MATH 611 – FINAL EXAM – December 18, 2000

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Show all work.

Problem #1 (15 points) Show $x = 4$ is a fixed point of the following iteration schemes:
a. $x_{n+1} = \frac{1}{4}(8x_n - x_n^2)$ **b.** $x_{n+1} = \frac{1}{3}(x_n^2 - 4)$ **c.** $x_{n+1} = \sqrt{3x_n + 4}$.
Analyze theoretically (don't calculate iterates) to determine which of these methods should converge to the root at $x = 4$ given a starting guess close enough. Which method should converge fastest?

Problem #2 (10 points)
Find the LU decomposition of $\begin{pmatrix} 1 & 3 \\ 4 & -1 \end{pmatrix}$ and use it to solve $\begin{aligned} x + 3y &= 9 \\ 4x - y &= 10 \end{aligned}$

Problem #3 (10 points)
Perform one step of Newton's Method for systems with initial guess (-1,2) on: $\begin{aligned} x^2 + xy - y^2 &= 5 \\ x^3 + y &= 9 \end{aligned}$

Problem #4 (10 points)
a. Find the Lagrange Polynomial through the points.

x	0	2	5	6
$f(x)$	-2	-4	53	112

b. Use the result of part **a** to approximate $f(1)$.

Problem #5 (10 points) Set up the system of equations for the cubic spline through the points in the previous problem given $S_0 = g''(0) = -6$ and $S_3 = g''(6) = 30$. DO NOT SOLVE.

Problem #6 (10 points)
The data points $(0, e^{0.1})$, $(1, e^2)$ and $(3, e^3)$ should lie approximately on a curve of the form $y = ae^{bx}$. Set up a system of linear "least squares" equations that would be useful for finding a and b. DO NOT SOLVE.

Problem #7 (10 points)
What does the difference scheme approximate and give error order? $\frac{1}{2h}\left[f(x+3h) - f(x-h) - 2f(x)\right]$.

Problem #8 (24 points) Consider the points

x	1	2	3	4	5
$f(x)$	0.01	0.69	1.10	1.39	1.61

a. Approximate $f'(3)$ using centered difference with $h = 2$.
b. Approximate $f'(3)$ using centered difference with $h = 1$.
c. Use Richardson extrapolation to improve the results in parts a and b.
d. Approximate $f''(3)$ using centered difference with $h = 1$.
e. Set up the finest possible Trapezoidal rule approximation of $\int_1^5 f(x)\,dx$. DON'T ADD UP.
f. Set up the coarsest possible Trapezoidal rule approximation of $\int_1^5 f(x)\,dx$. DON'T ADD UP.
g. Set up the finest possible Simpson's rule approximation of $\int_1^5 f(x)\,dx$. DON'T ADD UP.
h. Set up the coarsest possible Simpson's rule approximation of $\int_1^5 f(x)\,dx$. DON'T ADD UP.

Problem #9 (15 points)
Consider the ordinary differential equation $\frac{dy}{dx} = ty - 1$ with $y(0) = 3$.
a. Use Euler's method with $h = 2$ to approximate $y(2)$.
b. Use Modified Euler's method with $h = 2$ to approximate $y(2)$.
c. Use Fourth Order Runge Kutta method with $h = 2$ to approximate $y(2)$.
d. Use Taylor Series Method up to the x^3 term to approximate $y(2)$.
e. Use the result of part b along with the predictor-corrector method to approximate $y(4)$.

Problem #10 (6 points)
Is the multi-step scheme $y_{i+1} = \frac{1}{3}(2y_i + y_{i-1}) + \frac{4h}{3}f(t_i, y_i)$ for $y' = f(t, y)$ stable?