Math 111　　　Final Exam　　　May 7, 2012

Name: (Print)

Student ID:

Instructor's Name:

Signature*:

Your signature above affirms that this examination is completed in accordance with the NJIT Academic Integrity Code.

Problem	Score	Out of
1		16
2		12
3		32
4		12
5		12
6		20
7		16
8		12
9		20
10		28
11		20
Total		200

Read each problem carefully. Show your work clearly. No calculators.
(If you need more space, please use the back of the previous page and label clearly).

1. (16 points) Find the horizontal, vertical and slant asymptotes for the following functions:

    a) $f(x) = \dfrac{1 - 2x^2}{x - 1}$

    b) $f(x) = \dfrac{\sqrt{3x^2 + 7}}{x - \pi}$

2. (12 points) Find the equation of the line tangent to the curve $xe^{3y} - y^2 = 2$ at $(2, 0)$.

3. (32 points) Find $\dfrac{dy}{dx}$ for:

a) $y = x\sec x + x^2 \cos x$

b) $y = \displaystyle\int_{x^2}^{0} t^3 \sin t\, dt$

c) $y = \dfrac{\tan x}{1 - \sqrt{x}}$

d) $y = \sqrt[3]{\dfrac{e^{2x}\left(\tan^{-1} x\right)}{x^2 + 1}}$ use logarithmic differentiation

e) $y = \left(\sin^{-1}(x^3 + x)\right)^{3/2}$;

4. (12 points) Oil is spilling into the ocean from the hull of a damaged tanker creating a polluted circular region. How fast is the polluted area increasing when the radius of the circle is 60 feet and the radius is increasing at the rate of 0.5 ft/sec?

5. (12 points) Sketch a function, $f(x)$, about which the following is known: (One sketch is desired. The function satisfies ALL of the following conditions).

   a) $\left(-\sqrt{3}, 0\right)$, $\left(\sqrt{3}, 0\right)$, $(1, 2)$ and $(3, 6)$ lie on the curve $y = f(x)$.

   b) $x = 2$ and $y = x + 2$ are asymptotes of $y = f(x)$.

   c) $f(x)$ is continuous except at $x = 2$, where it does not exist;

   d) $f'(x) > 0$ on $(-\infty, 1)$ and $(3, \infty)$ and $f'(x) < 0$ on $(1, 2) \cup (2, 3)$.

   e) $f''(x) < 0$ on $(-\infty, 2)$ and $f''(x) > 0$ on $(2, \infty)$.
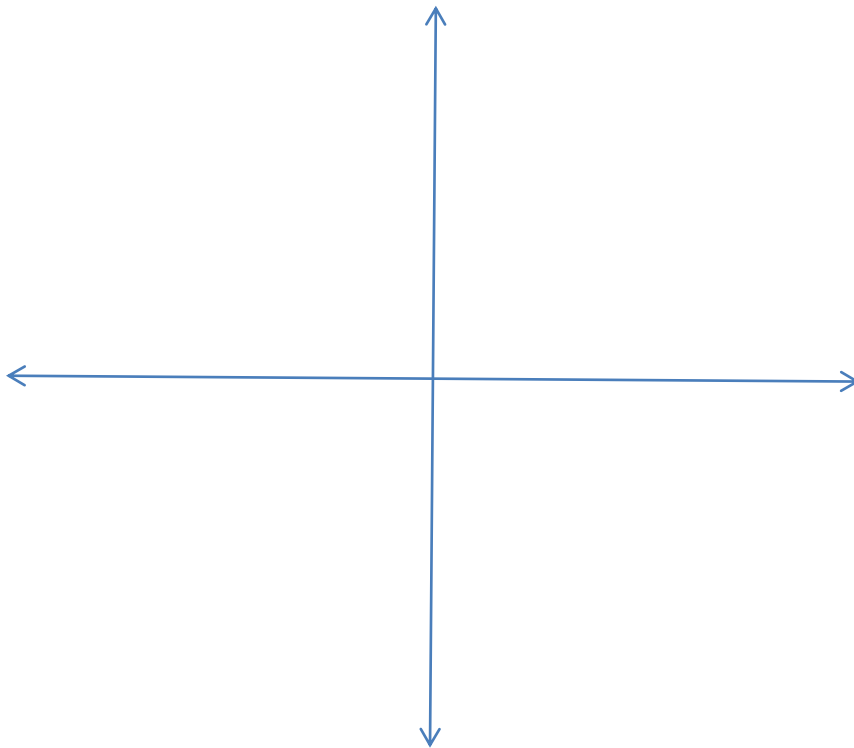
6. (20 points) Find all maxima, minima, points of inflection and asymptotes, if any for
   $y = x^3 - 3x + 2$. Sketch the curve.
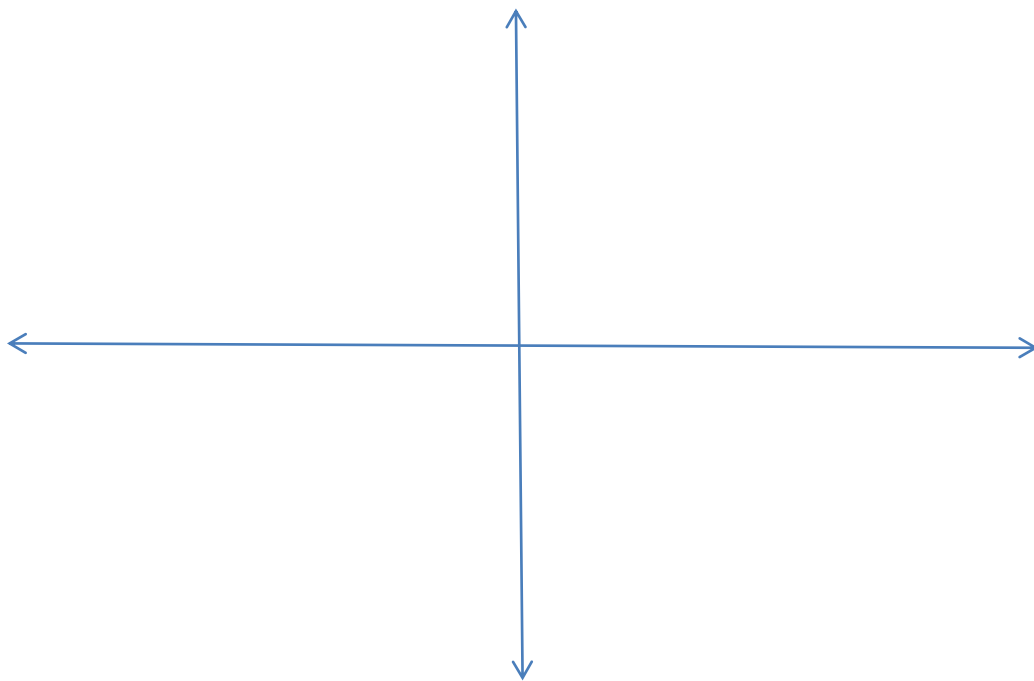
7. (16 points) Find the point on the line $x + 2y = 5$ at which the value of $x^2 + 4y$ is a minimum. Give this minimum value and justify that it is a minimum.

8. (12 points) A power company will lay a cable to connect a power station on the riverbank to a home on the other side of a 3 mile wide river. The home is 5 miles downstream of the power station. The cable will run from the power station under the water to a point on the other side of the river and then directly under the land the rest of the way to the home. If it costs $1000/mile to run cable under water and $250/mile to run cable under land, how much cable should the company lay under the land to minimize the cost? (SET UP A FUNCTION OF ONE VARIABLE THAT NEEDS TO BE MINIMIZED. DO NOT SOLVE). Be clear about what your variable represents and the domain on which it makes physical sense.

9. (20 points) Evaluate the following limits:

a. $\lim\limits_{x\to 0} \dfrac{\sin(e^x - 1)}{e^{2x} - \cos x}$

b. $\lim\limits_{x\to\infty} \dfrac{\sin(e^x - 1)}{e^{2x} - \cos x}$

c. $\lim\limits_{x\to 0^+} \left(x^2 + 1\right)^{2/\left(\ln(x+1)\right)}$

d. $\lim\limits_{x\to\infty} \left(x^2 + 1\right)^{2/\left(\ln(x+1)\right)}$

10. (28 points) Integrate the following:

a. $\int \dfrac{x^2 + \sqrt{x}}{x}\,dx$

b. $\int x \sqrt[3]{x-2}\,dx$

c. $\int_1^e \dfrac{1}{x}(1 + 7\ln x)^{-1/3}\,dx$

d. $\int_{1/2}^1 \dfrac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}\,dx$

11. (20 points) Find the area of the region bounded by $y^2 = x$ and $y = x - 6$.