Math 450H
Homework III: Due 10/12/04
Prof. Bukiet

1. Consider a two spring, two mass oscillator as described in class.
Theoretical part. Derive the equations of motion, assuming that the springs are linear and massless, and that there is no damping. Write the equations such that they describe the dynamics of the upper mass with respect to its equilibrium position and the dynamics of the distance between the two masses. Find expressions for the frequencies in the solution.
Experimental part. Set up a two spring, two mass oscillator. Measure the oscillation frequencies using the software in the lab. First, fix the accelerometer on the lower mass. Start the oscillations by

- Displacing the lower mass from its equilibrium position,
- Displacing the upper mass from its equilibrium position.
- Repeat the previous 2 steps with the accelerometer fixed on the upper mass.

Use values of m and k from the experimental part to find theoretical values for the frequencies of oscillation. Comment on the theoretical and experimental frequencies.

Solve the system of ODEs for the 4 sets of initial conditions above and comment on the amplitudes for the frequencies in each case compared to the experimentally measured amplitudes of each frequency.
Note: You will need to find spring constants separately, repeating the experiment from Homework I.
2. Solve numerically (using the code you wrote for HW 2, with modifications) the nonlinear pendulum problem:

$$\ddot{\theta} + \frac{g}{L} \sin \theta = 0 \ , \tag{1}$$

subject to the initial conditions: $\theta(0) = \theta_0$; $\dot{\theta}(0) = 0$. Use $g = 1000$ cm/s, $L = 10$ cm, and $\theta_0 = \pi/9, \pi/6, \pi/3$. Use a second-order (e.g., Improved Euler) or higher-order (e.g., Runge-Kutta) method, and choose a step size such that the maximum error is less than 10^{-5}. Do the following:

- From your numerical solution, determine the period T of the oscillations.
- Compare your result for T to the period of small oscillations for the given initial conditions (assuming the linearized ODE holds). Is there any obvious trend in the results as θ_0 is increased?
- Plot the (analytically evaluated) trajectory in the phase plane of this system for $\theta_0 = \pi/3$ for the linearized pendulum equation and plot the (numerically computed) trajectory for the non-linear pendulum system. How do the trajectories compare?

Your report should contain: Statement of the problems, explanation of the experiments, details of your analytical work, explanation of the numerical method you used, listing of the code, explanation of how the error criteria was enforced, answers to the questions, plot of the trajectory in the phase plane.