MATH 611 – FINAL EXAM – MAY 7, 1996

Instructor: B. Bukiet

Show all work.

Problem #1 (10 points) Find a single polynomial whose roots may be found using either of the following iteration schemes:

a. $x_{n+1} = \frac{2x_n^3 + 2x_n^2 + 2}{3x_n^2 + 4x_n - 1}$ **b.** $x_{n+1} = \frac{3x_n^3 + 5x_n^2 + 2x_n + 2}{4x_n^2 + 7x_n + 1}$.

Which scheme is better for finding the root at $x = 1$?

Problem #2 (20 points – 5 points each part) Consider a function $f(x)$ with the following values known:

x	-1	0	2
$f(x)$	9	4	12

a. Find the Lagrange Polynomial through the 3 points.

b. Use the result of part **a** to approximate $f(1)$ and write an expression for the error term.

c. Use the result of part **a** to approximate $f'(1)$.

d. Use the result of part **a** to approximate $\int_0^2 f(x)\,dx$.

Problem #3 (10 points)

Given $f'(x) = \frac{1}{3h}\,[f(x + 2h) - f(x - h)] + O(h)$.

If $g(x) = 2x^3 + x^2 + 1$, use the formula given to approximate $g'(-1)$ using $h = 2$. Do the same for $h = 1$. Improve the result using Richardson extrapolation.

Problem #4 (10 points)

Consider the ordinary differential equation $\frac{dy}{dt} = 2ty$ with $y(2) = 1$. Use Euler's method with $h = 2$ to approximate $y(6)$.

Problem #5 (10 points)

Is the multi-step scheme $w_{i+1} = \frac{1}{3}(2w_i + w_{i-1}) + \frac{4h}{3} f(t_i, w_i)$ for $y' = f(t, y)$ stable?

Problem #6 (10 points)

Set up (but **DO NOT SOLVE**) a system of first-order linear equations for solving the ODE Boundary Value Problem $y'' + y' + 3xy = 9x^2$ on $0 \le x \le 1$ with $y(0) = 1$ and $y(1) = 2$. Use grid spacing $h = \frac{1}{3}$.

Problem #7 (20 points)

Consider the system of linear equations
$$\begin{aligned} 4x \qquad\quad + z &= 5 \\ x + 4y + z &= 3 \\ x \qquad\quad + 4z &= -10 \end{aligned}$$

a. Use Gaussian Elimination with back substitution to solve the system.

b. Find the matrix that needs to be analyzed to determine whether Jacobi's iteration method will converge for this problem. (Do not analyze the relevant matrix, just find it).

c. Use one iteration of the Gauss-Seidel method, with starting guess $\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$

Problem #8 (10 points)

Use 2 iterations of the Power Method on $\begin{bmatrix} 5 & 0 & 5 \\ 0 & 1 & 0 \\ 5 & 0 & 5 \end{bmatrix}$ with initial guesses: $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$.

Discuss the results.