Hwk 2 Solutions

Section 1.3 # 3, 4b, 8a, 11, 20 (a-d)

3. Prove: If A and B are sets and X is a set s.t. $A \cup X = B \cup X$ and $A \cap X = B \cap X$, then $A = B$.

Pf.) We will show that $A \subset B$ {the proof of $B \subset A$ is identical}

let $y \in A$ and suppose $y \notin X$

then $y \in A \cup X$

$\Rightarrow y \in B \cup X$ ↓ Since $A \cup X = B \cup X$

$\Rightarrow y \in B$ ↓ Since $y \notin X$

Suppose $y \in X$.

then $y \in A \cap X$

$\Rightarrow y \in B \cap X$ ↓ Since $A \cap X = B \cap X$

$\Rightarrow y \in B$

In either case, if $y \in A \Rightarrow y \in B$. Thus $A \subset B$

Similarly we can show $B \subset A$

$\Rightarrow A = B$

4.b Find the largest ε s.t. S contains an ε-nbhd of x_0 ②

$x_0 = 2/3 \quad S = [1/2, 3/2]$

Thus we need ε s.t. $2/3 - \varepsilon > 1/2$ and $2/3 + \varepsilon < 3/2$

Clearly $\varepsilon = 1/6$ is the largest such value satisfying both.

8a). Prove: The intersection of finitely many open sets is open

Pf/. Let $S_i \; i = 1, \ldots n$ be open sets and $S = \bigcap_{i=1}^{n} S_i$

We need to show that all points of S are interior points

let $x \in S$

$\Rightarrow x \in S_i \quad \forall i = 1, \ldots n$

Since each S_i is open, x is an interior point of each

$\Rightarrow \exists \, \varepsilon_i$ s.t. $B_{\varepsilon_i}(x) \subset S_i$ for each i

Thus if we let $\varepsilon = \min_{i=1,\ldots n} \varepsilon_i$, then $B_\varepsilon(x) \subset S_i$

$\Rightarrow B_\varepsilon(x) \subset \bigcap_{i=1}^{n} S_i$

$\Rightarrow B_\varepsilon(x) \subset S$

$\Rightarrow x$ is an interior point of S

$\Rightarrow S$ is open.

11. Find the set of limit points of S, ∂S, $\bar{S}$, isolated pts and exterior of S.

(a) $S = (-\infty, -2) \cup (2,3) \cup \{4\} \cup (7, \infty)$

limit pts $= (-\infty, -2] \cup [2,3] \cup [7, \infty)$

$\partial S = \{-2, 2, 3, 4, 7\}$

$\bar{S} = S \cup \partial S = (-\infty, -2] \cup [2,3] \cup \{4\} \cup [7, \infty)$

exterior $= (-2, 2) \cup (3,4) \cup (4,7)$.

(b) $S = \mathbb{Z}$.

limit points $= \phi$

$\partial S = \mathbb{Z}$

$\bar{S} = \mathbb{Z}$

exterior $= \bigcup_{n=-\infty}^{\infty} (n, n+1)$

(c) $S = \bigcup \{ (n, n+1) : n = \text{integer} \} = \mathbb{Z}^c$.

limit points $= \mathbb{R}$.

$\partial S = \mathbb{Z}$

$\bar{S} = \mathbb{R}$.

Exterior $= \phi$.

(d) $S = \{ x : x = \frac{1}{n} \ \ n = 1, 2, 3 \ldots \}$

limit points : $\{0\}$

$\partial S = S$

$\bar{S} = S \cup \{0\}$

exterior $= (-\infty, 0) \cup (1, \infty) \cup \{ (\frac{1}{n+1}, \frac{1}{n}) : n = 1, 2, 3 \ldots \}$

20\. Counter-examples

a) The isolated points form a closed set.

False: The example from 11(d) shows this

$S = \{x : x = 1/n, n = 1, 2, 3, \dots\}$ has only isolated

Points, but it does not contain 0 which is a

limit point.

b) Every open set contains at least two points

False: Let $S = \emptyset$.

c) If S_1 and S_2 are arbitrary sets, then $\partial(S_1 \cup S_2) = \partial S_1 \cup \partial S_2$

False: Let $S_1 = (-\infty, 2]$ $S_2 = [0, \infty) \Rightarrow S_1 \cup S_2 = \mathbb{R}$

$\partial(S_1 \cup S_2) = \emptyset$

$\partial S_1 = \{2\}$
$\Rightarrow \partial S_1 \cup \partial S_2 = \{0, 2\} \neq \emptyset$
$\partial S_2 = \{0\}$

d). If S_1 and S_2 are arbitrary sets, then $\partial(S_1 \cap S_2) = \partial S_1 \cap \partial S_2$

False: Same sets as part (c)

$S_1 \cap S_2 = [0, 2] \Rightarrow \partial(S_1 \cap S_2) = \{0, 2\}$

but $\partial S_1 \cap \partial S_2 = \emptyset$.