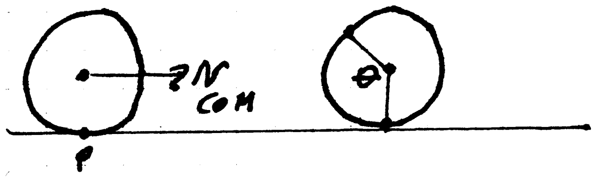
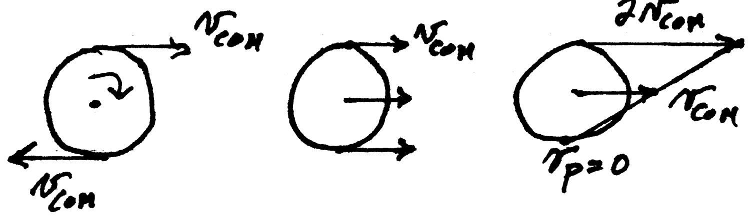
Rolling



$s = r\theta$

$\frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega$

$v_{com} = r\omega$



Kinetic Energy of Rolling

$KE = \frac{1}{2} I_p \omega^2$

$I_p = I_{com} + MR^2$ (Point of Rotation is P)

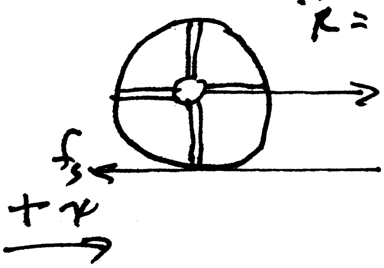
$KE = \frac{1}{2} I_{com} \omega^2 + \frac{1}{2} MR^2\omega^2$

$KE = \frac{1}{2} I_{com} \omega^2 + \frac{1}{2} M v_{com}^2$

EXAMPLE I
#8

$m = 10\,kg$
$R = 0.3\,m$

The wheel rolls smoothly on a horizontal surface & $a_{com} = 0.6\,m/s^2$



a) find the frictional force on the wheel (magnitude & direction)

$F - f_s = ma \qquad f_s = F - ma = 10 - 10(.6) = 4\,N$

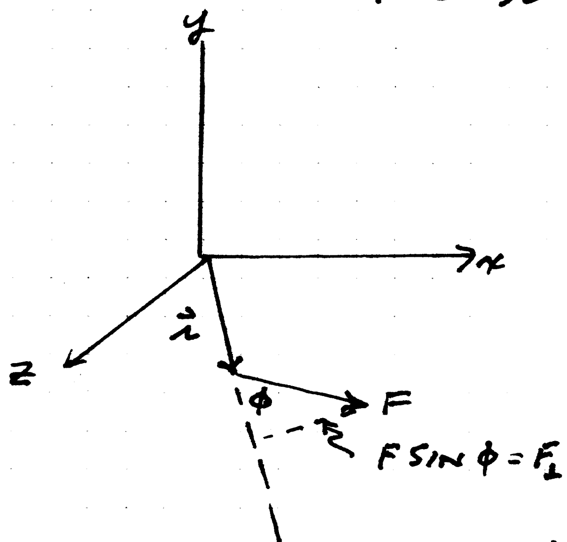
b) what is I_{com} $\qquad \alpha = \frac{a}{R} = \frac{0.6}{0.3} = 2\,rad/s^2$

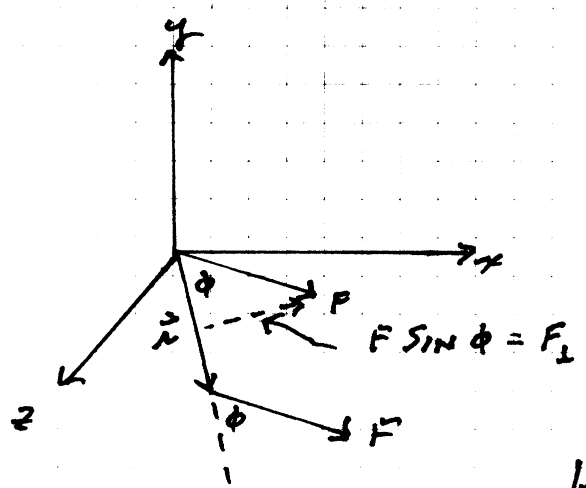
$\Sigma\tau = f_s R = 4(.3) = I_{com}\alpha \qquad I_{com} = \frac{1.2}{2} = 0.6\,kg\text{-}m^2$

ANGULAR MOMENTUM

TORQUE MAY BE WRITTEN AS

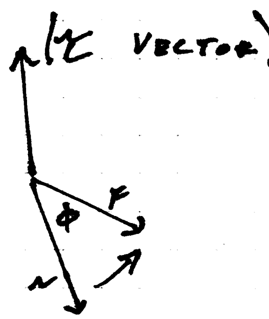
$$\vec{\tau} = \vec{r} \times \vec{F}$$





$$\tau = \vec{r}\times\vec{F} = rF\sin\phi$$

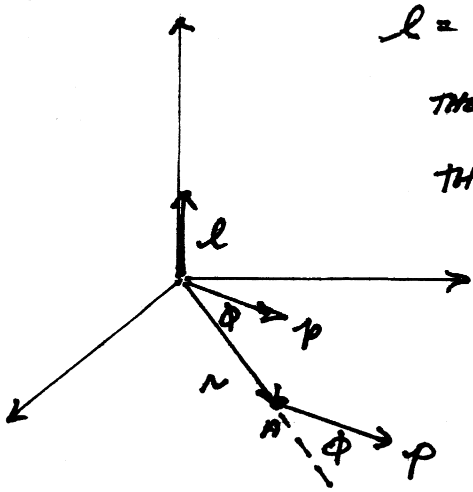
ROTATE $\vec{r}$ INTO $\vec{F}$ USING RIGHT HAND RULE



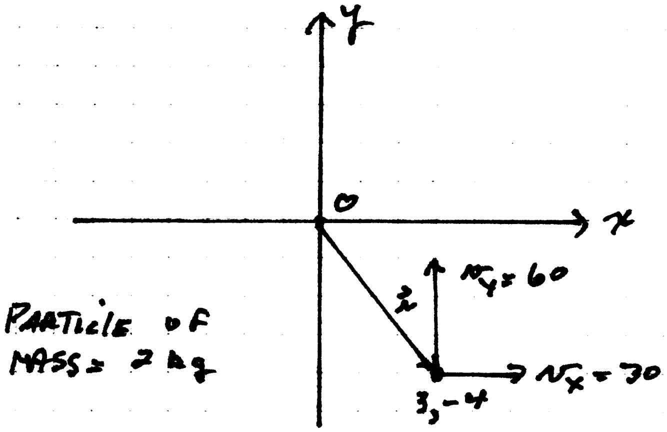
ANGULAR MOMENTUM

$$\vec{l} = \vec{r}\times\vec{p} = rp\sin\theta = r_\perp p = p_\perp$$

THE GEOMETRY OF $\vec{r}\times\vec{p}$ IS THE SAME AS IT WAS FOR $\vec{r}\times\vec{F}$



EXAMPLE II (#26P) A 2kg object moves in A PLANE WITH $v_x = 30$ m/s & $v_y = 60$ m/s AS IT PASSES A POINT $x = -3$m & $y = -4$ m. WHAT is $\vec{l}$ RELATIVE TO THE ORIGIN AT THIS MOMENT



FIND $\vec{l}$ OF THE PARTICLE AT THE POINT (3,-4) ABOUT O

$\vec{r} = 3i - 4j \qquad \vec{v} = 30i + 60j$

$$\vec{l} = \vec{r} \times \vec{p} = m\,\vec{r} \times \vec{v}$$

$$\vec{r} \times \vec{v} = (3i - 4j) \times (30i + 60j)$$

$$= 90\, i \times i + 180 \underbrace{i \times j}_{k} - 120 \underbrace{j \times i}_{-k} - 240\, j \times j$$

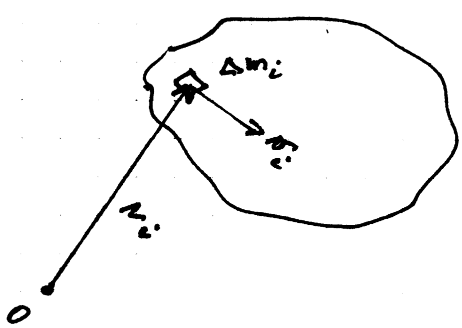
$$= k(180 - (-120)) = 300k$$

$$\vec{l} = m\,\vec{r} \times \vec{v} = 2(300)k = 600k$$

$$\vec{F}_{NET} = \frac{d\vec{p}}{dt} \qquad \begin{matrix} F \rightarrow \tau \\ p \rightarrow l \end{matrix} \qquad \vec{\tau}_{NET} = \frac{d\vec{l}}{dt}$$ FOR ONE PARTICLE

FOR A SYSTEM OF PARTICLES $\vec{L} = l_1 + l_2 + \cdots l_N$

$$\tau_{NET} = \frac{d\vec{L}}{dt}$$



$$l_i = r_i\, p_i \sin\theta = p_i\, r_{i\perp}$$

$$= \Delta m_i\, r_{i\perp}\, v_i$$

$$= \Delta m_i\, r_{i\perp}\,(r_{i\perp}\, \omega_i)$$

$$= \underbrace{(\Delta m_i\, r_{i\perp}^2)}_{I_i}\, \omega_i$$

$$l_i = I_i\, \omega_i$$

NEXT WE SUM UP OVER ALL THE MASSES Δm_i IN THE RIGID BODY

$$L_{TOTAL} = \sum_{i=1}^{N} l_i = \sum_{i=1}^{N} I_i \omega_i = \omega \sum_{i=1}^{N} I_i = \omega I$$

FOR MANY PARTICLES

$$\tau_{NET} = \frac{dL_{TOTAL}}{dt} = \frac{d}{dt}(I\omega)$$

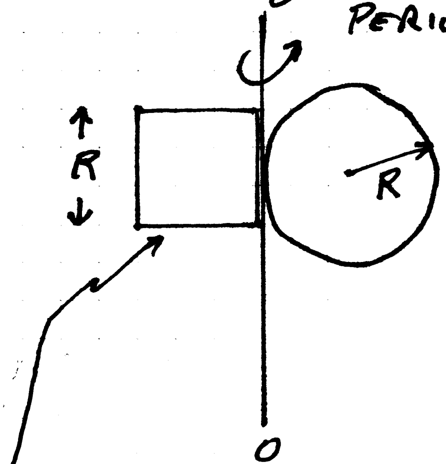
$$\text{IF } \tau_{NET} = 0 \quad \frac{d}{dt}(I\omega) = 0$$

$$I_i \omega_i = I_f \omega_f$$

EXAMPLE III (#36)

PERIOD OF ROTATION = 2.5 S

Hoop MASS is m = 2 kg R = 1/2



EACH BAR HAS A MASS m/3 & A Length R

a) find I ABOUT THE AXIS OF ROTATION

FOR THE Hoop $I_{DIA} = \frac{1}{2} mR^2$

$$I_{CIRCUM} = I_{DIA} + mR^2 = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2$$

FOR THE SQUARE

1. THE BAR ON THE AXIS HAS 0 CONTRIBUTION
2. THE OUTER BAR HAS A VALUE mR^2
3. EACH HORIZONTAL BAR YIELDS $I = \frac{1}{12}\left(\frac{m}{3}\right)R^2 + \frac{m}{3}\left(\frac{R}{2}\right)^2 = \frac{mR^2}{9}$

$$I_{TOTAL} = \underbrace{\frac{3}{2}mR^2}_{\text{Hoop}} + \underbrace{0 + \frac{mR^2}{3} + \frac{2}{9}mR^2}_{\text{SQUARE}} = 2.05\,m\ldots$$

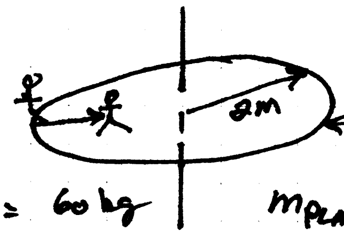
$= 2.05(2)(\ldots$

$= 1.03$

b) Determine the ANGULAR MOMENTUM ABOUT OO

$$L = I\omega = 1.03\left(\frac{1\,\text{Rev}}{2.5\,\text{Sec}} \times \frac{2\pi\,\text{RAD}}{1\,\text{Rev}}\right) = 2.58\ \text{kg-m}^2/\text{s}^2$$

EXAMPLE IV (54 P)



$m_{stud} = 60$ kg $\quad m_{PLATFORM} = 150$ kg

$\omega_{STUDENT} = 1.5$ RAD/SEC ON RIM

$$I_{PLAT} = 300 \text{ kg-m}^2 \Rightarrow \int r^2 dm = \sum \Delta m_i r_i^2$$

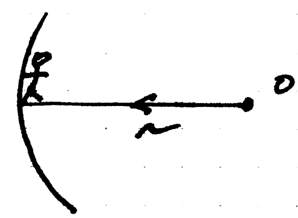
find ω WHEN THE STUDENT IS $\frac{R}{2}$ FROM THE CENTER

$$\tau_{NET} = \frac{d\vec{L}}{dt} \qquad L = I\omega$$

If $\tau = 0$ $\quad \frac{dL}{dt} = 0 \quad L = $ CONSTANT

$$I_i \omega_i = I_f \omega_f$$

$$I_{STUDENT} = \sum \Delta m_i r_i^2 = m_{ST} r^2$$



$$I_{\substack{STUDENT \\ INITIALLY}} = m_{STUDENT}\, r^2 = 60(2)^2 = 240 \text{ kg-m}^2$$

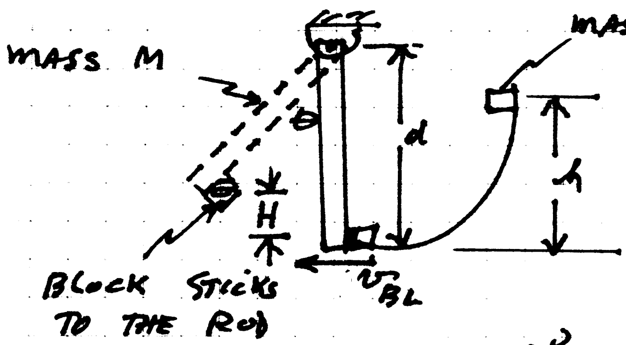
$$I_{\substack{STUDENT \\ r=1}} = m_{STUD}\, r^2 = 60(1)^2 = 60$$

$$I_i \omega_i = I_f \omega_f$$

$$(\underset{I_{PLAT}}{300} + \underset{I_{ST}}{240})(\underset{\omega}{1.5}) = (\underset{I_{PLAT}}{300} + \underset{I_{ST}}{60})\,\omega_f$$

$$\omega_f = \frac{540(1.5)}{360} = 2.25 \text{ RAD/S}$$

EXAMPLE V



MASS m SLIDES DOWN THE CHUTE & CAUSES THE VERTICAL ROD TO ROTATE - FIND THE MAXIMUM θ

FOR THE BLOCK SLIDING DOWN THE CHUTE

$$\cancel{W_{EXT}}^{\,0} = \Delta KE + \Delta U_g + \cancel{\Delta U_{SP}}^{\,0}$$

$$0 = \frac{1}{2} m v_{BL}^2 + mg(0 - h)$$

$$v_{BL} = \sqrt{2gh}$$

ANGULAR MOMENTUM

$$\vec{L} = \vec{r} \times \vec{p} = I\omega = \underbrace{m v d}_{L_{ROD}} = \underbrace{(I_{Rod} + md^2)\omega}_{L_{Rod + mass}}$$

$$I_{TOT} = \underbrace{I_{ROD}}_{\text{ABOUT ITS END}} + I_{BLOCK} = \frac{1}{3} M d^2 + m d^2$$

$$\left(\frac{1}{3} M d^2 + m d^2\right)\omega = d m \underbrace{\sqrt{2gh}}_{v_{BL}}$$

$$\omega = \frac{m d \sqrt{2gh}}{\frac{1}{3} M d^2 + m d^2}$$

NOW THE BLOCK & ROD RISE A DISTANCE $H = d(1 - \cos\theta)$

$$\underbrace{\Delta KE}_{\text{OF THE ROD \& MASS } m} = \underbrace{\Delta U_g}_{\text{OF THE ROD \& MASS } m}$$

$$\frac{1}{2} I_{TOT}\, \omega^2 = \underbrace{mgH}_{BLOCK} + \underbrace{MgH/2}_{ROD}$$

$$\frac{1}{2}\left(\frac{1}{3} M d^2 + m d^2\right)\left\{\frac{m d \sqrt{2gh}}{\frac{1}{3} M d^2 + m d^2}\right\}^2 = \left(m + \frac{M}{2}\right) g H \quad (H = d(1 - \cos\theta$$

$$\theta = \cos^{-1}\left(1 - \frac{m^2 h}{\left(m + \frac{M}{2}\right)\left(m + \frac{M}{3}\right)}\right)$$