Math 450H
Homework IV: Due 11/17/05

Prof. Bukiet

Topic: Double Mass Spring and Nonlinear Springs

1a. Consider a two spring, two mass oscillator as described in class.
Find the values of k for two different springs by matching the analytical frequencies with experimental frequencies measured by Logger Pro, as in the previous homework. (I.e., use one spring and several masses and average the results). Next, set up the two mass, two springs system. Find the frequencies of the resulting motion using the analytical formulas derived in class. Perform the experiment with the Logger Pro setup and print graphs (acceleration and FFT) for 3 cases: 1 with the lower frequency having the larger amplitude, one with the higher frequency dominating and one where the 2 frequencies are near each other in magnitude. Describe how you varied the initial conditions to get each result.

1b. Solve the system of ODEs for 3 sets of initial conditions that give results similar to those in each of the cases in 1a. above. Use the Runge-Kutta method and plot the $x(t)$ graphs and provide the initial conditions that led to them.

2. Consider a non-linear spring that satisfies $F = -kx^3$.

- Find a quantity (an expression) that is conserved (i.e. an energy equation) for the motion of the system.
- For the initial conditions $x(0) = 0$ and $x'(0) = 8$ with $k = m = 1$ plot a phase diagram for the motion. (Draw arrows incdicating direction).
- Use the Runge-Kutta method to solve for $x(t)$ and $x'(t)$ during one circuit in the phase plane. What is the period of the oscillation?
- Make a plot that presents the kinetic and potential energies vs x.
- Suppose there is linear damping (cx'). Now, energy is lost in time. How does the derivation in the first part of this problem for energy change?
- Use Runge-Kutta to find the value of c such that in one revolution (with initial conditions as above) in the phase plane with the above initial conditions, the velocity is halved. How much (what fraction) of the energy has been lost in this time? Has the period of the oscillation changed?