Math 450H
Homework II: Due 09/28/04
Prof. Bukiet

1. Set up a spring/mass system. Notice that maximum value of acceleration (as measured using the computer software) decreases from one period to the next. Let's assume that the accelerometer is measuring position for the analysis.

Obviously the ODE describing the dynamics is $mx'' + cx' + kx = 0$.

(a). Write the solution to this ODE. Show that the time (period) between consecutive maximums of $x(t)$ is a constant.

(b). Show that the ratio of the amplitudes of consecutive maximums is a constant and find this ratio (in terms of m, c, and k).

(c). Use the LoggerPro software to measure the period.

(d). Use the analytic period and the measured period to compute c (you should know k and m from the experiment).

(e). Measure the amplitudes of 10 consecutive maxima using the LoggerPro system. Fit, using Least squares to an appropriate function to Amplitude vs. time to approximate c.

(f). For both methods of computing c, is $c^2 \ll 4mk$?

2. (a). Write the ODE $mx'' + cx' + kx = f(t)$ as a first order system.

(b). Write a computer program to solve the system using Euler's method, Improved Euler and Fourth Order Runge Kutta and solve the system where $m = k = 1$ and $c = 0.1$ with initial conditions $x(0) = 1$ and $x'(0) = 1$. Let $f(t) = 0$. Solve of $0 \leq t \leq 6\pi$. How fine does the grid spacing have to be in each case such that the solution is correct to 10^{-5} in each case? Discuss how this relates to the order of accuracy of each of the methods. Now let $f(t) = \frac{1}{2+\cos(t)}$ and solve using your Runge Kutta code with the same step size. Numerical methods are useful for problems like this that are hard/impossible to solve analytically.

Your report should contain clear explanations of your work. Text should be written in full sentences and all details should be presented. A hard copy of your numerical method should be included. You should make clear how the error criteria was enforced.