absorptions, and of electron scattering (other scattering terms—e.g., Rayleigh scattering—may also be added). To calculate the *spontaneous thermal emission* (non-LTE) we use the rates derived in equations (5-55) and (5-62) to write

$$\eta_{\nu} = (2h\nu^{3}/c^{2}) \left[\sum_{i} \sum_{j>i} n_{j}(g_{i}/g_{j})\alpha_{ij}(\nu) + \sum_{i} n_{i}^{*}\alpha_{i\kappa}(\nu)e^{-h\nu/kT} + \sum_{\kappa} n_{e}n_{\kappa}\alpha_{\kappa\kappa}(\nu, T)e^{-h\nu/kT} \right]$$

$$(7-2)$$

Mihalas 1978 (Stellar Atmospheres, 2nd Ed.)

*alpha is the respective cross section in cm² (see Rutten 2003 lecture notes)

OPTICALLY THICK RADIATIVE TRANSFER: CONTINUUM OPACITY

choices: photosphere of O star, outer atmosphere of O star, photosphere of Vega, photosphere of the Sun

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OPTICALLY THICK RADIATIVE TRANSFER: CONTINUUM OPACITY

For a flare atmosphere (open circles):

OPTICALLY THICK RADIATIVE TRANSFER: CONTINUUM OPACITY

"dissolved level continuum" opacity (see Tremblay & Bergeron 2009, ApJ) for hydrogen

OPTICALLY THICK RADIATIVE TRANSFER: EMISSIVITY

- erg / s / cm³ / s.r. / Angstrom
- bound-free for hydrogen (LTE):

$$\begin{split} \eta_{\lambda} &= \frac{6.48 \times 10^{-14}}{4\pi \lambda^2} \frac{n_e n_p}{T^{3/2} n^3} e^{\frac{1.58 \times 10^5}{n^2 T} - \frac{1.44 \times 10^8}{\lambda T}} \\ \eta_{\lambda} &\propto \frac{1}{\lambda^2} e^{\frac{-hc}{\lambda kT}} \end{split} \text{(for a given principal quantum number n and temperature T)} \end{split}$$

CONTINUUM FORMED AT T=10,000 K OVER LOW OR HIGH OPTICAL DEPTH?

- Photospheric-like or chromospheric-like?
- Measure the Balmer jump ratio = F_{3600} / F_{4170}
- Measure a blue-to-red continuum ratio = F₄₁₇₀ / F₆₀₀₀

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- Photospheric-like or chromospheric-like?
- Measure the Balmer jump ratio = F_{3600} / F_{4170}
- Measure a blue-to-red continuum ratio = F_{4170} / F_{6000}

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FLARE HEATING PRESCRIPTION IN RHD CODES

- Bright redshifts, bright white-light (optical) co-spatial and cotemporal with hard X-ray emission: nonthermal electron distributions
- See Prof. Chen's previous lecture on modeling hard X-ray emission to infer power-law index (δ), low energy cutoff (keV), and power in nonthermal electrons (erg / s).
- In every flare loop, one must specify a nonthermal energy flux (erg / s / cm²) injected at the top of the loop: e.g., F11, 3.5F11, ... F13?
- See Prof. Longcope's lecture on stochastic and shock acceleration mechanisms; radiative-hydro models do not model reconnection and initial particle acceleration/injection.

INPUT PARAMETERS TO RADYN

Quite highly collimated / "beamed"

THE FOKKER-PLANCK EQUATION

- additional reading:
 - McTiernan & Petrosian 1990, Allred et al. 2015
 - https://hesperia.gsfc.nasa.gov/hessi/flarecode/efluxdoc.pdf
 - ► $f(x, p, t) \rightarrow f(z, E, mu)$, units of [number/cm³/keV/steradian]
 - time-independent solution to the Fokker-Planck equation where df(z,E,mu)/dt = 0
 - re-solved when the atmosphere has changed state

PARTIALLY IONIZED CHROMOSPHERE

Energy loss rate due to elastic Coulomb collisions with ambient electrons and electrons bound to atoms.

A ratio of about three (Hawley & Fisher 1994, Mott & Massey 1965)
 *Electron beam heating [erg/s/cm³, left axis] here is at t=0s for E_c=25 keV, 5F11, δ=4.2

CALCULATING BEAM ENERGY DEPOSITION RATES

TERMS CAN BE TURNED ON OR OFF IN NUMERICAL F-P SOLVER

The F-P solver in RADYN written by Joel Allred (NASA/GSFC)

! bfield: (nz) Magnetic field strength (Gauss) at each z. ! thermE: (nz) Energy (eV) at which a particle is considered thermalized. Particles with energies below this will not contribute to the number flux. ! res: (nmu, nE, nz) If icntl(9).eq.1, then output the normalized residual after the solver has been run. Otherwise, not used. ! tol: When the average change between iterations is below tol, the solver is assumed to have converged. Typical value is 1e-3. ! theta: Must be between 0 to 1. Relative weight of the new solution. This can be used to damp changes in the iterative solver. $phi(n+1) = phi(n) * (1-theta) + new_phi * theta.$ Small values will slow changes and can be used to damp oscillations produced by the non-linear return current terms. If no oscillations occur, then large values will speed up convergence. A typical value is 0.5, but this can be lowered if the return current causes oscillations in the solution. ! icntl: (10) Parameters which control how the code is run. Setting the following switches to zero turns off the corresponding terms: icntl(1): Energy loss from collisions. icntl(2): Pitch angle diffusion from collisions. icntl(3): Energy loss and scattering due to synchrotron emission. icntl(4): Pitch angle scattering due to magnetic mirroring. icntl(5): Energy loss and scattering due to return currents. icntl(6): Top of loop boundary condition = 0: Inject flux given in ftop for positive mu. Negative mu is allowed to vary according to the solver. This is effectively an OPEN boundary for negative mu. = 1: Inject flux given in ftop for positive mu, and reflect back downward the flux for negative mu. This is a REFLECTING condition. Recommended for typical flare situations.

MOVIE OF ATMOSPHERIC EVOLUTION AND CONTRIBUTION FUNCTION

Simulation from Kowalski et al. 2017

SUMMARY

- For modeling radiation from the chromosphere, need to include optical depth (absorption, scattering).
- Many sources of opacity.
- Modeling continuum opacity and emergent intensity provides an important and powerful diagnostic of the optical depth at T~10,000 K in flares.
- On Monday: will do an IDL-based lab with RADYN model output.
 - analyze the physics of chromospheric evaporation and condensation that result from electron beam heating.
 - Will look at a contribution function!