Math 481 Mideterm 1

Spring 2019

Prof. Bose

1. Prove the Principle of Nested Sets: If S_1, S_2, ... are closed nonempty subsets of R^n such that $S_1 \supset S_2 \supset \ldots \supset S_r \supset \ldots$ and $\lim_{r\to\infty} d(S_r) = 0$, then the intersection

$$I = \cap_{r=1}^{\infty} S_r$$

contains exactly one point.

2. Suppose $f : R^n \to R$. Define what it means for f to be differentiable at a point $\mathbf{X_0}$.

3. Suppose $f : R^n \to R$ and that f, f_x. f_y and f_{xy} exist on a neighborhood N of (x_0, y_0), and f_{xy} is continuous at that point. If $A(h,k) = f(x_0 + h, y_0 + k) - f(x_0 + h, y_0) - f(x_0, y_0 + k) + f(x_0, y_0)$ where h, k are sufficiently small (be specific in your answer), show that $A(h,k) = f_{xy}(\hat{x}, \hat{y})hk$ for a point $(\hat{x}, \hat{y})$ (again be specific where this point is located).

4. If $T : R^n \to R^m$, define what it means for T to be an affine transformation. Explain why a transformation $F : R^n \to R^m$ that is differentiable at $\mathbf{X_0}$ can be approximated by an affine transformation in a neighborhood of $\mathbf{X_0}$.

5. Find the largest subset of R^2 on which

$$f(x,y) = \frac{x^2 + y^2}{x - y}$$

is continuously differentiable. Justify your answer by quoting the appropriate theorem.

6. Prove that if $h(u,v) = f(u^2 + v^2)$ then $vh_u = uh_v$.

7. Use the chain rule to find $H'(U)$ where $H(U) = F(G(U))$ where

$$F(x,y) = \begin{bmatrix} x^2 - y^2 \\ \frac{y}{x} \end{bmatrix}, \qquad G(u,v) = \begin{bmatrix} v\cos u \\ v\sin u \end{bmatrix}$$